Lecture 18 Section 5.5 Some Area Problems

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Quiz 1
What is today?

a. Monday

b. Wednesday

c. Friday

d. None of these

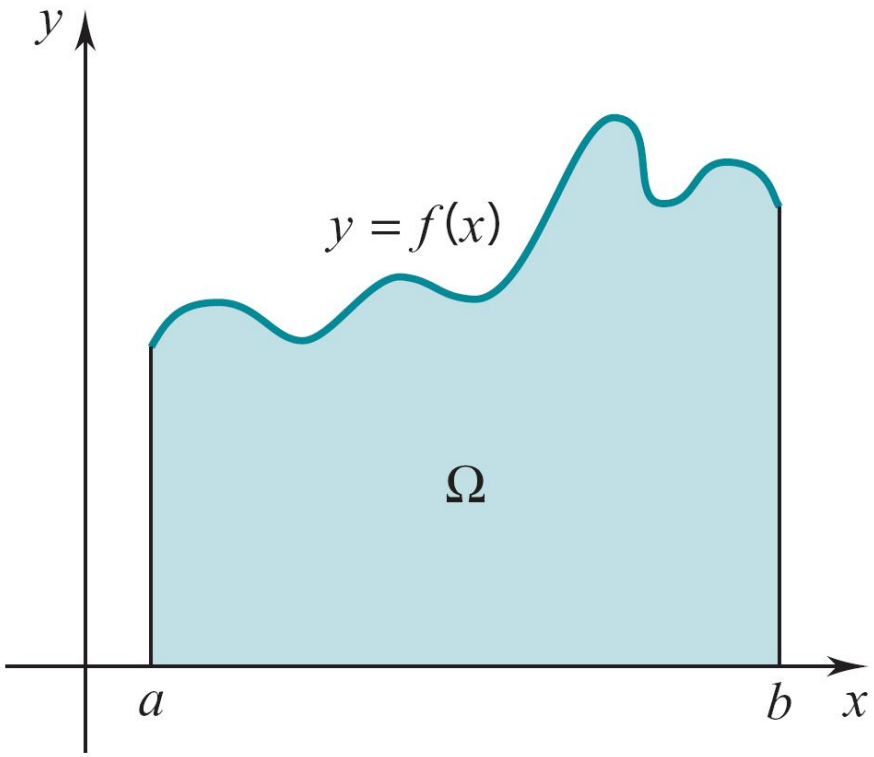
1 Section 5.5 Some Area Problems

1.1 Area below the graph of a Nonnegative f

Area below the graph of a Nonnegative f

$$f(x) \geq 0 \quad \text{for all } x \text{ in } [a, b].$$

$$\Omega = \text{ region below the graph of } f.$$



$$\text{Area of } \Omega = \int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

Fundamental Theorem of Integral Calculus

Theorem 1. *In general,*

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

where $F(x)$ is an antiderivative of $f(x)$.

Function	Antiderivative
x^r	$\dfrac{x^{r+1}}{r+1}$ (r a rational number $\neq -1$)
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$
$\sec x \tan x$	$\sec x$
$\csc^2 x$	$-\cot x$
$\csc x \cot x$	$-\csc x$

Quiz 2

Give the value of $\displaystyle\int_{-1}^{1} \left[x^3 - 2x^2 + \sin(\pi x)\right] dx$.

