Lecture 18

Section 5.5 Some Area Problems

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Quiz 1

What is today?

a. Monday

b. Wednesday

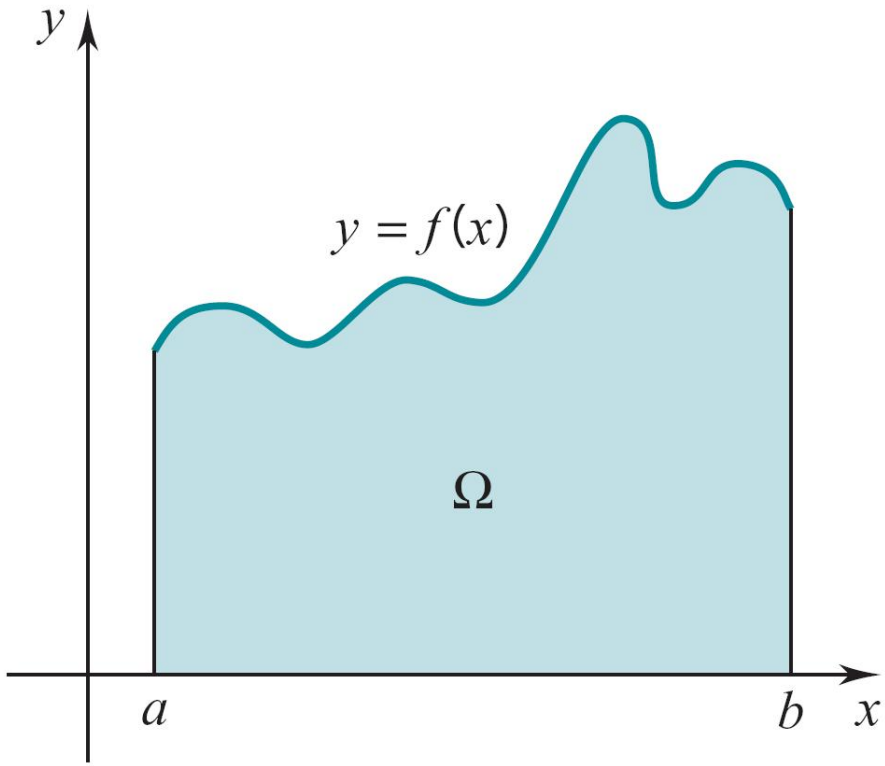
c. Friday

d. None of these

Area below the graph of a Nonnegative f

$f(x) \geq 0$ for all x in $[a, b]$.

$\Omega =$ region below the graph of f.



$$\text{Area of } \Omega = \int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

Fundamental Theorem of Integral Calculus

Theorem

In general,

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

where $F(x)$ is an antiderivative of $f(x)$.

Function	Antiderivative
x^r	$\frac{x^{r+1}}{r+1}$ (r a rational number $\neq -1$)
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$
$\sec x \tan x$	$\sec x$
$\csc^2 x$	$-\cot x$
$\csc x \cot x$	$-\csc x$

Quiz 2

Give the value of $\int_{-1}^{1} \left[x^3 - 2x^2 + \sin(\pi x)\right] dx$.

