Lecture 22 Section 6.2 Volume by Parallel Cross Section

Section 6.3 Volume by the Shell Method

Jiwen He

Test 3

- Test 3: Dec. 4-6 in CASA
- Material - Through 6.3.

Final Exam

- Final Exam: Dec. 14-17 in CASA

Review for Test 3

- Review for Test 3 by the College Success Program.
- Friday, November 21 2:30–3:30pm in the basement of the library by the C-site.

Online Quizzes

- Online Quizzes are available on CourseWare.

Quiz 1

What is today?

a. Monday

b. Wednesday

c. Friday

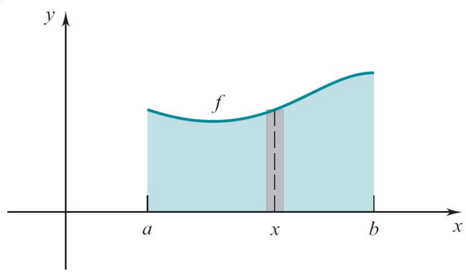
d. None of these

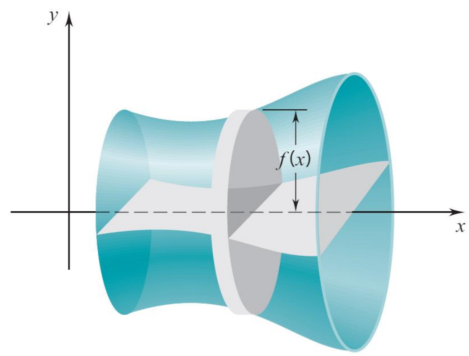
1 Volume by Parallel Cross Section; Discs and Washers

1.1 Solid of Revolution: Disk Method

Solid of Revolution About the x-Axis: Disk

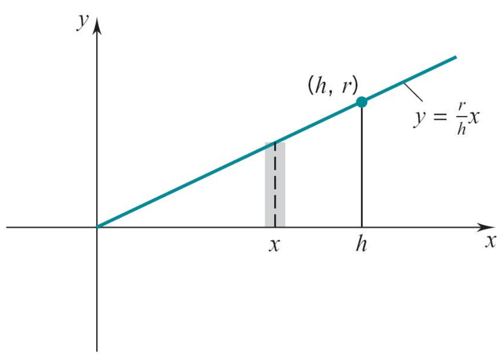
Cylinder Volume: $\pi[f(x_i^*)]^2\Delta x_i$ Riemann Sum: $\sum \pi[f(x_i^*)]^2\Delta x_i$

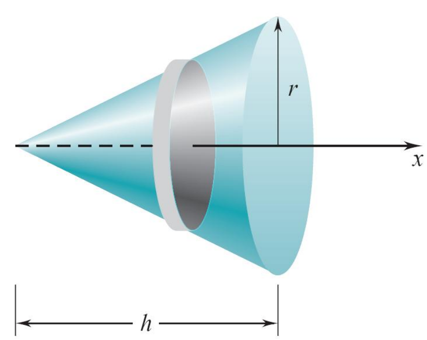




$$V = \int_a^b \pi[f(x)]^2\,dx = \lim_{\|P\|\to 0} \sum \pi[f(x_i^*)]^2\Delta x_i.$$

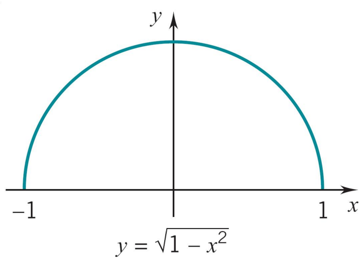
Example
Example 1. Find the volume of the cone shown in the figure below.