a. $\dfrac{1}{2}$

b. $\dfrac{4}{3}$

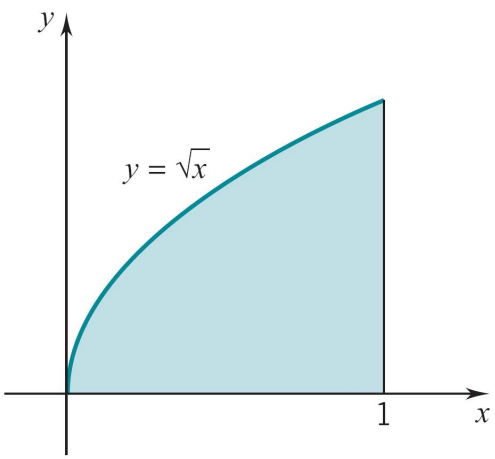
c. $-\dfrac{4}{3}$

d. $-\dfrac{1}{2}$

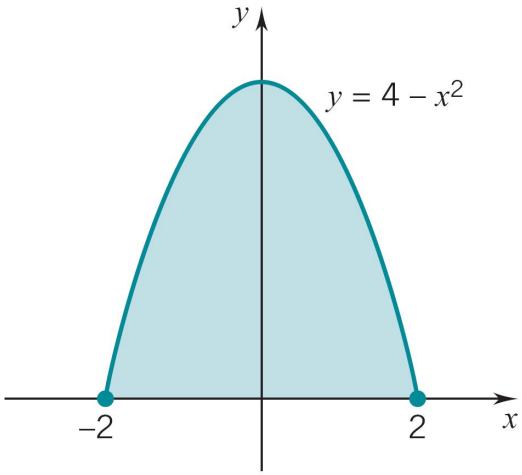
e. None of these

Example 1

Example 2. Find the area below the graph of the square-root function from $x = 0$ to $x = 1$.



Example 2
Example 3. Find the area bounded above by the curve $y = 4 - x^2$ and below by the x-axis.



Quiz 3
Give the area bounded between the x-axis and the graph of $y = x^2 + 1$ for

$-1 \leq x \leq 2$.

a. 5

b. 4

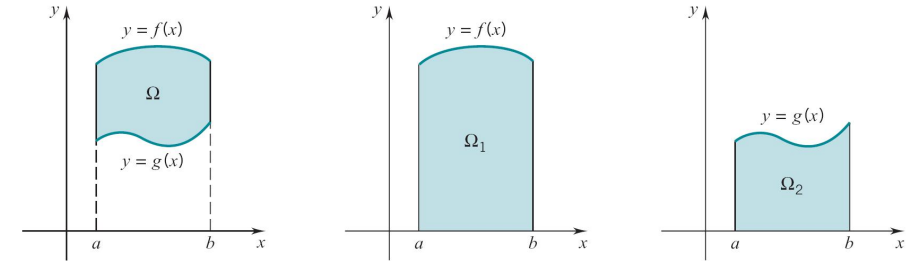
c. 3

d. 2

e. None of these

1.2 Area between the graphs of f and g

Area between the graphs of two Nonnegative f and g



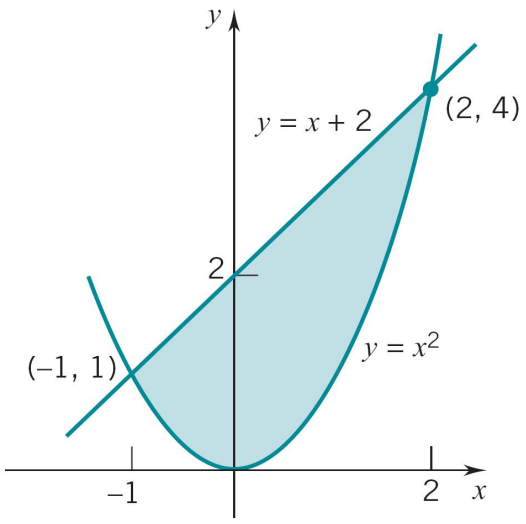
area of Ω = area of Ω_1 – area of Ω_2

$f(x) \geq g(x) \geq 0$ for all x in $[a, b]$.

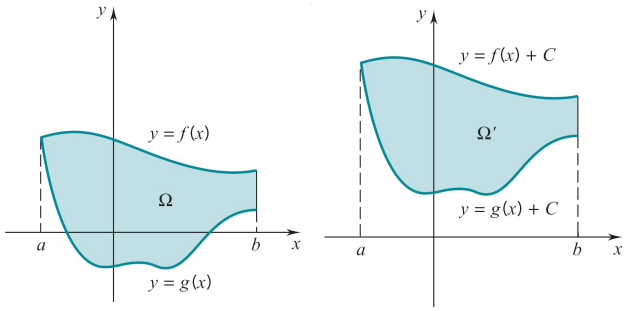
$\Omega =$ region between the graphs of f (Top) and g (Bottom).

$$\text{Area of } \Omega = \int_a^b \left[\text{ Top } - \text{ Bottom } \right] dx = \int_a^b \left[f(x) - g(x) \right] dx.$$

Example 3
Example 4. Find the area bounded above by $y = x + 2$ and below by $y = x^2$.