a. $\frac{1}{2}$

b. $\frac{4}{3}$

c. $-\frac{4}{3}$

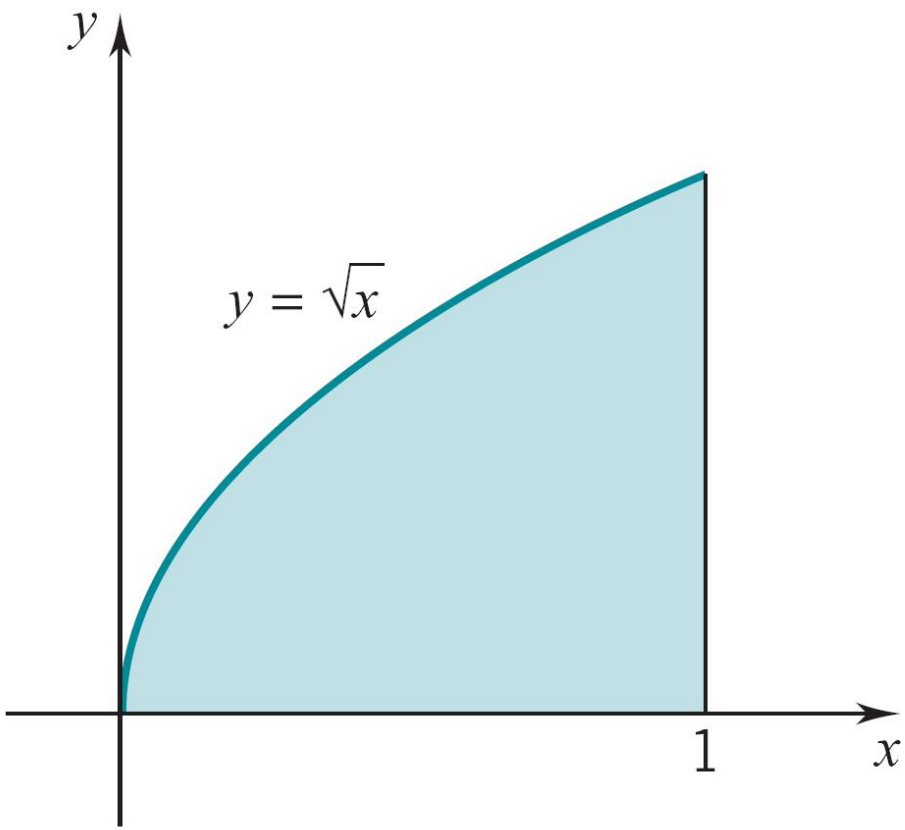
d. $-\frac{1}{2}$

e. None of these

Example 1

Example

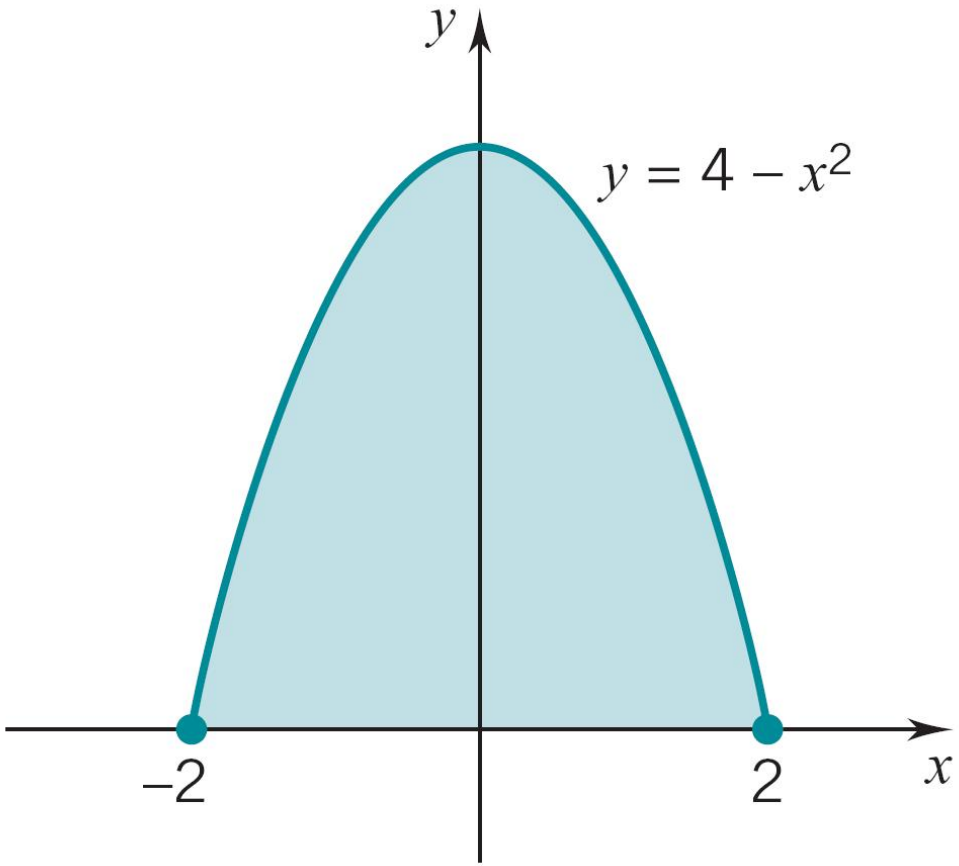
Find the area below the graph of the square-root function from $x = 0$ to $x = 1$.



Example 2

Example

Find the area bounded above by the curve $y = 4 - x^2$ and below by the x-axis.



Quiz 3

Give the area bounded between the x-axis and the graph of $y = x^2 + 1$ for $-1 \leq x \leq 2$.

a. 5

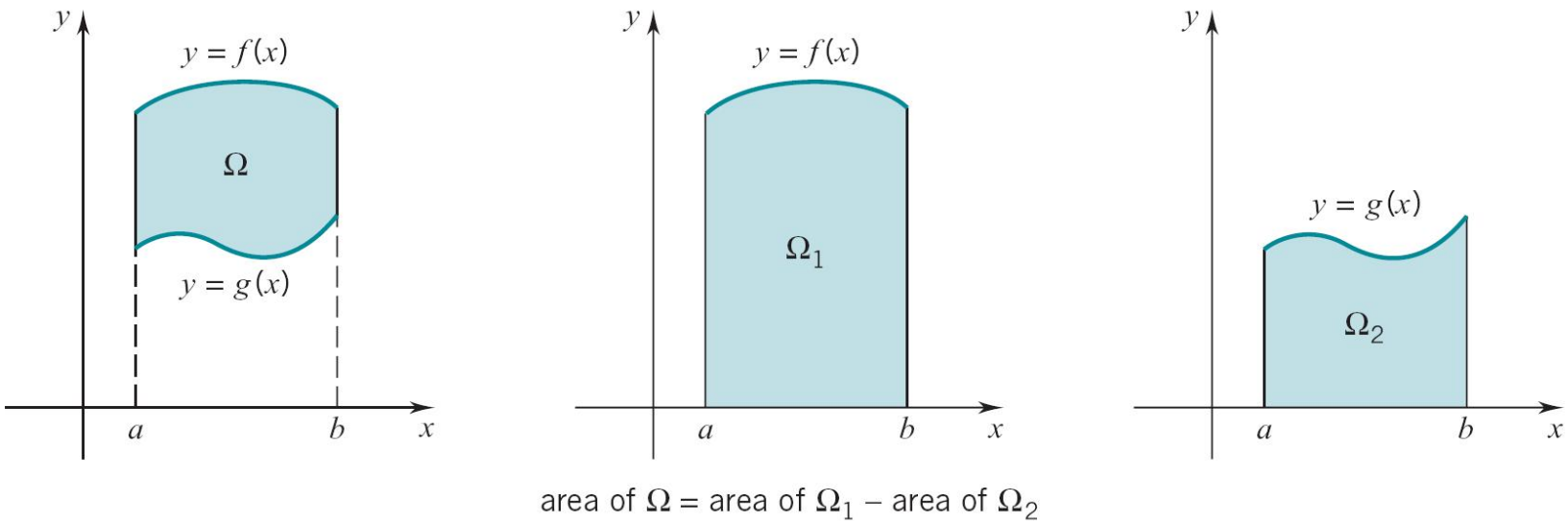
b. 4

c. 3

d. 2

e. None of these

Area between the graphs of two Nonnegative f and g



area of Ω = area of Ω_1 – area of Ω_2

$f(x) \geq g(x) \geq 0$ for all x in $[a, b]$.

