Lecture 21 Section 6.1 More on Area Section 6.2 Volume by Parallel Cross Section

Jiwen He

Test 3

- Test 3: Dec. 4-6 in CASA

Final Exam

- Final Exam: Dec. 14-17 in CASA

Review for Test 3

- Review for Test 3 by the College Success Program.
- Friday, November 21 2:30–3:30pm in the basement of the library by the C-site.

Online Quizzes

- Online Quizzes are available on CourseWare.

Quiz 1
What is today?

a. Monday

b. Wednesday

c. Friday

d. None of these

The Average value of f

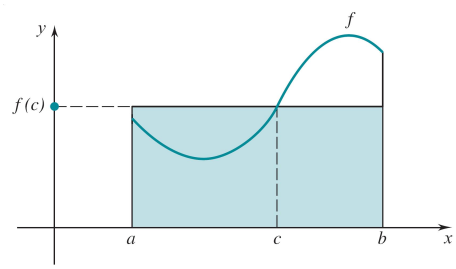
Let f_{avg} denote the average or mean value of f on $[a, b]$. Then

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x)\, dx.$$

The First Mean-Value Theorems for Integrals

If f is continous on $[a, b]$, then there is at least one number c in (a, b) for which

$$f(c) = f_{\text{avg}}.$$



Quiz 2

The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. Give the average value of f on the interval $[d, b]$ with $d = 3$ and $b = 6$.

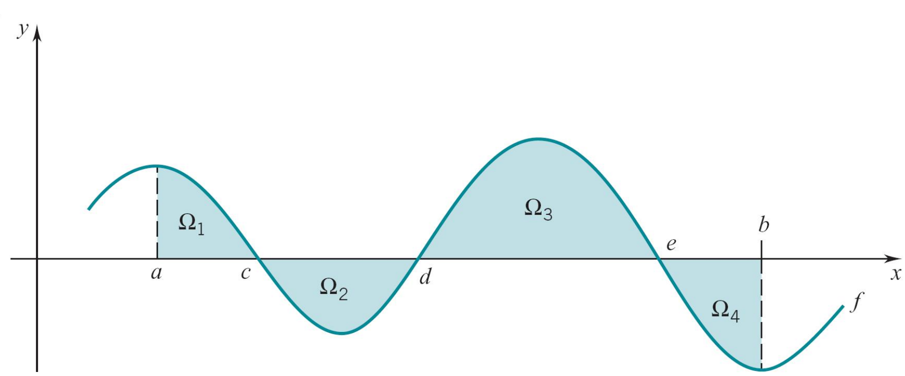
a. 1/3

b. 2/3

c. 1

d. 4/3

e. None of these



Quiz 3

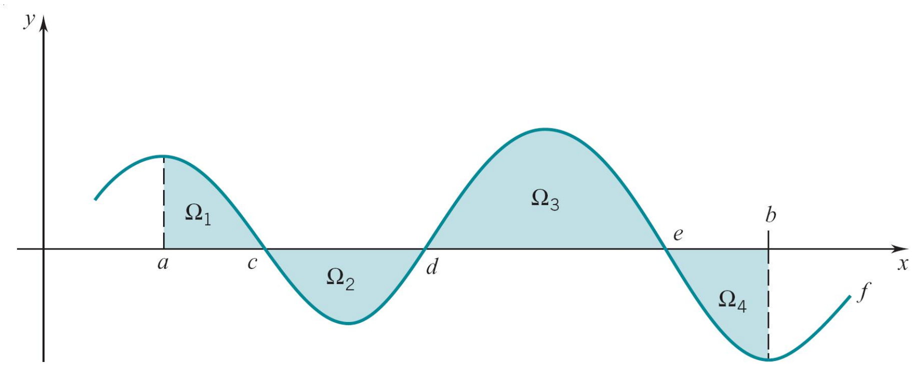
The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. How many values of c satisfy the conclusion of the mean value theorem for integrals on the interval $[d, b]$ with $d = 3$ and $b = 6$.

a. 1

b. 2

c. 3

d. None of these



Quiz 4

The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. Give the average value of f on the interval $[a, b]$ with $a = 1$ and $b = 6$.