Area between the graphs of f and g

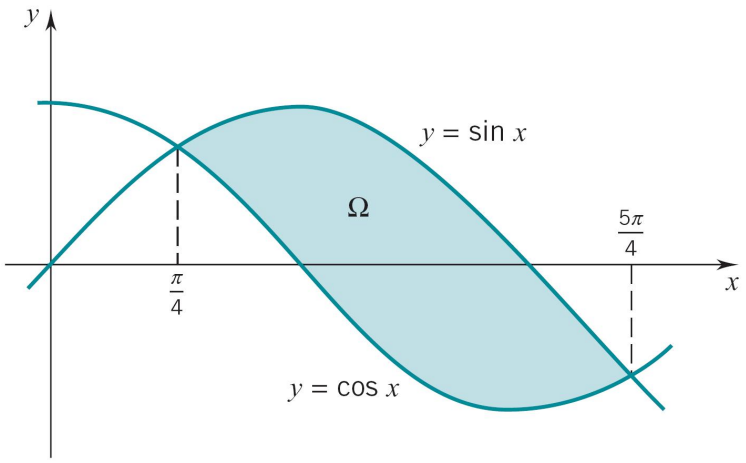


$f(x) \geq g(x)$ for all x in $[a, b]$.

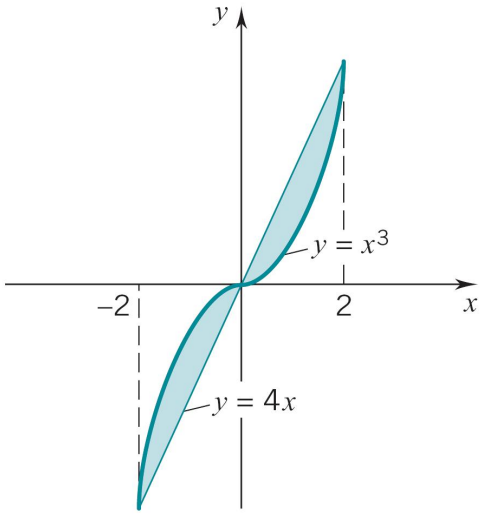
$\Omega =$ region between the graphs of f (Top) and g (Bottom).

$$\text{Area of } \Omega = \int_a^b \big[\ \text{Top} \ - \ \text{Bottom}\ \big]\, dx = \int_a^b \big[f(x) - g(x)\big]\, dx.$$

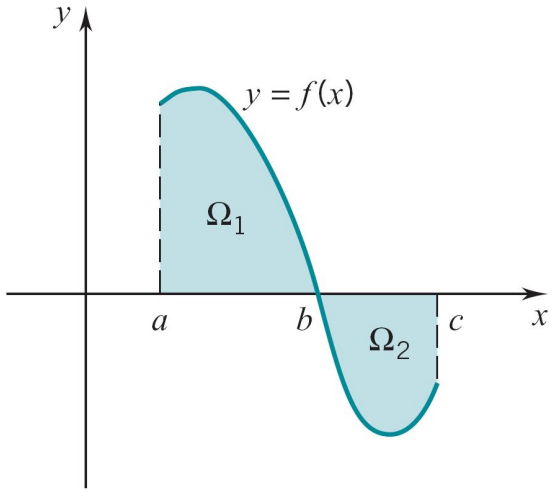
Example 4
Example 5. Find the area of the region shown in the figure below.



Example 5
Example 6. Find the area between $y = 4x$ and $y = x^3$ from $x = -2$ to $x = 2$.



Example 6
Example 7. Use integrals to represent the area of the region $\Omega = \Omega_1 \cup \Omega_2$ shaded in the figure below.