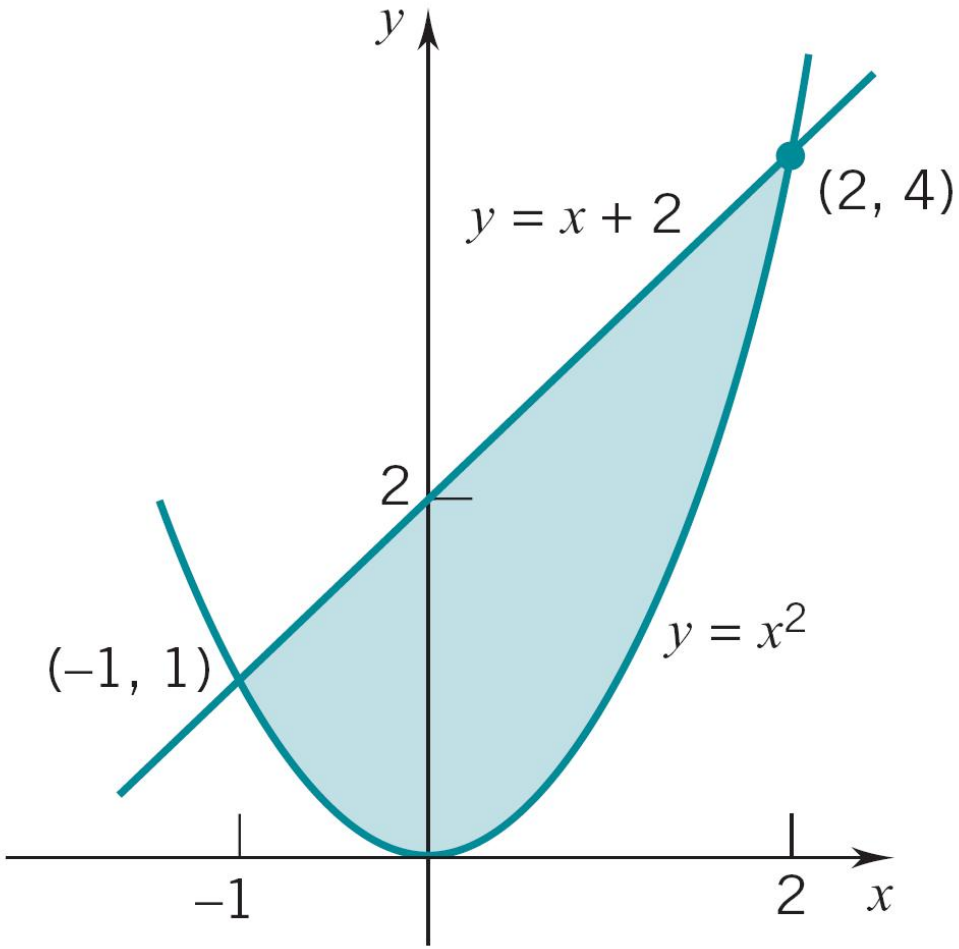
$\Omega =$ region between the graphs of f (Top) and g (Bottom).

$$\text{Area of } \Omega = \int_a^b \left[\text{ Top } - \text{ Bottom } \right] dx = \int_a^b \left[f(x) - g(x) \right] dx.$$

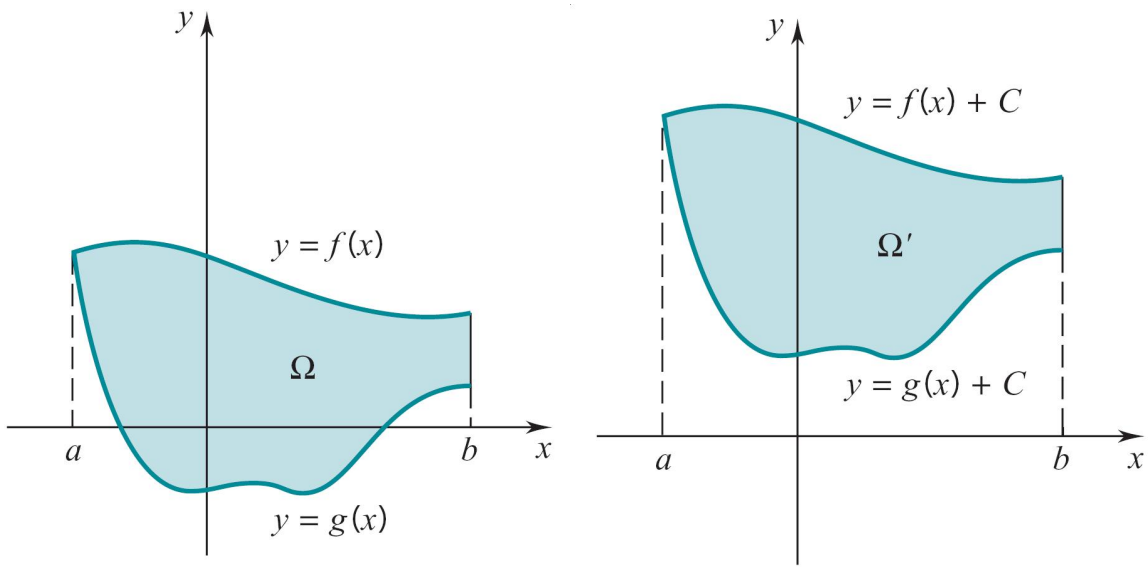
Example 3

Example

Find the area bounded above by $y = x + 2$ and below by $y = x^2$.



Area between the graphs of f and g



$f(x) \geq g(x)$ for all x in $[a, b]$.