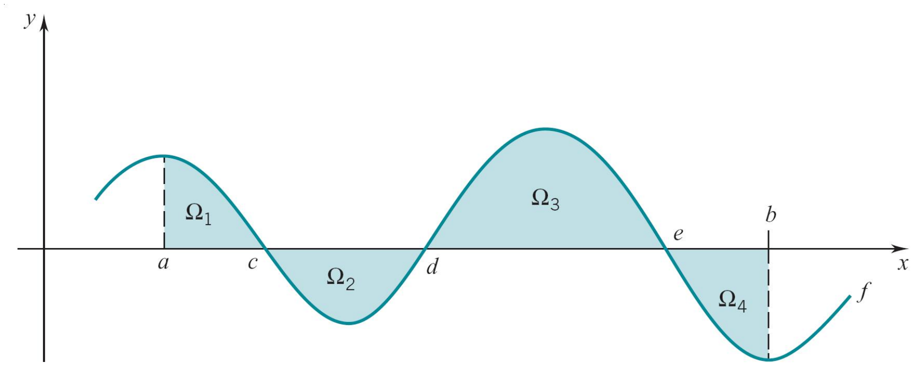
a. $1/5$

b. $2/5$

c. $3/5$

d. $4/5$

e. None of these



Quiz 5

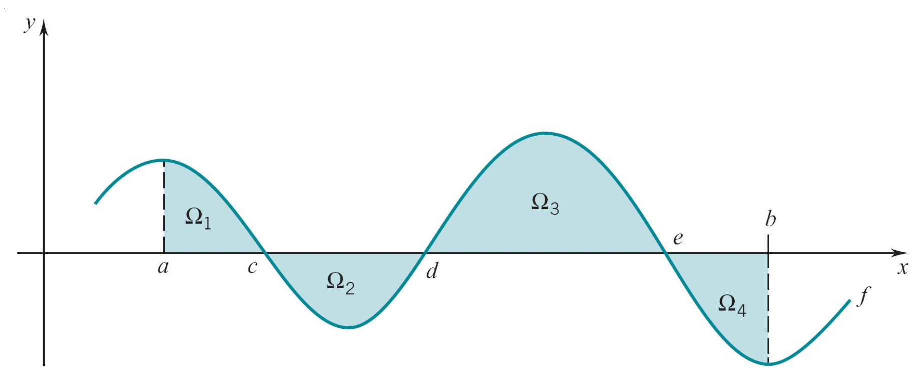
The graph of $y = f(x)$ is shown below, where Ω_1 has area 2, Ω_2 has area 3, Ω_3 has area 8, and Ω_4 has area 4. How many values of c satisfy the conclusion of the mean value theorem for integrals on the interval $[a, b]$ with $d = 1$ and $b = 6$.