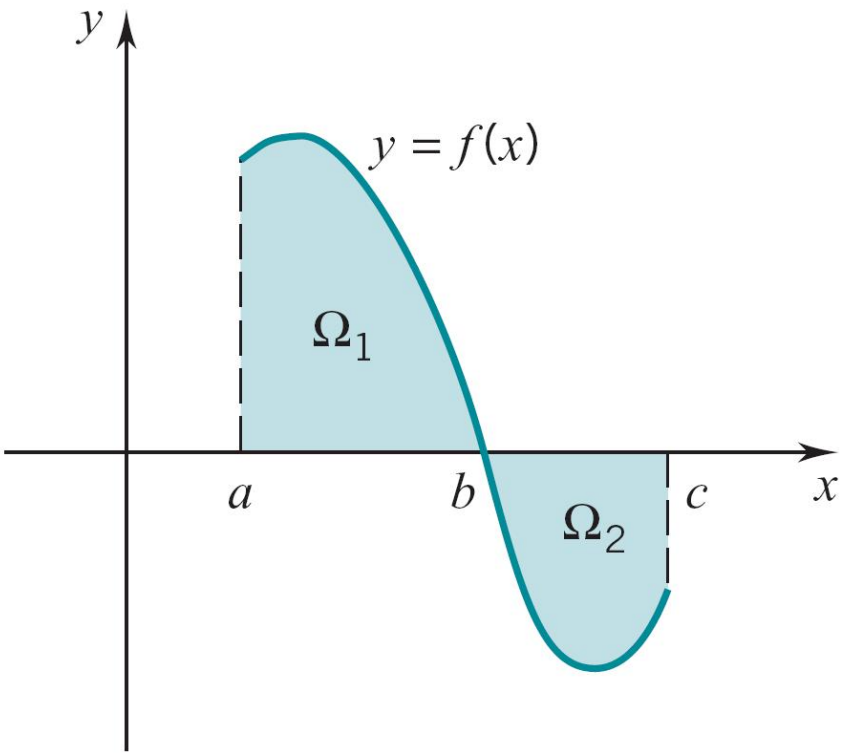


1.3 Signed Area

$\int_a^c f(x)\,dx$ as Signed Area

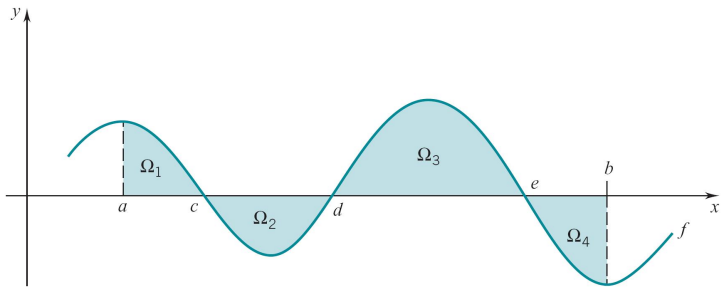
$$f(x) \geq 0 \quad \text{for all } x \text{ in } [a, b]$$
$$\int_a^b f(x)\,dx = \text{Area of } \Omega_1$$

$$f(x) \leq 0 \quad \text{for all } x \text{ in } [b, c]$$
$$\int_b^c f(x)\,dx = -\text{Area of } \Omega_2$$



$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \text{Area of } \Omega_1 - \text{Area of } \Omega_2$$
$$= \text{Area above the } x\text{-axis} - \text{Area below the } x\text{-axis.}$$