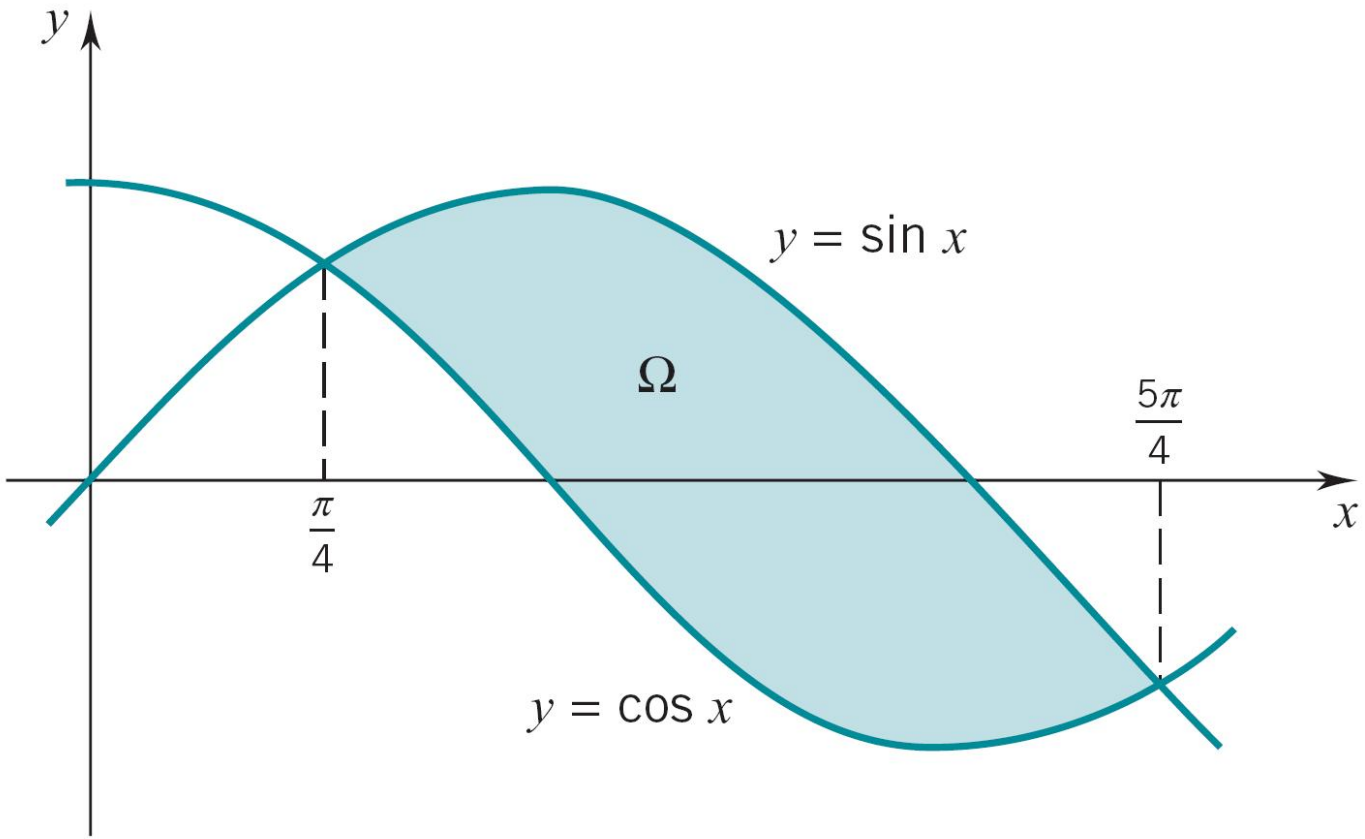
$\Omega =$ region between the graphs of f (Top) and g (Bottom).

$$\text{Area of } \Omega = \int_a^b \big[\ \text{Top} \ - \ \text{Bottom}\ \big]\, dx = \int_a^b \big[f(x) - g(x)\big]\, dx.$$

Example 4

Example

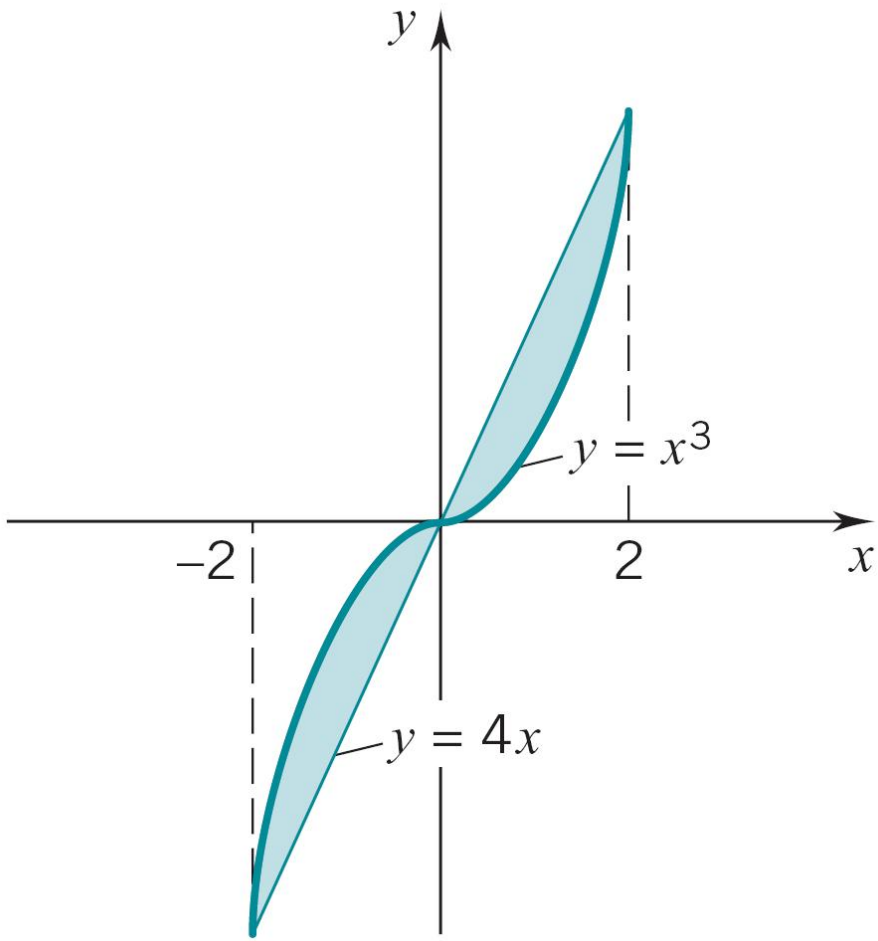
Find the area of the region shown in the figure below.