a. 1

b. 2

c. 3

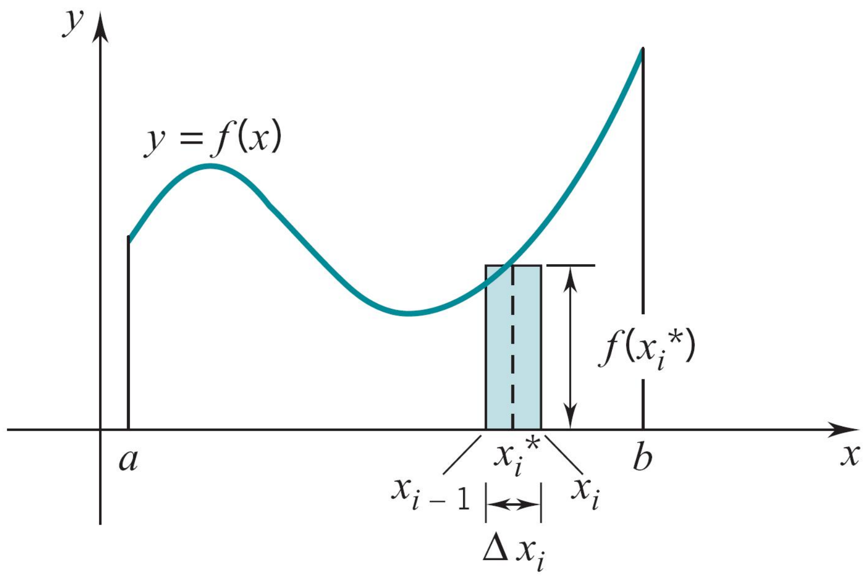
d. None of these



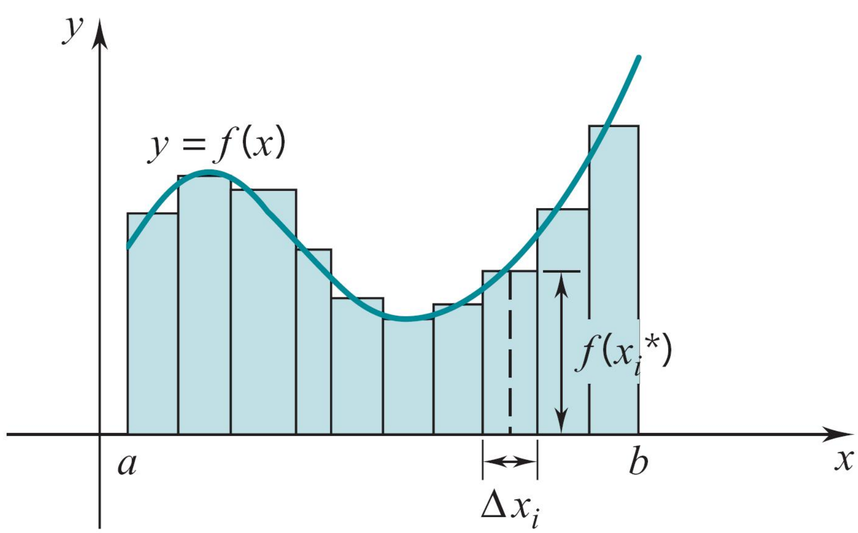
1 Section 6.1 More on Area

Representative Rectangle, Riemann Sum and Area: $f \geq 0$

$$\text{Rectangle: } f(x_i^*)\Delta x_i$$



Riemann Sum: $\sum f(x_i^*)\Delta x_i$

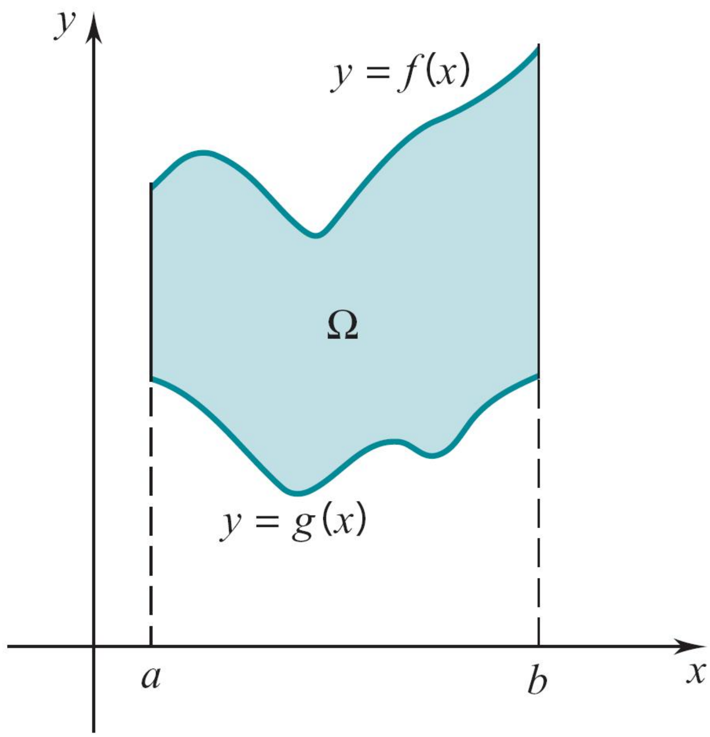


$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum f(x_i^*)\Delta x_i.$$

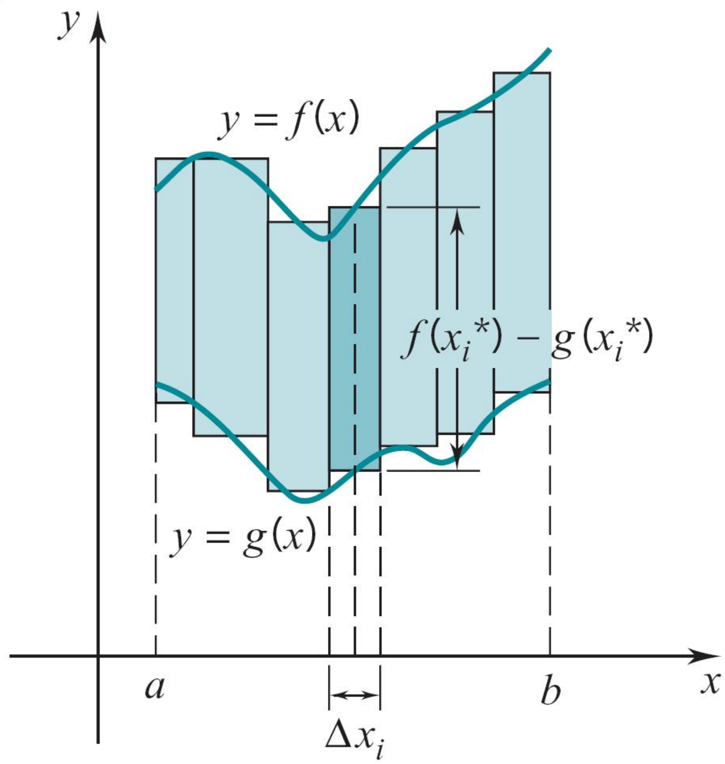
$$\text{area } = \int_a^b f(x)\,dx \approx \sum f(x_i^*)\Delta x_i.$$

Area by Integration with Respect to x: $f(x) \geq g(x)$

Rectangle Area $[f(x_i^*) - g(x_i^*)]\Delta x_i$



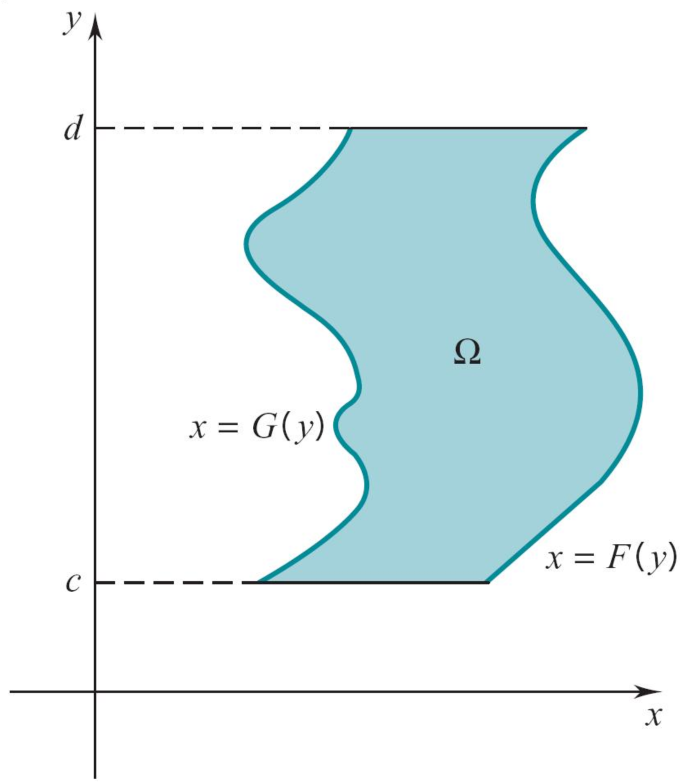
Riemann Sum $\sum [f(x_i^*) - g(x_i^*)]\Delta x_i$



