Lecture 16

Section 4.8 Some Curve Sketching

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Test 2

- Test 2: November 1-4 in CASA
- Loggin to CourseWare to reserve your time to take the exam.

Review for Test 2

- Review for Test 2 by the College Success Program.
- Friday, October 24 2:30–3:30pm in the basement of the library by the C-site.

Help Session for Homework

- Homework Help Session by Prof. Morgan.
- Tonight 8:00 - 10:00pm in 100 SEC

Online Quizzes

- Online Quizzes are available on CourseWare.
- If you fail to reach 70% during three weeks of the semester, I have the option to drop you from the course!!!.

Dropping Course

- Tuesday, November 4, 2008
- Last day to drop a course or withdraw with a "W" (must be by 5 pm)

Quiz 1

The graph of $f(x)$ is shown below. Give the number of critical values of f.

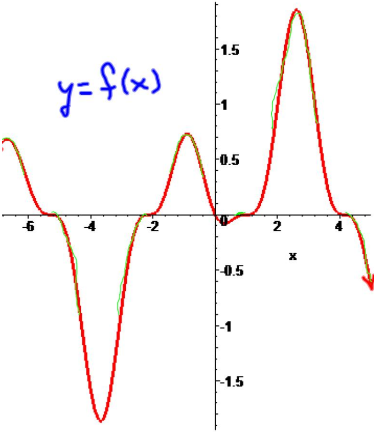
a. 5

b. 7

c. 8

d. 9

e. None of these



Quiz 2

The graph of $f(x)$ is shown below. Give the number of the points of inflection of the graph of f.

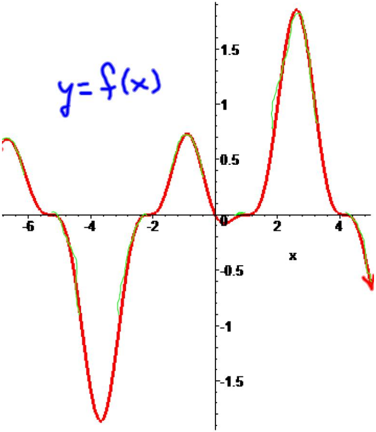
a. 8

b. 10

c. 12

d. 14

e. None of these



Quiz 3

The graph of $f(x)$ is shown below. Give the number of local minima for f.

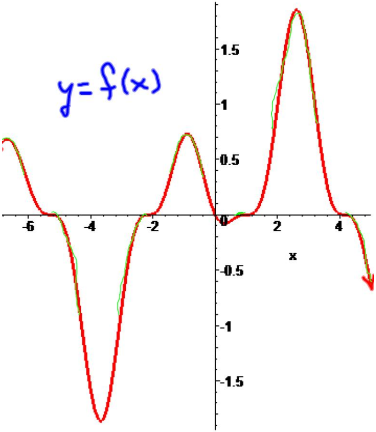
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 4

The graph of $f(x)$ is shown below. Give the number of local maxima for f.

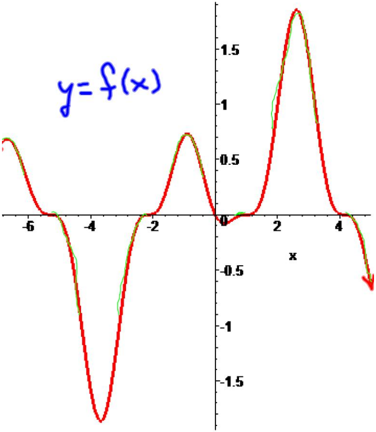
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 5

The graph of $f'(x)$ is shown below. Give the number of local maxima for f.

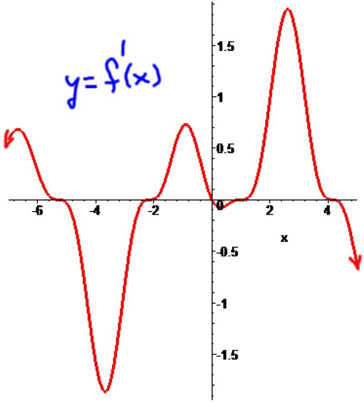
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 6

The graph of $f'(x)$ is shown below. Give the number of local minima for f.

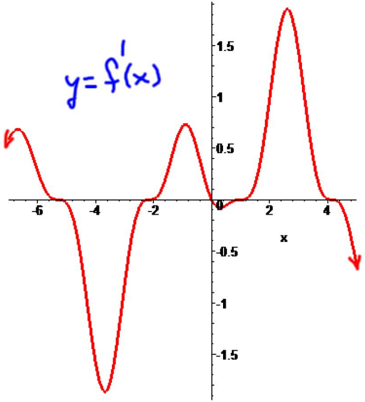
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 7

The graph of $f'(x)$ is shown below. Give the number of critical numbers for f.