Example 5

Example

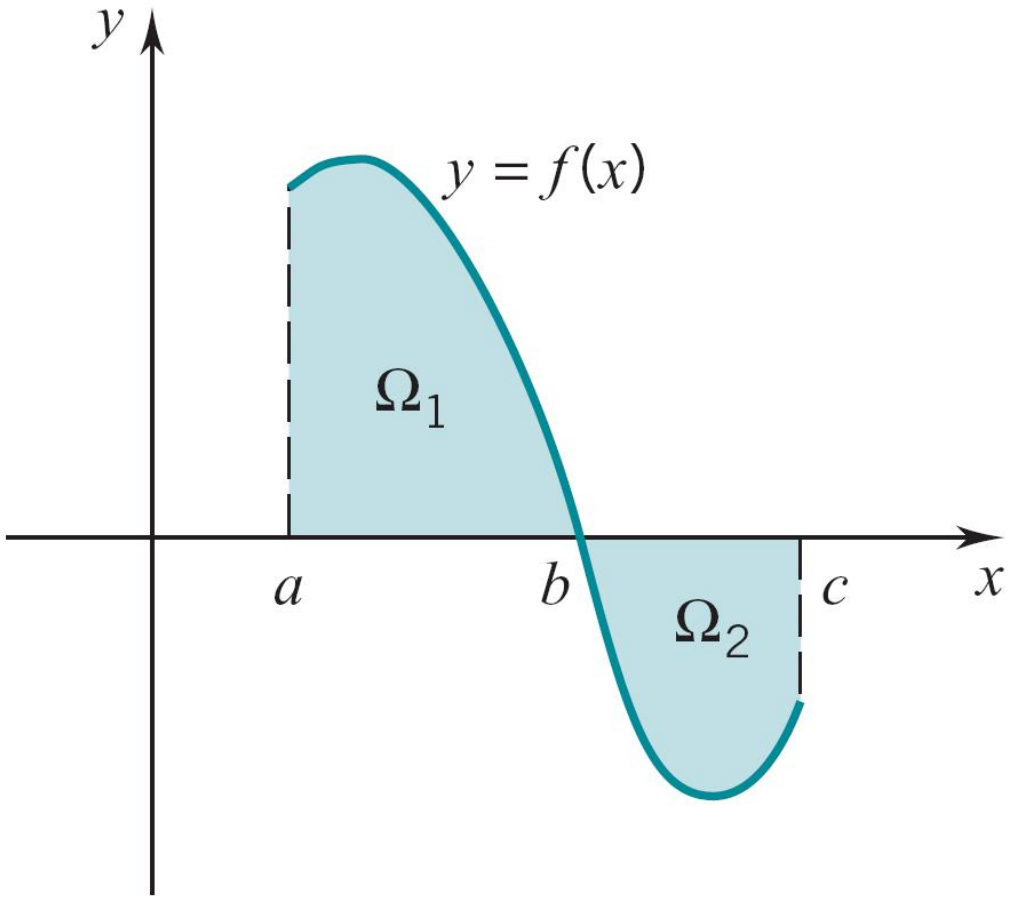
Find the area between $y = 4x$ and $y = x^3$ from $x = -2$ to $x = 2$.



Example 6

Example

Use integrals to represent the area of the region $\Omega = \Omega_1 \cup \Omega_2$ shaded in the figure below.