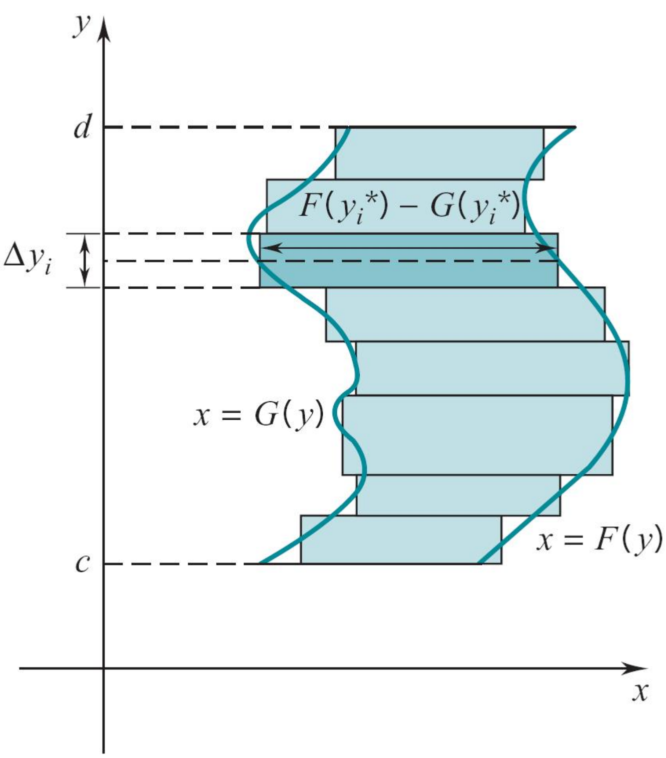
$$\text{area}(\Omega) = \int_a^b [f(x) - g(x)]\, dx = \lim_{\|P\| \to 0} \sum [f(x_i^*) - g(x_i^*)] \Delta x_i.$$

Area by Integration with Respect to y: $F(y) \geq G(y)$

Rectangle Area $[F(y_i^*) - G(y_i^*)]\Delta y_i$

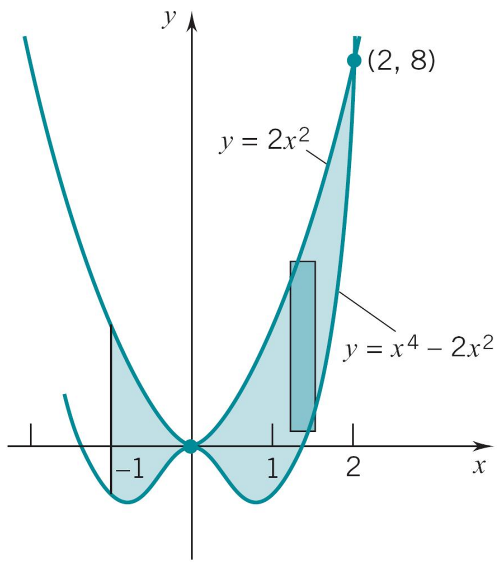


Riemann Sum $\sum[F(y_i^*) - G(y_i^*)]\Delta y_i$

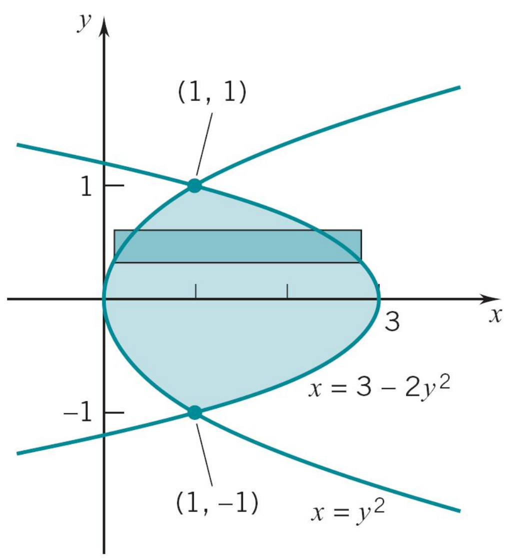


$$\text{area}(\Omega) = \int_c^d [F(y) - G(y)]\, dy = \lim_{\|P\| \to 0} \sum [F(y_i^*) - G(y_i^*)] \Delta y_i.$$

