Lecture 21

Section 6.1 More on Area
Section 6.2 Volume by Parallel Cross Section

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Test 3

- Test 3: Dec. 4-6 in CASA

Final Exam

- Final Exam: Dec. 14-17 in CASA

Review for Test 3

- Review for Test 3 by the College Success Program.
- Friday, November 21 2:30–3:30pm in the basement of the library by the C-site.

Online Quizzes

- Online Quizzes are available on CourseWare.

Quiz 1

What is today?

a. Monday

b. Wednesday

c. Friday

d. None of these

The Average value of f

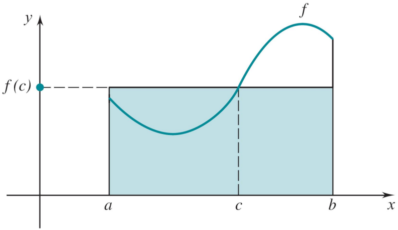
Let f_{avg} denote the average or mean value of f on $[a, b]$. Then

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

The First Mean-Value Theorems for Integrals

If f is continous on $[a, b]$, then there is at least one number c in (a, b) for which

$$f(c) = f_{\text{avg}}.$$



Quiz 2

The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. Give the average value of f on the interval $[d, b]$ with $d = 3$ and $b = 6$.

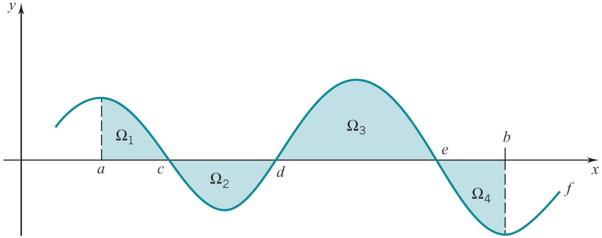
a. $1/3$

b. $2/3$

c. 1

d. $4/3$

e. None of these



Quiz 3

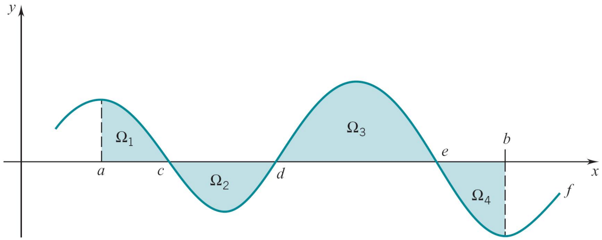
The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. How many values of c satisfy the conclusion of the mean value theorem for integrals on the interval $[d, b]$ with $d = 3$ and $b = 6$.

a. 1

b. 2

c. 3

d. None of these



Quiz 4

The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. Give the average value of f on the interval $[a, b]$ with $a = 1$ and $b = 6$.

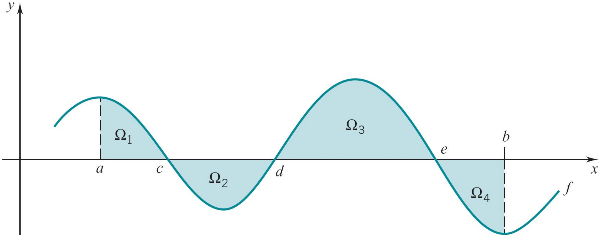
a. $1/5$

b. $2/5$

c. $3/5$

d. $4/5$

e. None of these



Quiz 5

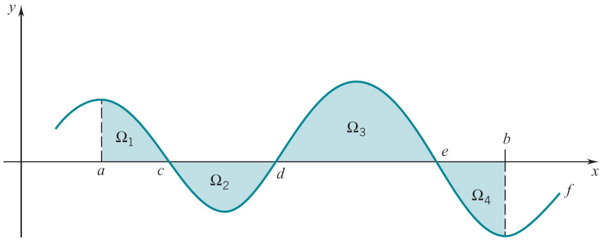
The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. How many values of c satisfy the conclusion of the mean value theorem for integrals on the interval $[a, b]$ with $d = 1$ and $b = 6$.