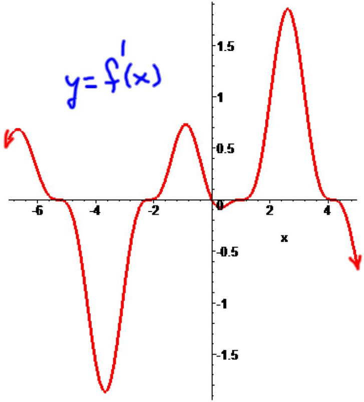
a. 1

b. 2

c. 3

d. 4

e. None of these



Quiz 8

The graph of $f'(x)$ is shown below. Give the number of inflection points for f.

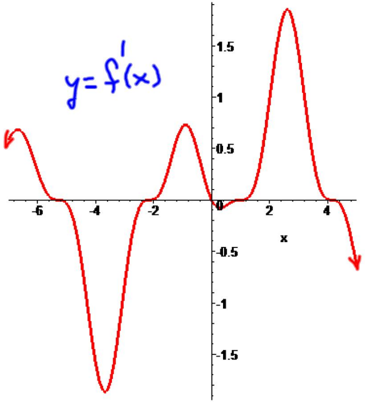
a. 3

b. 4

c. 5

d. 6

e. None of these



Example 1

Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

Step 1: Domain of f

(i) Determine the domain of f;

(ii) Identify endpoints;

(iii) Find the vertical asymptotes;

(iv) Determine the behavior of f as $x \to \pm\infty$;

(v) Find the horizontal asymptotes.

Example 1 (cont.)

Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

Step 2: Intercepts

(i) Determine the y-intercept of the graph:

- The y-intercept is the value of $f(0)$;

(ii) Determine the x-intercepts of the graph:

- The x-intercepts are the solutions of the equation $f(x) = 0$.

Example 1 (cont.)

Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

Step 3: Symmetry and Periodicity

(i) Symmetry:

(a) If f is an even function, i.e., $f(-x) = f(x)$, then the graph is symmetric w.r.t. the y-axis;

(b) If f is an odd function, i.e., $f(-x) = -f(x)$, then the graph is symmetric w.r.t. the origin.

(ii) Periodicity:

- If f is periodic with period p, then the graph replicates itself on intervals of length p.

Example 1 (cont.)

Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

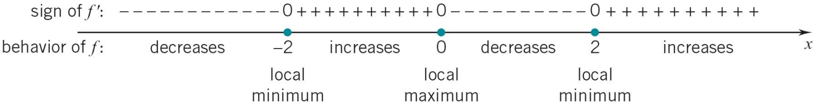
Step 4: First Derivative f'

(i) Calculate f';

(ii) Determine the critical numbers of f;

(iii) Examine the sign of f' to determine the intervals on which f increases and the intervals on which f decreases;

(iv) Determine vertical tangents and cusps.



Example 1 (cont.)

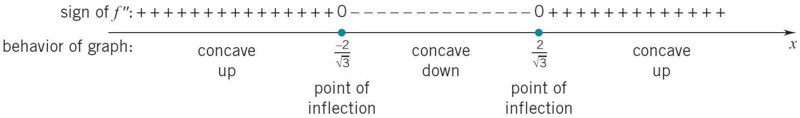
Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

Step 5: Second Derivative f''

(i) Calculate f'';

(ii) Examine the sign of f'' to determine the intervals on which the graph is concave up and the intervals on which the graph is concave down;

(iii) Determine the points of inflection.



Example 1 (cont.)

Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

Step 6: Preliminary sketch

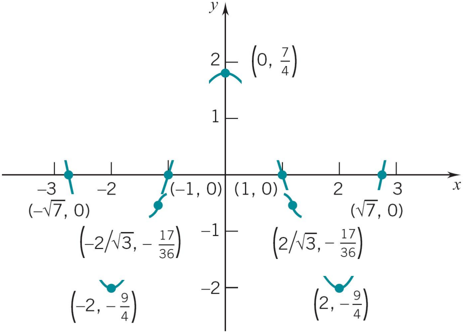
Plot the points of interest:

(i) intercept points,

(ii) extreme points

- local extreme points,
- endpoint extreme points,
- absolute extreme points,

(iii) and points of inflection.



Example 1 (cont.)

Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{7}{4}$

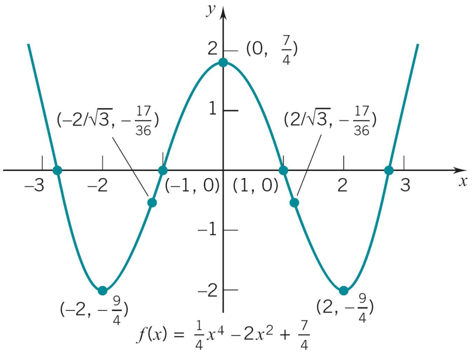
Step 7: Sketch the graph

(i) Symmetry: sketch the graph for $x \geq 0$;

(ii) Connect the points of the preliminary sketch;

(iii) Make sure the curve "rises", "falls", and "bends" in the proper way;

(iv) Obtain the graph for $x \leq 0$ by a reflection in the y-axis.



Example 2

Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

Step 1: Domain of f

(i) Determine the domain of f;

(ii) Identify endpoints;

(iii) Find the vertical asymptotes;

(iv) Determine the behavior of f as $x \to \pm\infty$;

(v) Find the horizontal asymptotes.

Example 2 (cont.)

Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

Step 2: Intercepts

(i) Determine the y-intercept of the graph:

- The y-intercept is the value of $f(0)$;

(ii) Determine the x-intercepts of the graph:

- The x-intercepts are the solutions of the equation $f(x) = 0$.

Example 2 (cont.)

Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

Step 3: Symmetry and Periodicity

(i) Symmetry:

(a) If f is an even function, i.e., $f(-x) = f(x)$, then the graph is symmetric w.r.t. the y-axis;

(b) If f is an odd function, i.e., $f(-x) = -f(x)$, then the graph is symmetric w.r.t. the origin.

(ii) Periodicity:

- If f is periodic with period p, then the graph replicates itself on intervals of length p.

Example 2 (cont.)

Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

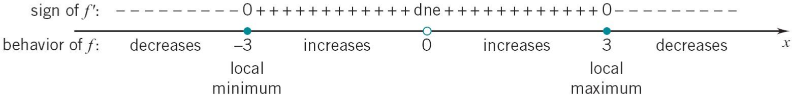
Step 4: First Derivative f'

(i) Calculate f';

(ii) Determine the critical numbers of f;

(iii) Examine the sign of f' to determine the intervals on which f increases and the intervals on which f decreases;

(iv) Determine vertical tangents and cusps.



Example 2 (cont.)

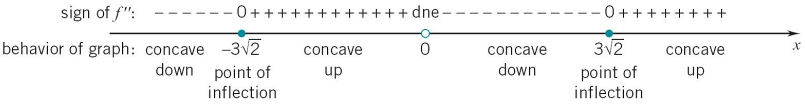
Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

Step 5: Second Derivative f''

(i) Calculate f'';

(ii) Examine the sign of f'' to determine the intervals on which the graph is concave up and the intervals on which the graph is concave down;

(iii) Determine the points of inflection.



Example 2 (cont.)

Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

Step 6: Preliminary sketch

Plot the points of interest:

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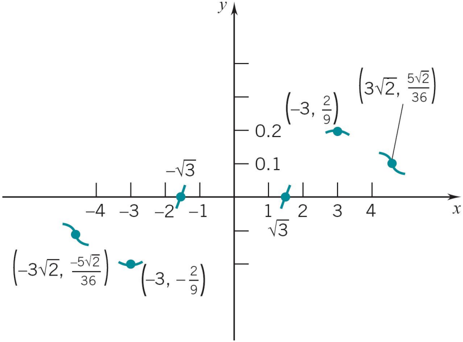


Example 2 (cont.)

Sketch the graph of $f(x) = \dfrac{x^2 - 3}{x^3}$

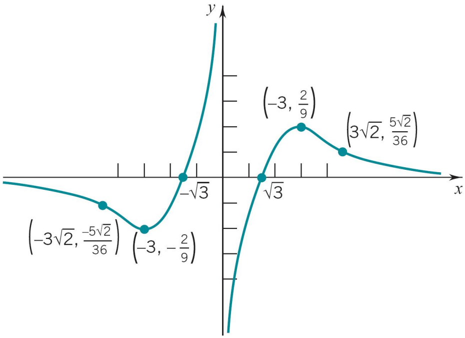
Step 7: Sketch the graph

(i) Symmetry: sketch the graph for $x \geq 0$;

(ii) Connect the points of the preliminary sketch;

(iii) Make sure the curve "rises", "falls", and "bends" in the proper way;

(iv) Obtain the graph for $x \leq 0$ by symmetry w.r.t the origin.