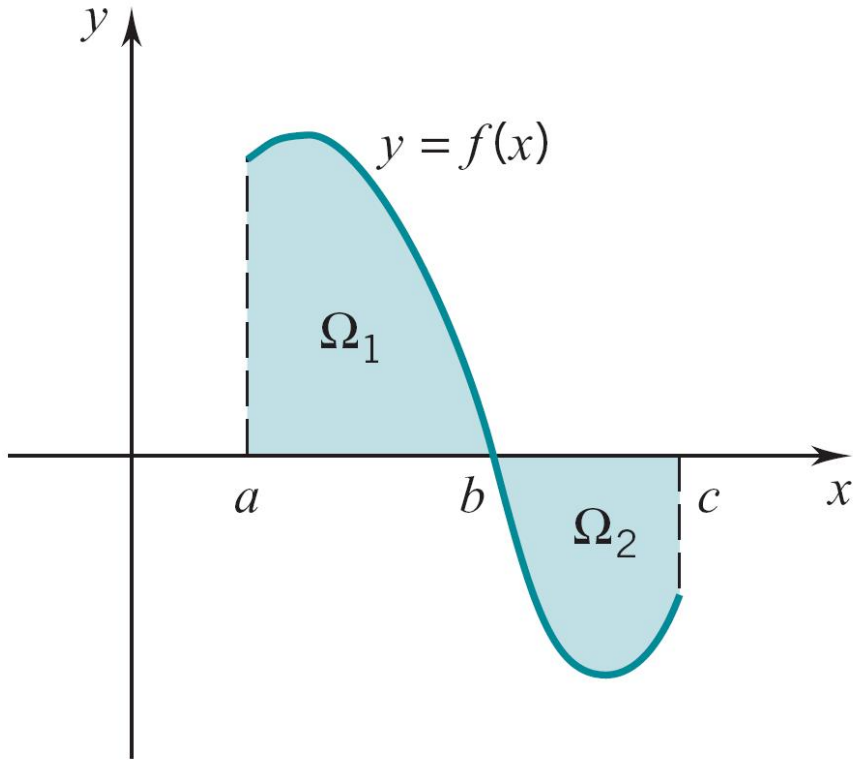
$\int_a^c f(x)\,dx$ as Signed Area

$f(x) \geq 0$ for all x in $[a, b]$

$$\int_a^b f(x)\,dx = \text{Area of } \Omega_1$$

$f(x) \leq 0$ for all x in $[b, c]$

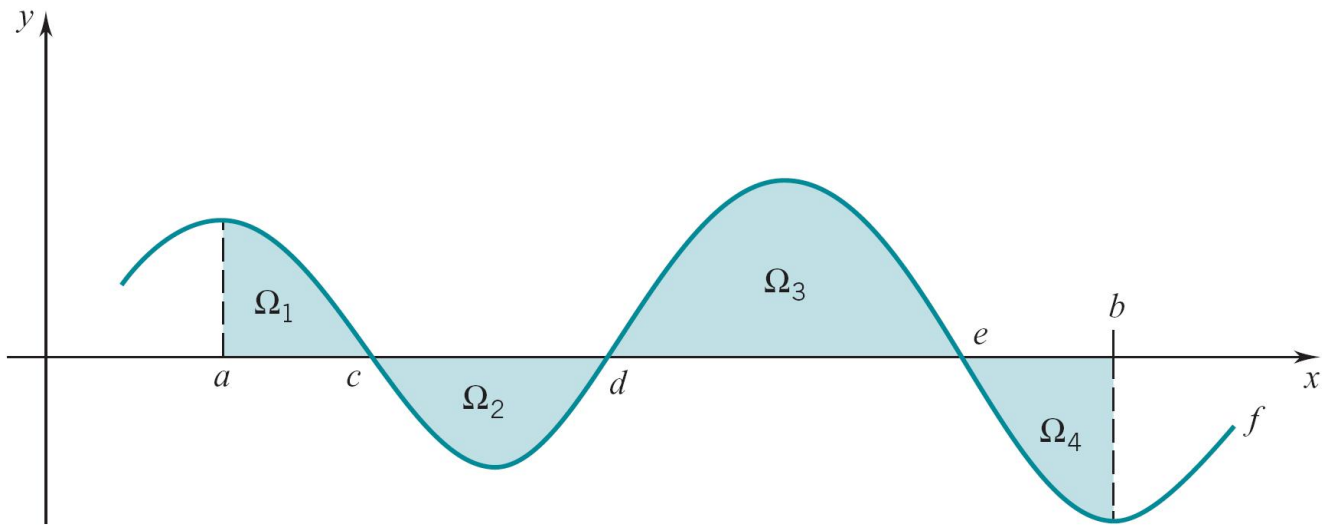
$$\int_b^c f(x)\,dx = -\text{Area of } \Omega_2$$



$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \text{Area of } \Omega_1 - \text{Area of } \Omega_2$$

$$= \text{Area above the } x\text{-axis} - \text{Area below the } x\text{-axis.}$$

$\int_a^b f(x)\,dx$ as Signed Area



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^d f(x)\,dx + \int_d^e f(x)\,dx + \int_e^b f(x)\,dx$$

$$= \text{Area of } \Omega_1 - \text{Area of } \Omega_2 + \text{Area of } \Omega_3 - \text{Area of } \Omega_4$$

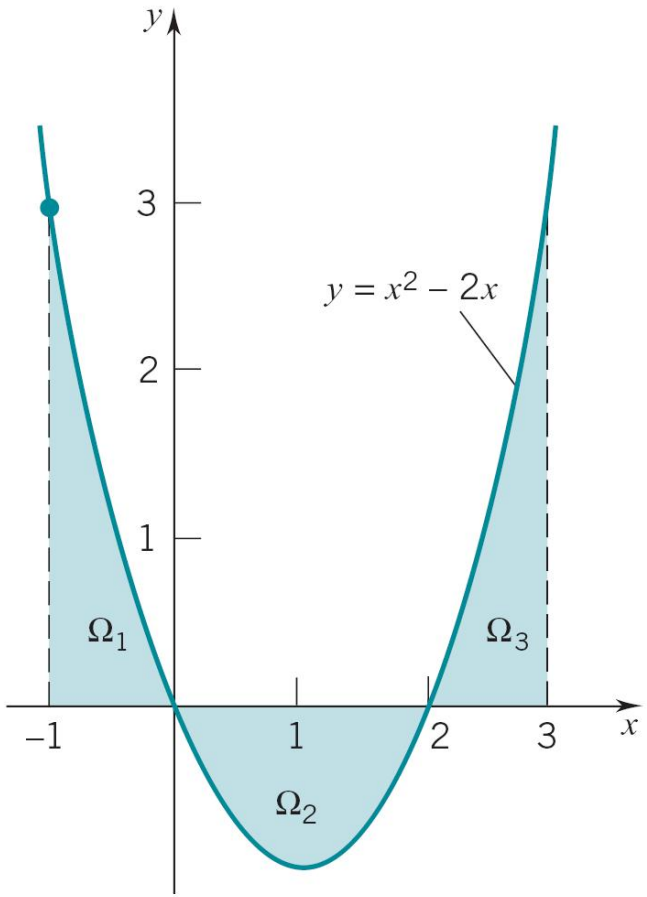
$$= \left[\text{Area of } \Omega_1 + \text{Area of } \Omega_3\right] - \left[\text{Area of } \Omega_2 + \text{Area of } \Omega_4\right]$$

$$= \text{Area above the } x\text{-axis} - \text{Area below the } x\text{-axis.}$$