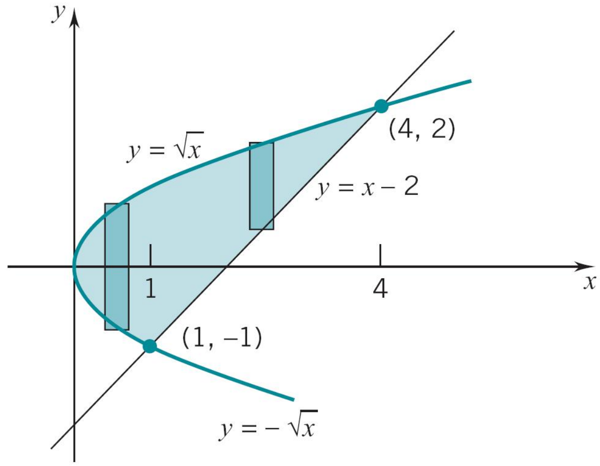
Example
Example 1. Find the area of the shaded region shown in the figure below.



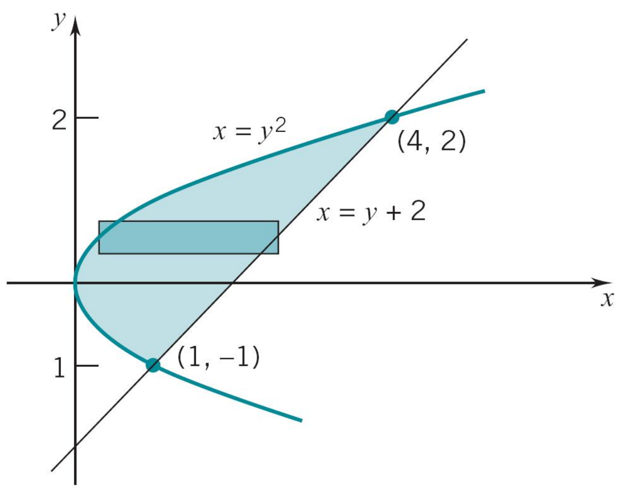
Example
Example 2. Find the area of the shaded region shown in the figure below.



Example
Example 3. Find the area of the shaded region shown in the figure below by integrating with respect to x.



Example
Example 4. Find the area of the shaded region shown in the figure below by integrating with respect to y.



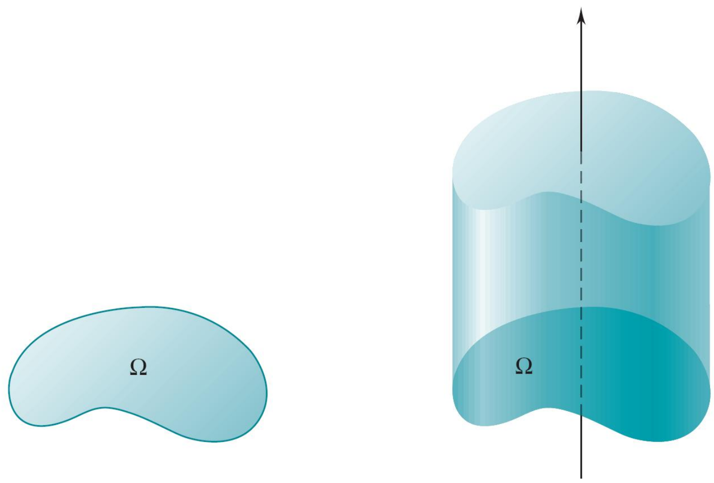
2 Volume by Parallel Cross Section

2.1 Volume by Parallel Cross Section

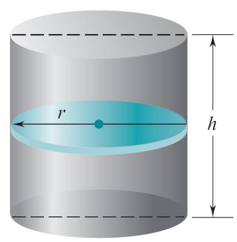
Right Cylinder with Cross Section

Volume of a Right Cylinder with Cross Section

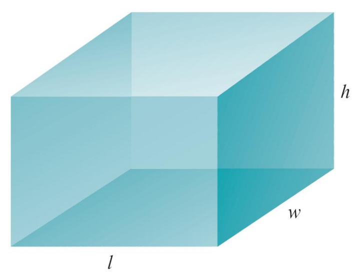
$$V = A \cdot h = (\text{cross-sectional area}) \cdot \text{height}$$