Example 3

Sketch the graph of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

Step 1: Domain of f

(i) Determine the domain of f;

(ii) Identify endpoints;

(iii) Find the vertical asymptotes;

(iv) Determine the behavior of f as $x \to \pm\infty$;

(v) Find the horizontal asymptotes.

Example 3 (cont.)

Sketch the graph of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

Step 2: Intercepts

(i) Determine the y-intercept of the graph:

- The y-intercept is the value of $f(0)$;

(ii) Determine the x-intercepts of the graph:

- The x-intercepts are the solutions of the equation $f(x) = 0$.

Example 3 (cont.)

Sketch the graph of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

Step 3: Symmetry and Periodicity

(i) Symmetry:

(a) If f is an even function, i.e., $f(-x) = f(x)$, then the graph is symmetric w.r.t. the y-axis;

(b) If f is an odd function, i.e., $f(-x) = -f(x)$, then the graph is symmetric w.r.t. the origin.

(ii) Periodicity:

- If f is periodic with period p, then the graph replicates itself on intervals of length p.

Example 3 (cont.)

Sketch the graph of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

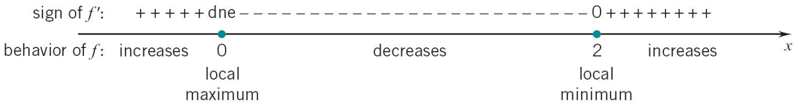
Step 4: First Derivative f'

(i) Calculate f';

(ii) Determine the critical numbers of f;

(iii) Examine the sign of f' to determine the intervals on which f increases and the intervals on which f decreases;

(iv) Determine vertical tangents and cusps.



Example 3 (cont.)

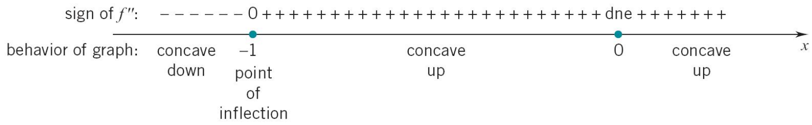
Sketch the graph of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

Step 5: Second Derivative f''

(i) Calculate f'';

(ii) Examine the sign of f'' to determine the intervals on which the graph is concave up and the intervals on which the graph is concave down;

(iii) Determine the points of inflection.



Example 3 (cont.)

Sketch the graph of $f(x) = \dfrac{3}{5}x^{5/3} - 3x^{2/3}$

Step 6: Preliminary sketch

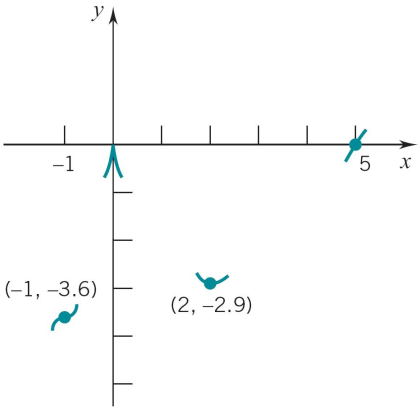
Plot the points of interest:

(i) intercept points,

(ii) extreme points

- local extreme points,
- endpoint extreme points,
- absolute extreme points,

(iii) and points of inflection.



Example 3 (cont.)

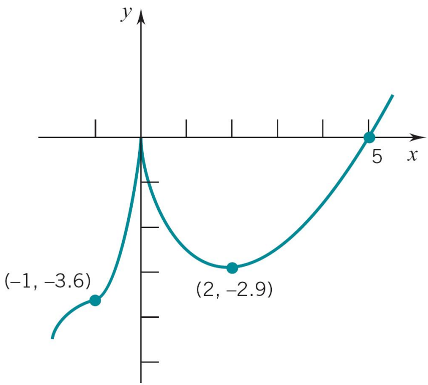
Sketch the graph of $f(x) = \frac{3}{5}x^{5/3} - 3x^{2/3}$

Step 7: Sketch the graph

(i) Neither symmetry and nor periodicity;

(ii) Connect the points of the preliminary sketch;

(iii) Make sure the curve "rises", "falls", and "bends" in the proper way.



Example 4

Sketch the graph of $f(x) = \sin 2x - 2\sin x$.

Step 1: Domain of f

(i) Determine the domain of f;

(ii) Identify endpoints;

(iii) Find the vertical asymptotes;

(iv) Determine the behavior of f as $x \to \pm\infty$;

(v) Find the horizontal asymptotes.