Example 7

Example

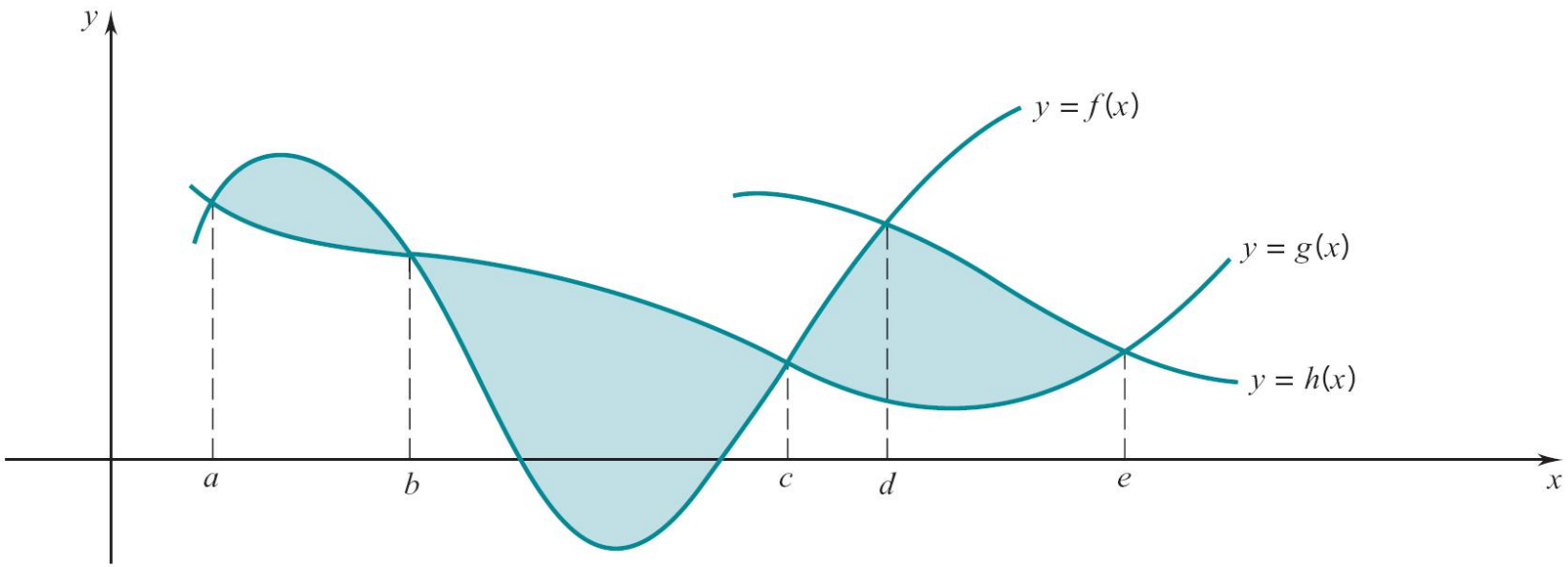
Evaluate $\int_{-1}^{3} (x^2 - 2x)\, dx$ and interpret the result in terms of areas.



Example 8

Example

Use integrals to represent the area of the region shaded in the figure below.



Quiz 4

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_{-1}^{3} f(x)\,dx$.

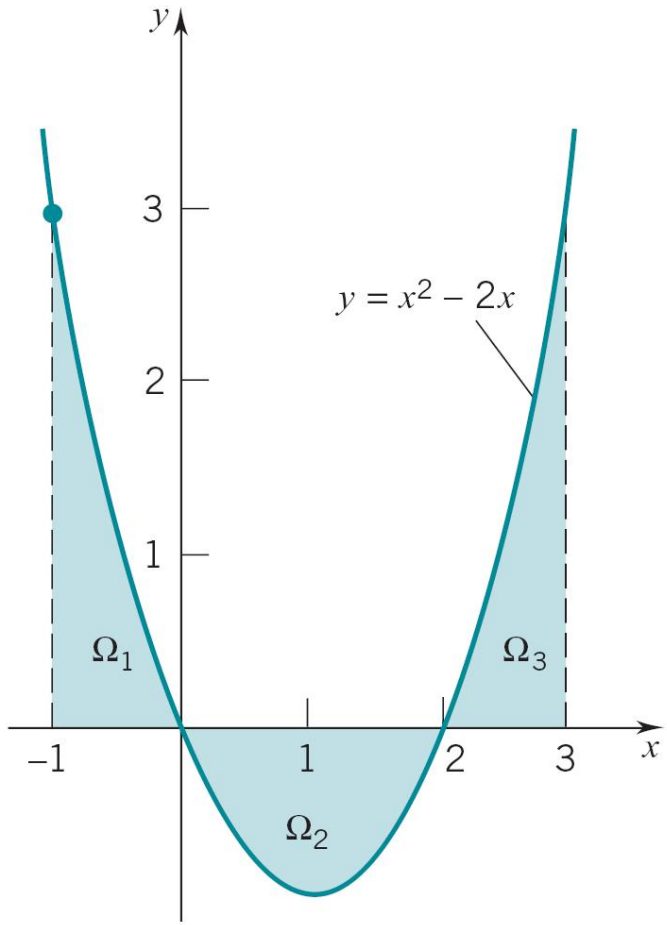
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 5

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_{-1}^{2} f(x)\,dx$.

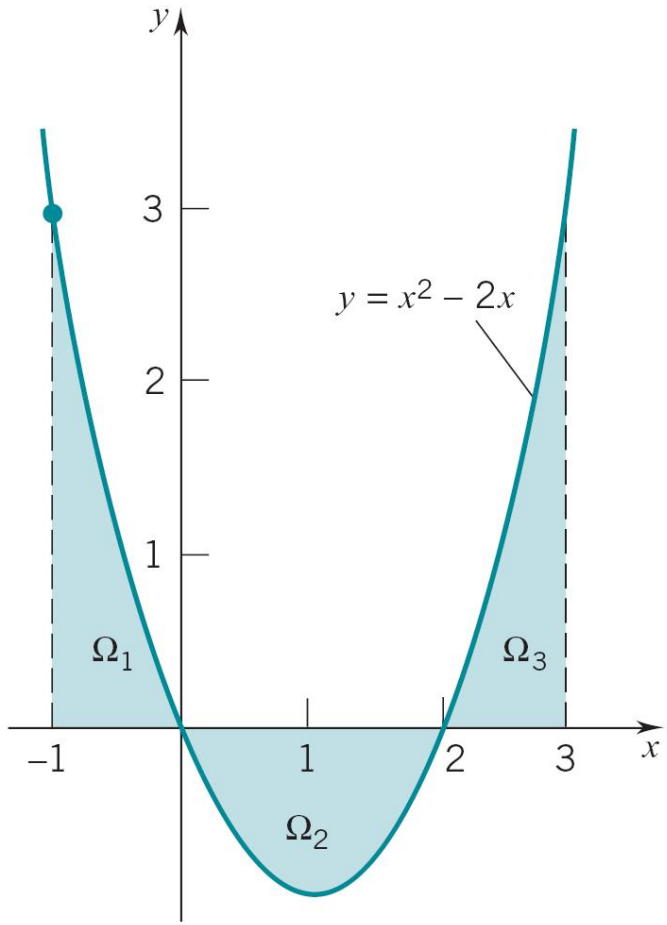
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 6

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_0^2 f(x)\,dx$.