a. 1

b. 2

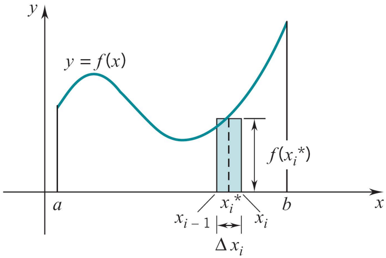
c. 3

d. None of these

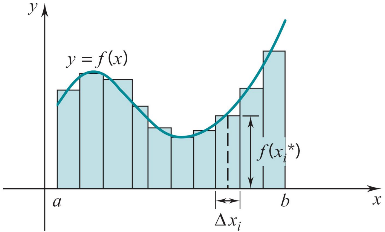


Representative Rectangle, Riemann Sum and Area: $f \geq 0$

Rectangle: $f(x_i^*)\Delta x_i$



Riemann Sum: $\sum f(x_i^*)\Delta x_i$



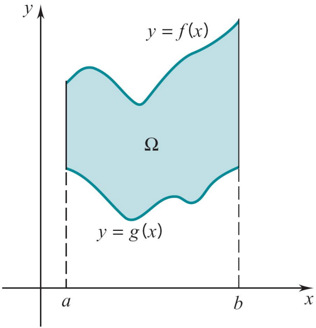
$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum f(x_i^*)\Delta x_i.$$

$$\text{area} = \int_a^b f(x)\,dx \approx \sum f(x_i^*)\Delta x_i.$$

Area by Integration with Respect to x: $f(x) \geq g(x)$

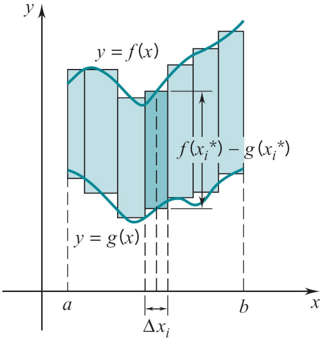
Rectangle Area

$[f(x_i^*) - g(x_i^*)]\Delta x_i$



Riemann Sum

$\sum[f(x_i^*) - g(x_i^*)]\Delta x_i$

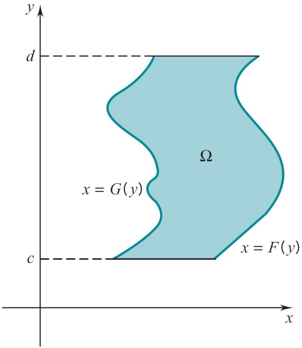


$$\text{area}(\Omega) = \int_a^b [f(x) - g(x)]\,dx = \lim_{\|P\|\to 0} \sum[f(x_i^*) - g(x_i^*)]\Delta x_i.$$

Area by Integration with Respect to y: $F(y) \geq G(y)$

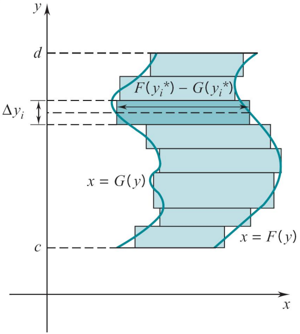
Rectangle Area

$[F(y_i^*) - G(y_i^*)]\Delta y_i$



Riemann Sum

$\sum[F(y_i^*) - G(y_i^*)]\Delta y_i$

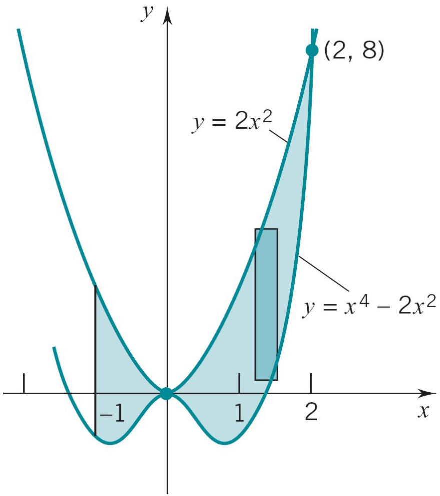


$$\text{area}(\Omega) = \int_c^d [F(y) - G(y)]\,dy = \lim_{\|P\|\to 0} \sum[F(y_i^*) - G(y_i^*)]\Delta y_i.$$