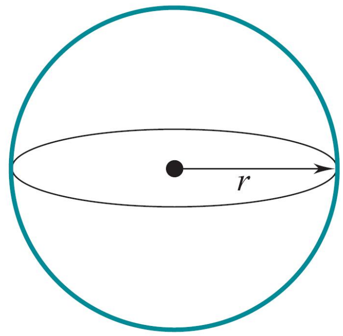




Example
Example 2. Find the volume of a sphere of radius r by revolving about the x-axis the region below the graph of

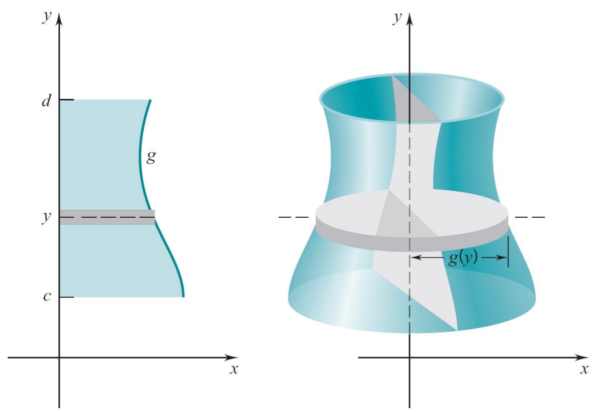
$$f(x) = \sqrt{r^2 - x^2}, \quad -r \le x \le r.$$





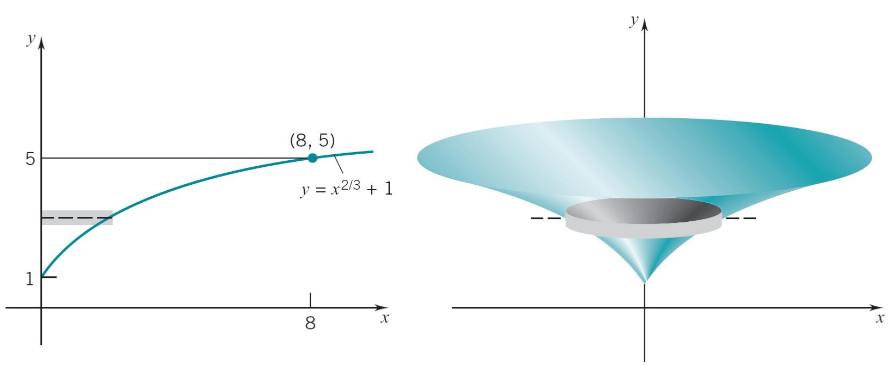
Solid of Revolution About the y-Axis: Disk

Cylinder Volume: $\pi[g(y_i^*)]^2\Delta y_i$ Riemann Sum: $\sum \pi[g(y_i^*)]^2\Delta y_i$



$$V = \int_c^d \pi[g(y)]^2\,dy = \lim_{\|P\|\to 0} \sum \pi[g(y_i^*)]^2 \Delta y_i.$$

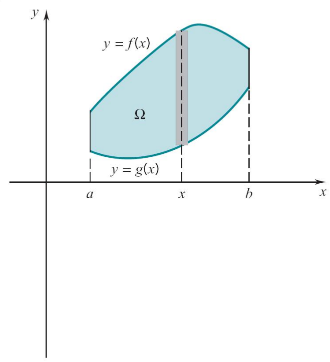
Example
Example 3. Find the volume of the solid shown in the figure below.

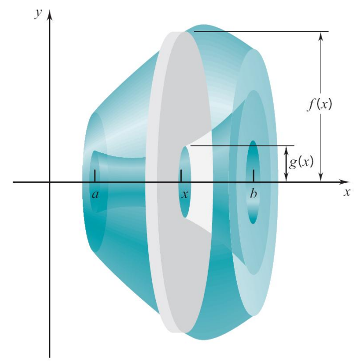


1.2 Solid of Revolution: Washer Method

Solid of Revolution About the x-Axis: Washer

Cylinder Volume: $\pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$ [1ex] Riemann Sum: $\sum \pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$ [1ex]