Example 4 (cont.)

Sketch the graph of $f(x) = \sin 2x - 2\sin x$

Step 2: Intercepts

(i) Determine the y-intercept of the graph:
- The y-intercept is the value of $f(0)$;

(ii) Determine the x-intercepts of the graph:
- The x-intercepts are the solutions of the equation $f(x) = 0$.

Example 4 (cont.)

Sketch the graph of $f(x) = \sin 2x - 2\sin x$

Step 3: Symmetry and Periodicity

(i) Symmetry:

(a) If f is an even function, i.e., $f(-x) = f(x)$, then the graph is symmetric w.r.t. the y-axis;

(b) If f is an odd function, i.e., $f(-x) = -f(x)$, then the graph is symmetric w.r.t. the origin.

(ii) Periodicity:

- If f is periodic with period p, then the graph replicates itself on intervals of length p.

Example 4 (cont.)

Sketch the graph of $f(x) = \sin 2x - 2\sin x$

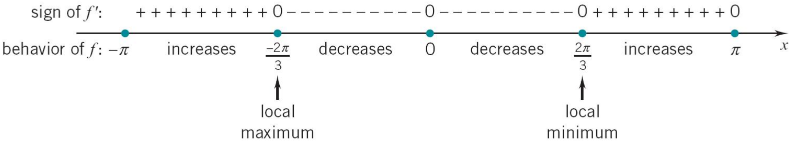
Step 4: First Derivative f'

(i) Calculate f';

(ii) Determine the critical numbers of f;

(iii) Examine the sign of f' to determine the intervals on which f increases and the intervals on which f decreases;

(iv) Determine vertical tangents and cusps.



Example 4 (cont.)

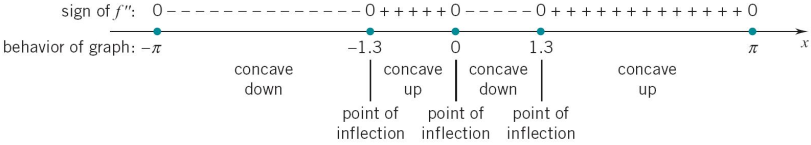
Sketch the graph of $f(x) = \sin 2x - 2\sin x$

Step 5: Second Derivative f''

(i) Calculate f'';

(ii) Examine the sign of f'' to determine the intervals on which the graph is concave up and the intervals on which the graph is concave down;

(iii) Determine the points of inflection.



Example 4 (cont.)

Sketch the graph of $f(x) = \sin 2x - 2\sin x$

Step 6: Preliminary sketch

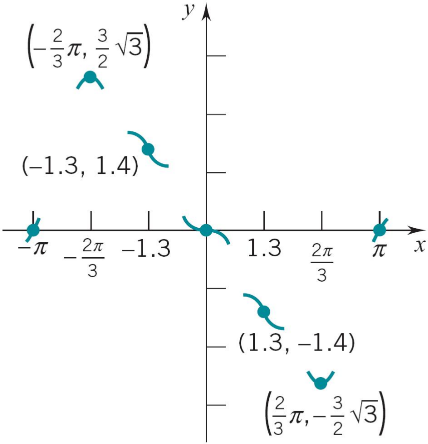
Plot the points of interest:

(i) intercept points,

(ii) extreme points

- local extreme points,
- endpoint extreme points,
- absolute extreme points,

(iii) and points of inflection.



Example 4 (cont.)

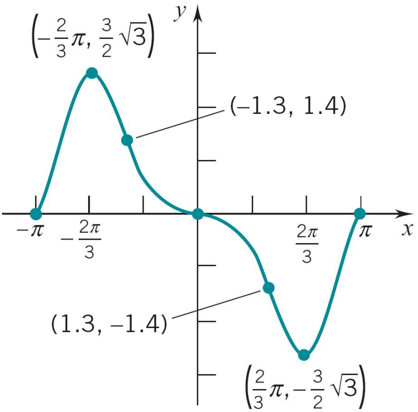
Sketch the graph of $f(x) = \sin 2x - 2\sin x$

Step 7: Sketch the graph

(i) Symmetry: sketch the graph on the interval $[-\pi, \pi]$;

(ii) Connect the points of the preliminary sketch;

(iii) Make sure the curve "rises", "falls", and "bends" in the proper way;



Example 4 (cont.)

Sketch the graph of $f(x) = \sin 2x - 2\sin x$

Step 7: Sketch the graph

(iv) Obtain the complete graph by replicating itself on intervals of length 2π.