Example

Example

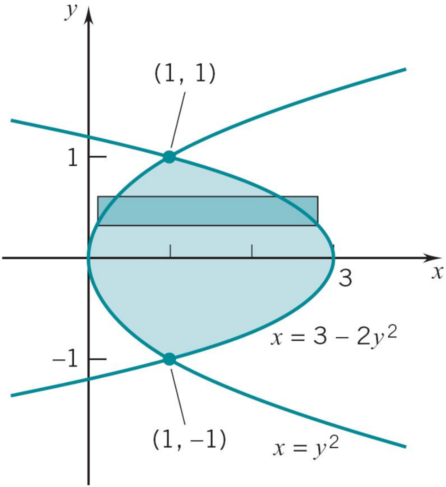
Find the area of the shaded region shown in the figure below.



Example

Example

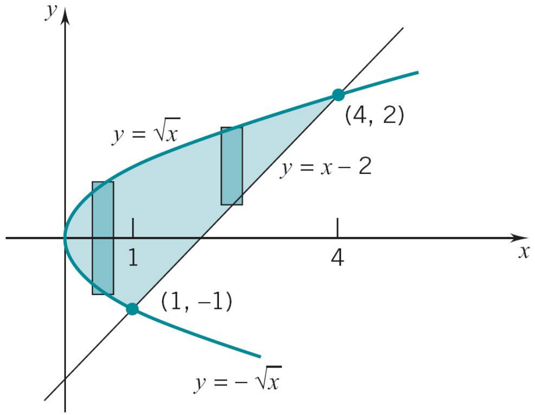
Find the area of the shaded region shown in the figure below.



Example

Example

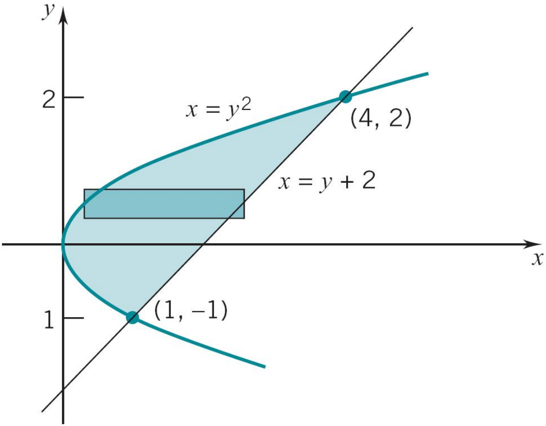
Find the area of the shaded region shown in the figure below by integrating with respect to x.



Example

Example

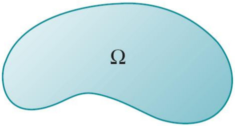
Find the area of the shaded region shown in the figure below by integrating with respect to y.

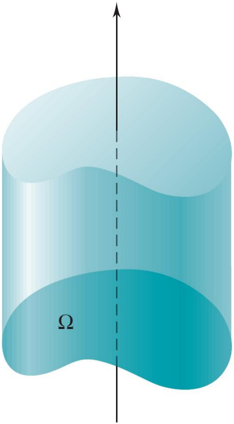


Right Cylinder with Cross Section

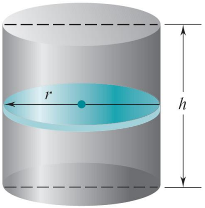
Volume of a Right Cylinder with Cross Section