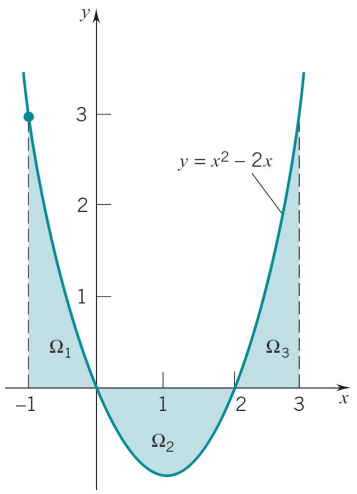
$\int_a^b f(x)\,dx$ as Signed Area



$$\begin{aligned}\int_a^b f(x)\,dx &= \int_a^c f(x)\,dx + \int_c^d f(x)\,dx + \int_d^e f(x)\,dx + \int_e^b f(x)\,dx \\ &= \text{Area of } \Omega_1 - \text{Area of } \Omega_2 + \text{Area of } \Omega_3 - \text{Area of } \Omega_4 \\ &= \big[\text{Area of } \Omega_1 + \text{Area of } \Omega_3\big] - \big[\text{Area of } \Omega_2 + \text{Area of } \Omega_4\big] \\ &= \text{Area above the } x\text{-axis} - \text{Area below the } x\text{-axis.}\end{aligned}$$

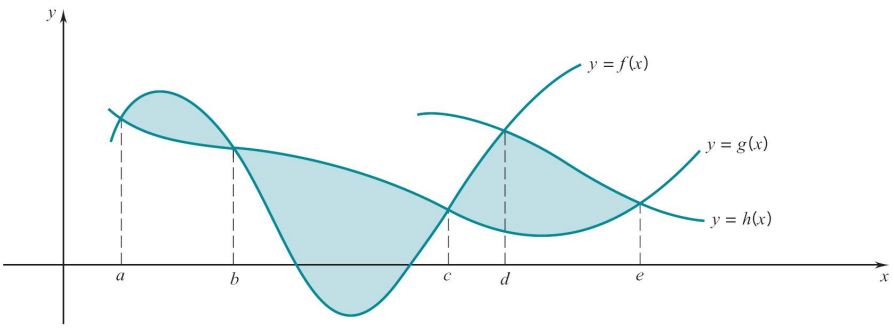
Example 7

Example 8. Evaluate $\int_{-1}^{3} \left(x^2 - 2x\right)\,dx$ and interpret the result in terms of areas.



Example 8

Example 9. Use integrals to represent the area of the region shaded in the figure below.



Quiz 4

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_{-1}^{3} f(x)\,dx$.

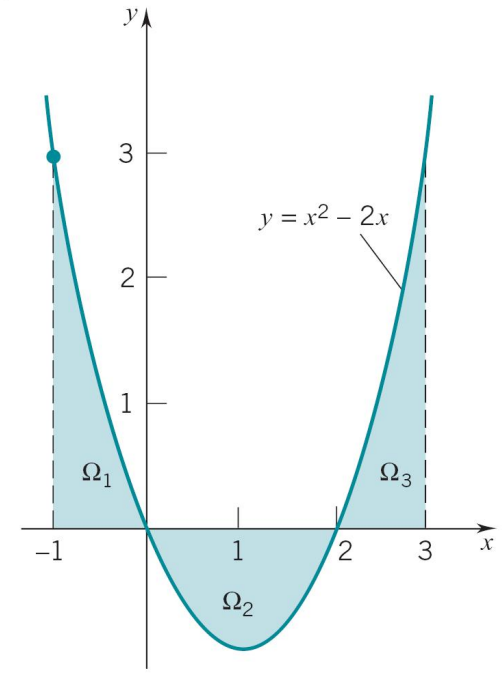
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 5

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_{-1}^{2} f(x)\,dx$.

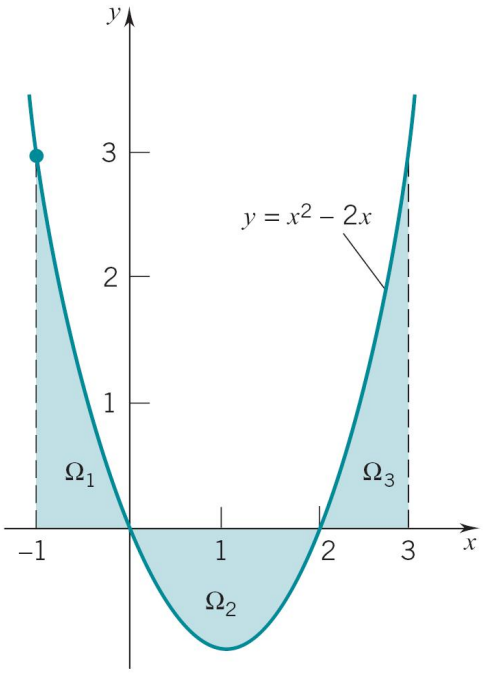
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 6

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_0^2 f(x)\,dx$.