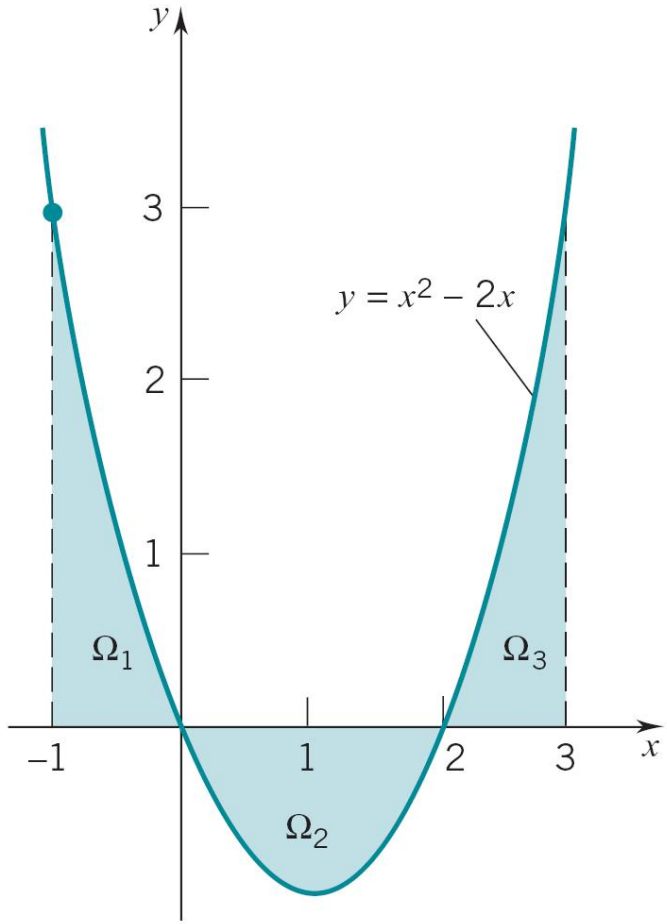
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 7

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_2^3 f(x)\,dx$.

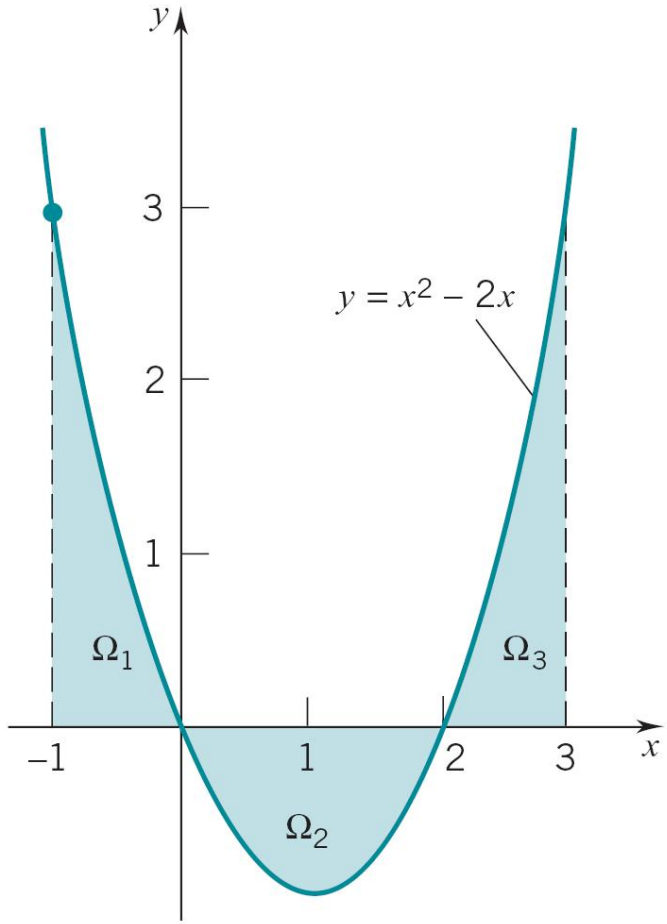
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 8

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_0^3 f(x)\,dx$.

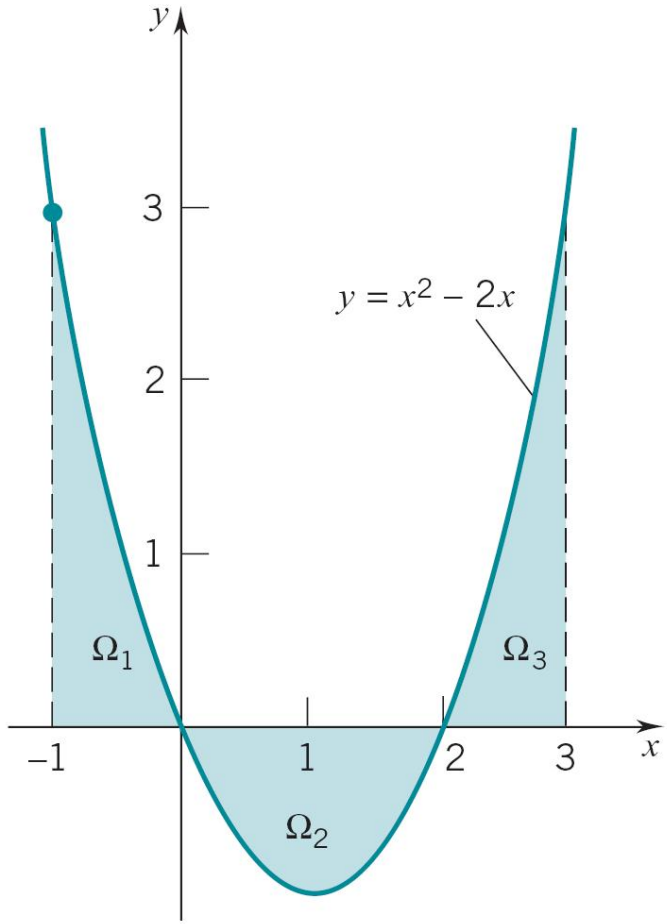
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 9

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give the area bounded between the x-axis and $y = f(x)$ from $x = -1$ to $x = 3$.

a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these