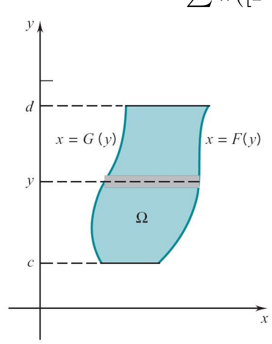


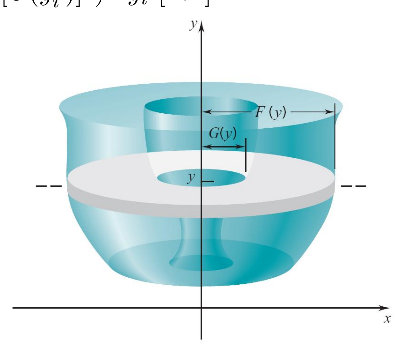


$$V = \int_a^b \pi([f(x)]^2 - [g(x)]^2)\,dx = \lim_{\|P\|\to 0} \sum \pi([f(x_i^*)]^2 - [g(x_i^*)]^2)\Delta x_i$$

Solid of Revolution About the y-Axis: Washer

Cylinder Volume: $\pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$ [1ex] Riemann Sum: $\sum \pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$ [1ex]

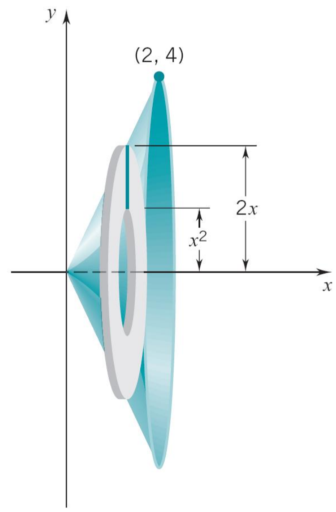




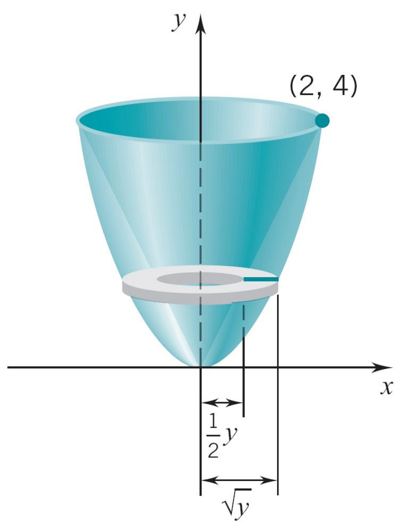
$$V = \int_c^d \pi([F(y)]^2 - [G(y)]^2)\,dy = \lim_{\|P\|\to 0} \sum \pi([F(y_i^*)]^2 - [G(y_i^*)]^2)\Delta y_i$$

Example

Example 4. Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the x-axis.



Example
Example 5. Find the volume of the solid generated by revolving the region between $y = x^2$ and $y = 2x$ about the y-axis.