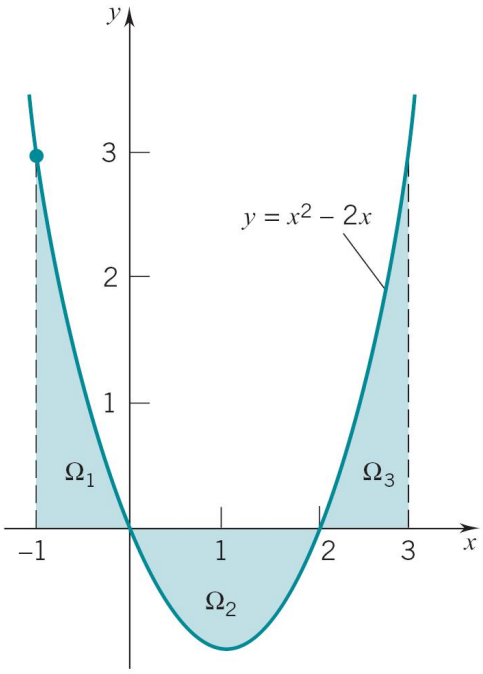
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 7

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_2^3 f(x)\,dx$.

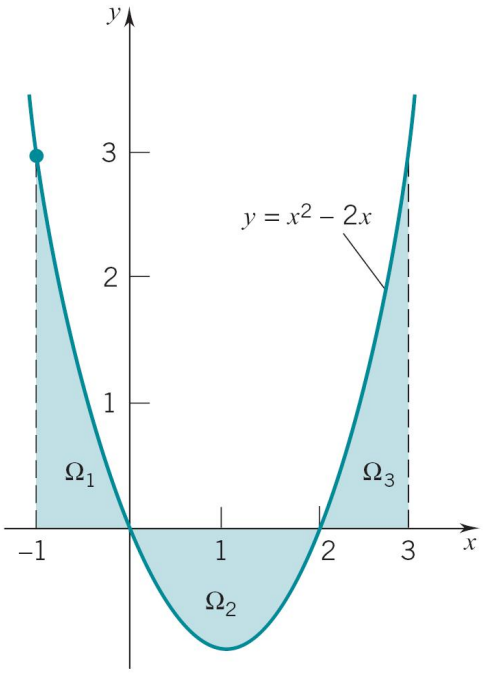
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 8

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give $\int_0^3 f(x)\,dx$.

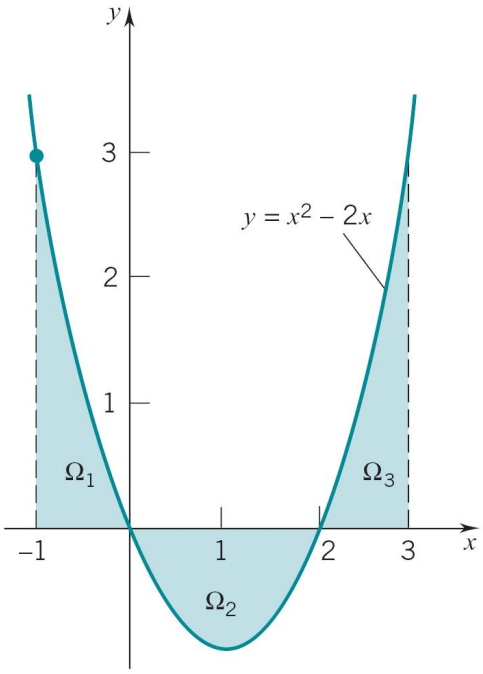
a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these



Quiz 9

The graph of $y = f(x)$ is shown below. Ω_1 has area $\frac{4}{3}$, Ω_2 has area $\frac{4}{3}$, and Ω_3 has area $\frac{4}{3}$. Give the area bounded between the x-axis and $y = f(x)$ from $x = -1$ to $x = 3$.

a. 0

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 4

e. None of these