$$V = A \cdot h = (\text{cross-sectional area}) \cdot \text{height}$$

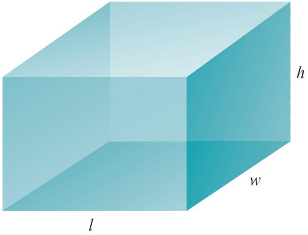




Right Circular Cylinder and Rectangular Box



$V = \pi r^2 h$ = (cross-sectional area) · height



$V = l \cdot w \cdot h$ = (cross-sectional area) · height

Volume of a Right Circular Cylinder

$$V = \pi r^2 \cdot h = \text{(cross-sectional area)} \cdot \text{height}$$

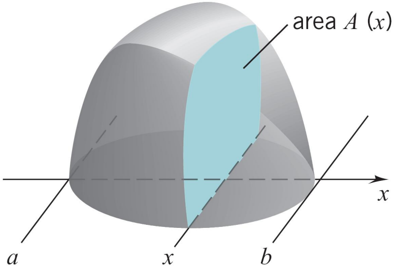
Volume of a Rectangular Box

$$V = l \cdot w \cdot h = \text{(cross-sectional area)} \cdot \text{height}$$

Volume by Parallel Cross Section

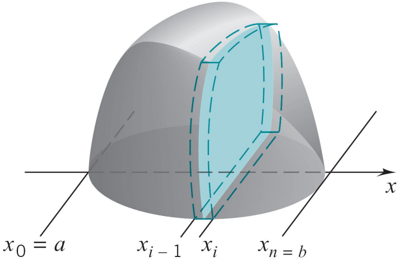
Cylinder Volume

$A(x_i^*)\Delta x_i$



Riemann Sum

$\sum A(x_i^*)\Delta x_i$

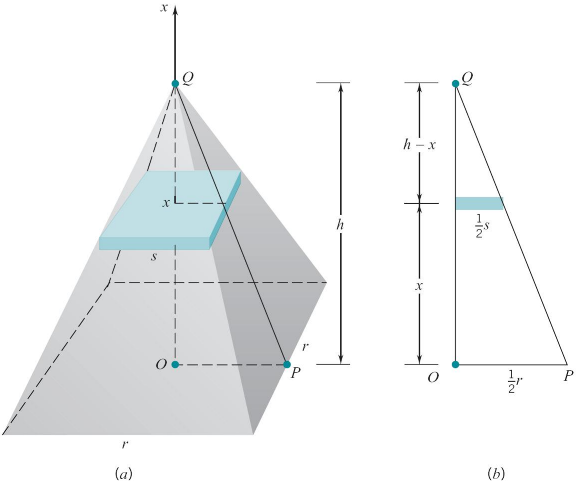


$$V = \int_a^b A(x)\,dx = \lim_{\|P\|\to 0} \sum A(x_i^*)\Delta x_i$$

Example

Example

Find the volume of the pyramid shown in the figure below.



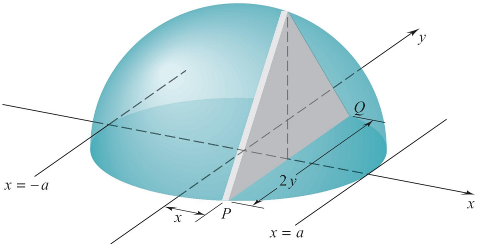
Example

Example

The base of a solid is the region bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

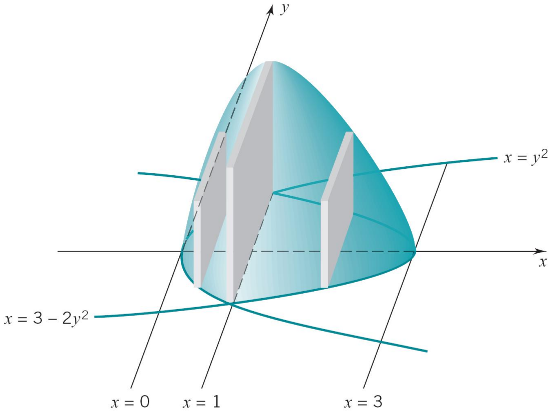
Find the volume of the solid given that each cross section is an isosceles triangle with base in the region and altitude equal to one-half the base.