Right Circular Cylinder and Rectangular Box



$V = \pi r^2 h$ = (cross-sectional area) · height



$V = l \cdot w \cdot h$ = (cross-sectional area) · height

Volume of a Right Circular Cylinder

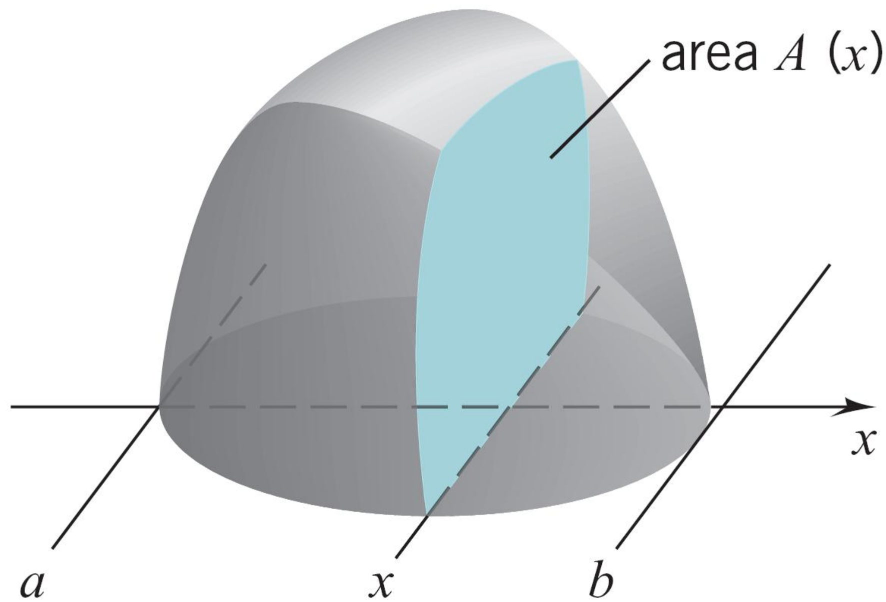
$$V = \pi r^2 \cdot h = (\text{cross-sectional area}) \cdot \text{height}$$

Volume of a Rectangular Box

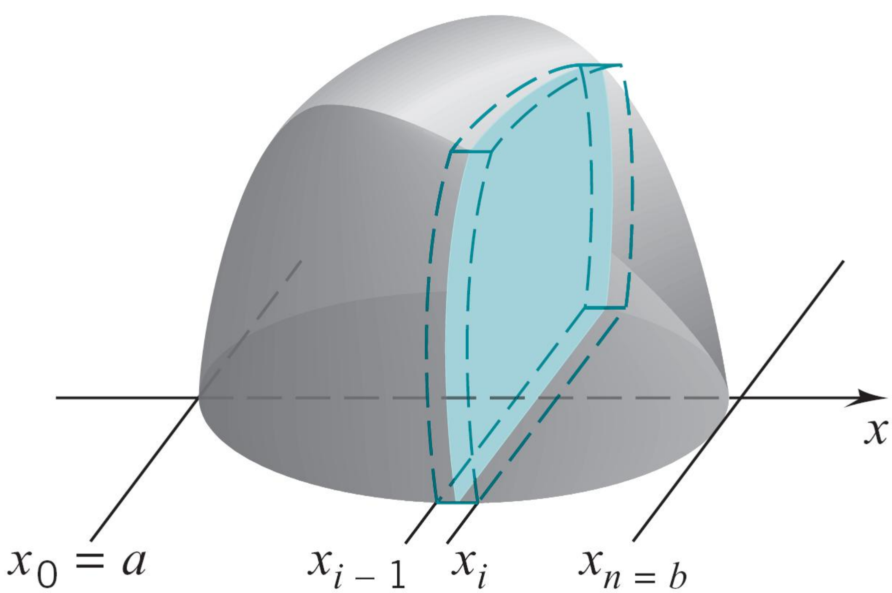
$$V = l \cdot w \cdot h = (\text{cross-sectional area}) \cdot \text{height}$$

Volume by Parallel Cross Section

$$\text{Cylinder Volume } A(x_i^*)\Delta x_i$$

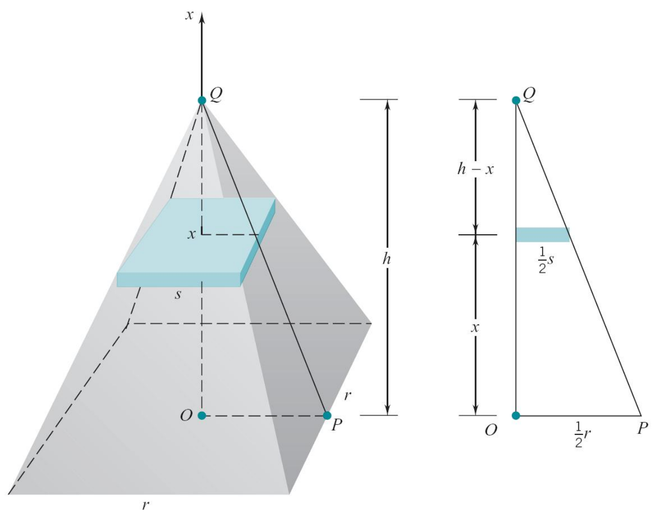


Riemann Sum $\sum A(x_i^*)\Delta x_i$



$$V = \int_a^b A(x)\,dx = \lim_{\|P\|\to 0} \sum A(x_i^*)\Delta x_i$$

Example
Example 5. Find the volume of the pyramid shown in the figure below.

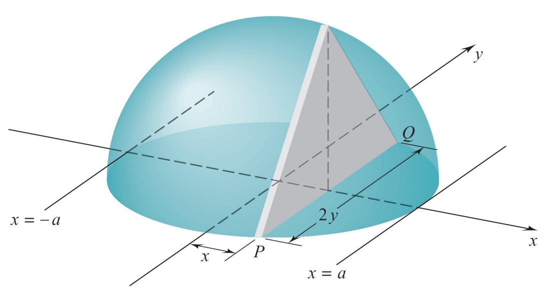


(a) (b)

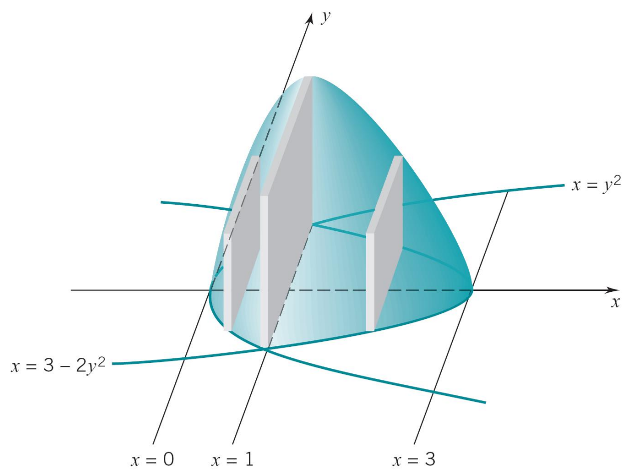
Example
Example 6. The base of a solid is the region bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Find the volume of the solid given that each cross section is an isosceles triangle with base in the region and altitude equal to one-half the base.



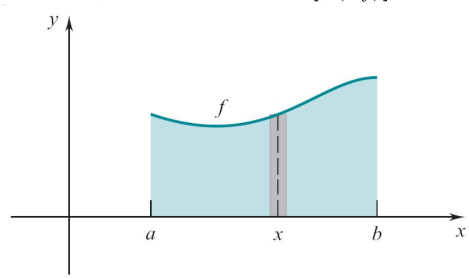
Example
Example 7. The base of a solid is the region between the parabolas $x = y^2$ and $x = 3 - 2y^2$. Find the volume of the solid given that the cross sections are squares.

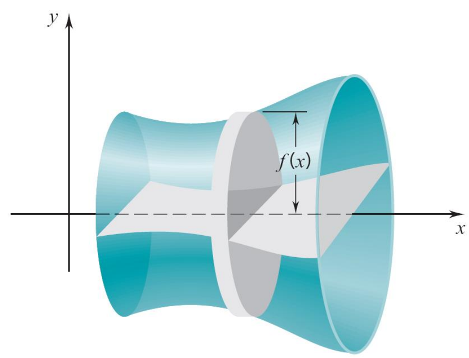


2.2 Solid of Revolution: Disk Method

Solid of Revolution About the x-Axis

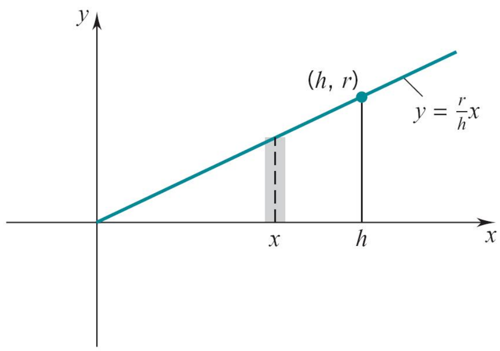
Cylinder Volume: $\pi[f(x_i^*)]^2\Delta x_i$ Riemann Sum: $\sum \pi[f(x_i^*)]^2\Delta x_i$

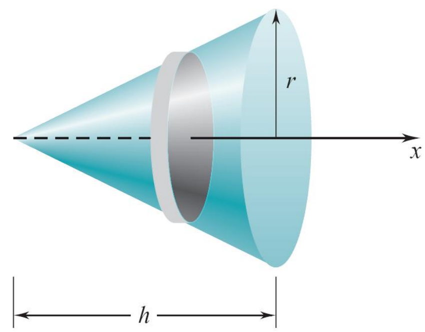




$$V = \int_a^b \pi[f(x)]^2\,dx = \lim_{\|P\|\to 0} \sum \pi[f(x_i^*)]^2\Delta x_i.$$

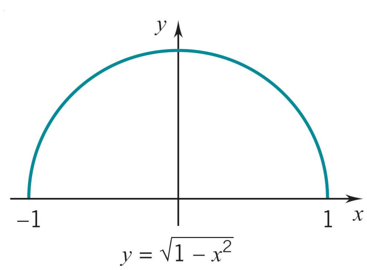
Example
Example 8. Find the volume of the cone shown in the figure below.

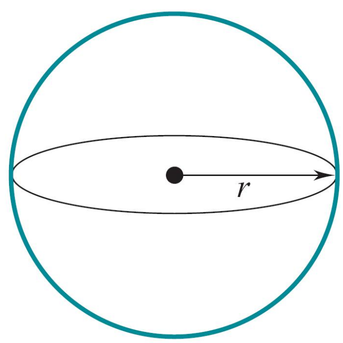




Example
Example 9. Find the volume of a sphere of radius r by revolving about the x-axis the region below the graph of