Example

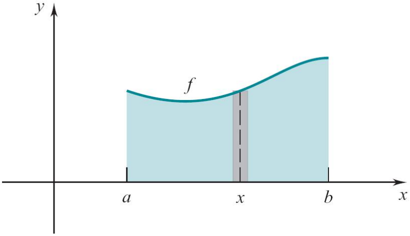
Example

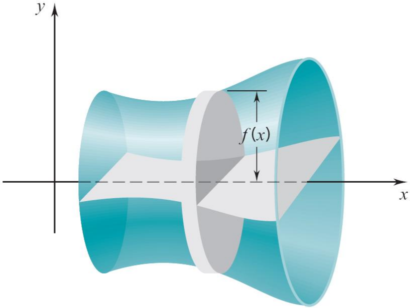
The base of a solid is the region between the parabolas $x = y^2$ and $x = 3 - 2y^2$. Find the volume of the solid given that the cross sections are squares.



Solid of Revolution About the x-Axis

Cylinder Volume: $\pi[f(x_i^*)]^2\Delta x_i$ Riemann Sum: $\sum \pi[f(x_i^*)]^2\Delta x_i$



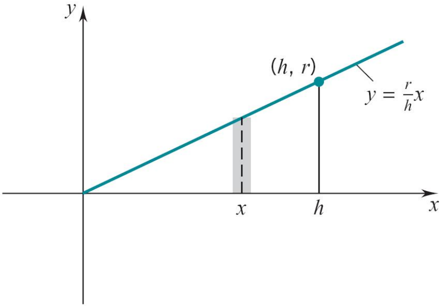


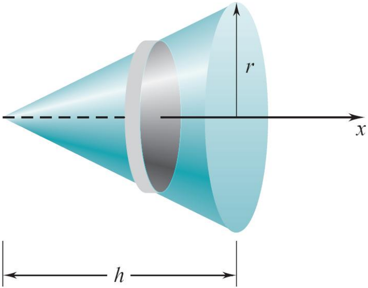
$$V = \int_a^b \pi[f(x)]^2\,dx = \lim_{\|P\|\to 0} \sum \pi[f(x_i^*)]^2\Delta x_i.$$

Example

Example

Find the volume of the cone shown in the figure below.



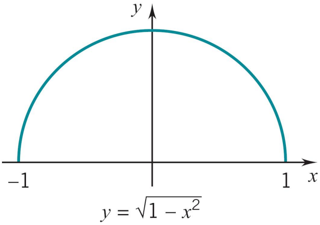


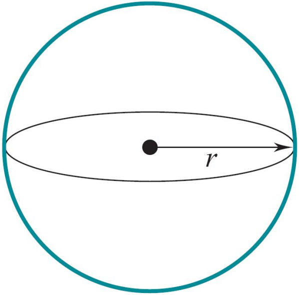
Example

Example

Find the volume of a sphere of radius r by revolving about the x-axis the region below the graph of

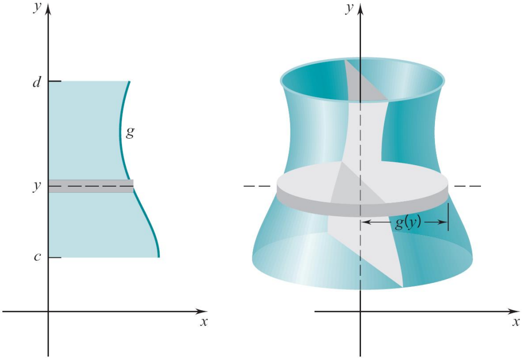
$$f(x) = \sqrt{r^2 - x^2}, \quad -r \le x \le r.$$