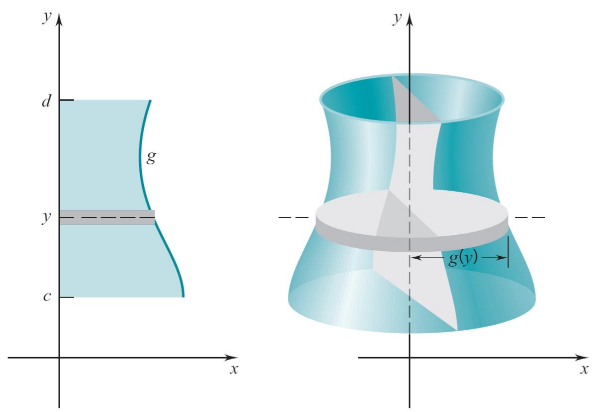
$$f(x) = \sqrt{r^2 - x^2}, \quad -r \leq x \leq r.$$





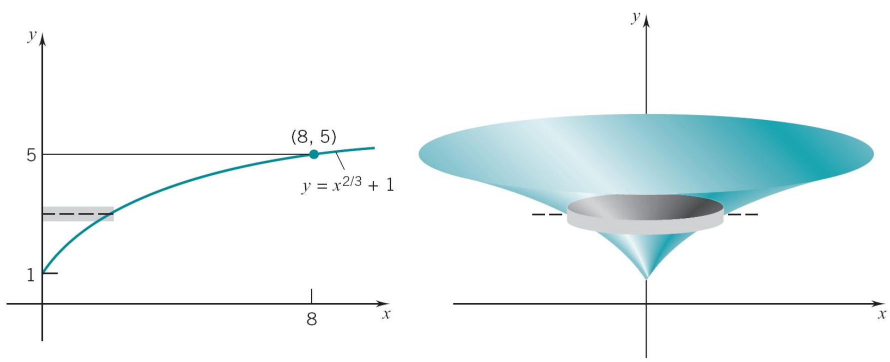
Solid of Revolution About the y-Axis

Cylinder Volume: $\pi[g(y_i^*)]^2\Delta y_i$ Riemann Sum: $\sum \pi[g(y_i^*)]^2\Delta y_i$



$$V = \int_c^d \pi[g(y)]^2\, dy = \lim_{\|P\|\to 0} \sum \pi[g(y_i^*)]^2 \Delta y_i.$$

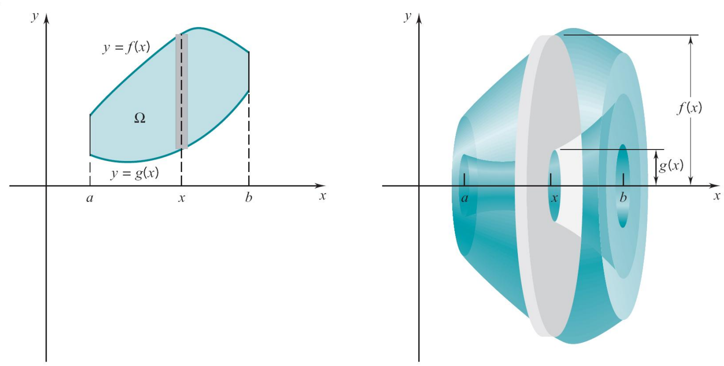
Example
Example 10. Find the volume of the solid shown in the figure below.



2.3 Solid of Revolution: Washer Method

Solid of Revolution About the x-Axis

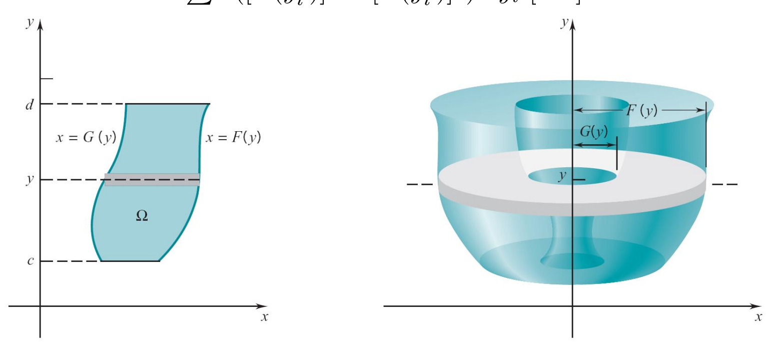
Cylinder Volume: $\pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$ [1ex] Riemann Sum: $\sum \pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$ [1ex]



$$V = \int_a^b \pi([f(x)]^2 - [g(x)]^2)\,dx = \lim_{\|P\|\to 0} \sum \pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$$

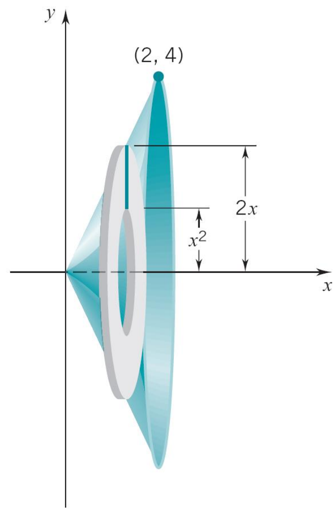
Solid of Revolution About the y-Axis

Cylinder Volume: $\pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$ [1ex] Riemann Sum: $\sum \pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$ [1ex]



$$V = \int_c^d \pi([F(y)]^2 - [G(y)]^2)\,dy = \lim_{\|P\|\to 0} \sum \pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$$

Example
Example 11. Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the x-axis.



Example
Example 12. Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the y-axis.