Solid of Revolution About the y-Axis

Cylinder Volume: $\pi[g(y_i^*)]^2\Delta y_i$ Riemann Sum: $\sum \pi[g(y_i^*)]^2\Delta y_i$

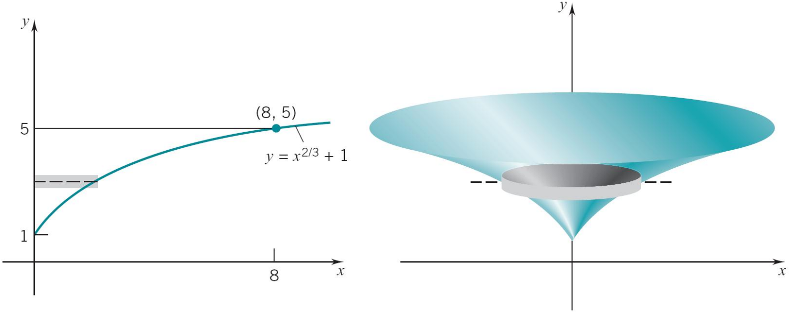


$$V = \int_c^d \pi[g(y)]^2\,dy = \lim_{\|P\|\to 0} \sum \pi[g(y_i^*)]^2\Delta y_i.$$

Example

Example

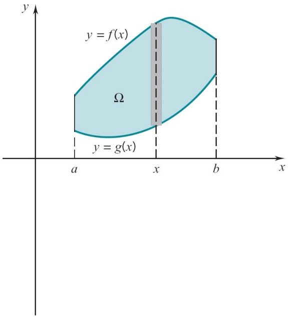
Find the volume of the solid shown in the figure below.

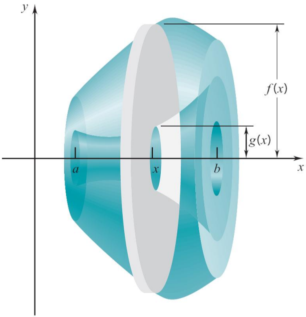


Solid of Revolution About the x-Axis

Cylinder Volume: $\pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$

Riemann Sum: $\sum \pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$



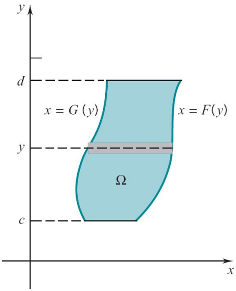


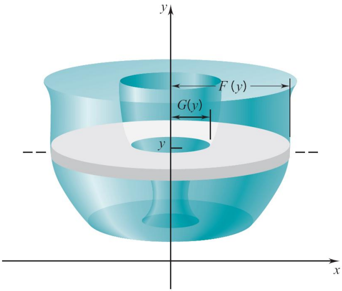
$$V = \int_a^b \pi([f(x)]^2 - [g(x)]^2)\,dx = \lim_{\|P\| \to 0} \sum \pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$$

Solid of Revolution About the y-Axis

Cylinder Volume: $\pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$

Riemann Sum: $\sum \pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$



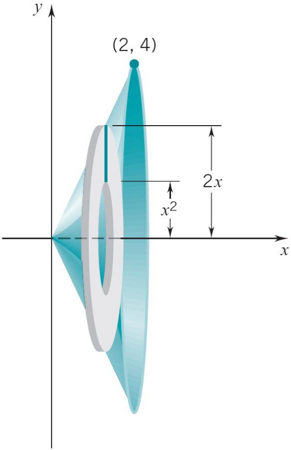


$$V = \int_c^d \pi([F(y)]^2 - [G(y)]^2)\,dy = \lim_{\|P\|\to 0} \sum \pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$$

Example

Example

Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the x-axis.



Example

Example

Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the y-axis.