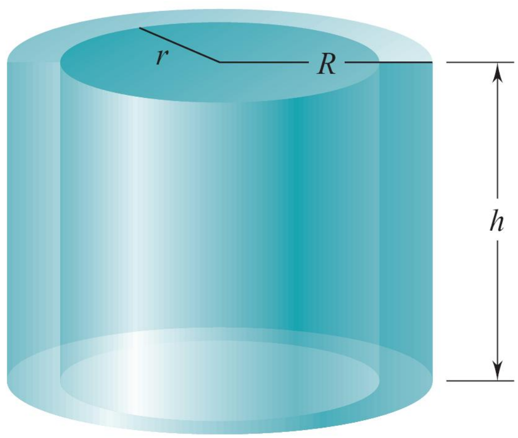
2 Volume by the Shell Method

2.1 Solid of Revolution: Shell Method

Volume of a Cylindrical Shell

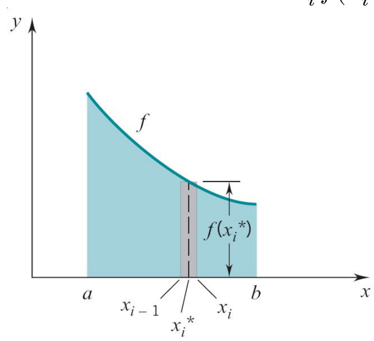
Volume of a Cylindrical Shell

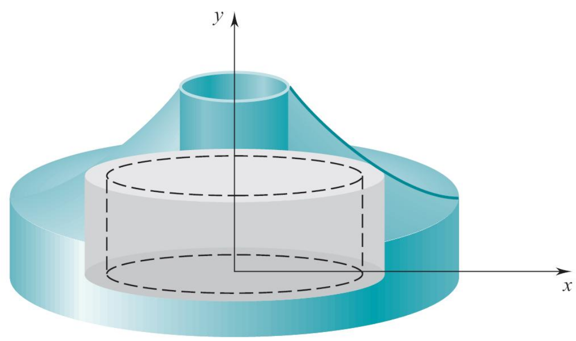
$$V = \pi R^2 h - \pi r^2 h = \pi h(R+r)(R-r).$$



Solid of Revolution About the y-Axis: Shell

Shell Volume: $2\pi x_i^* f(x_i^*)\Delta x_i$ Riemann Sum: $\sum 2\pi x_i^* f(x_i^*)\Delta x_i$

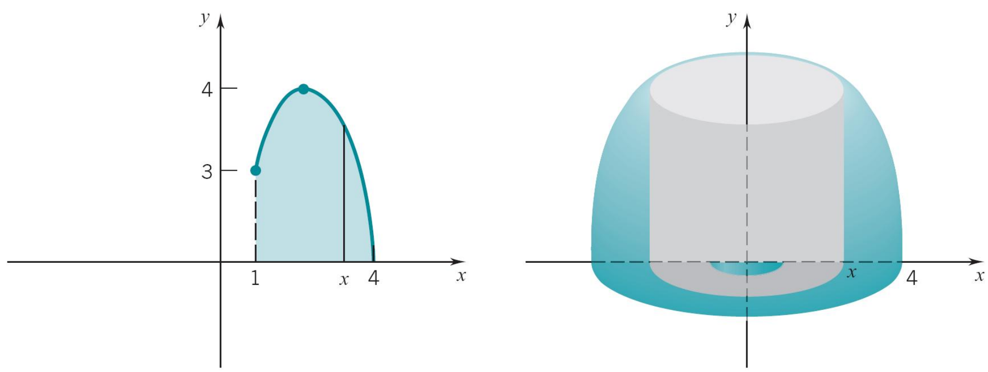




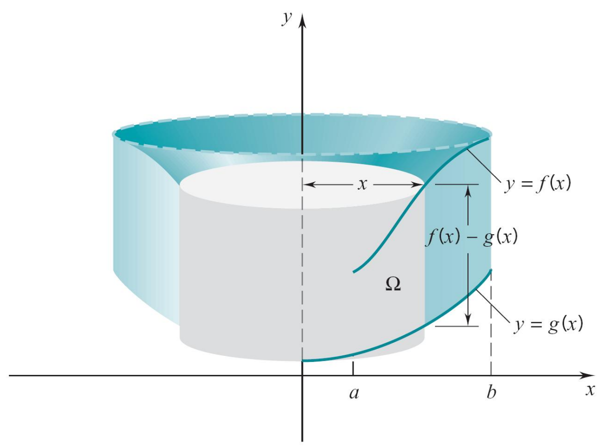
$$V = \int_a^b 2\pi\, x\, f(x)\, dx = \lim_{\|P\|\to 0} \sum 2\pi\, x_i^*\, f(x_i^*)\Delta x_i.$$

The integrand $2\pi\, x\, f(x)$ is the lateral area of the cylinder.

Example
Example 6. Find the volume of the solid generated by revolving about the y-axis the region bounded by $f(x) = 4x - x^2$ and the x-axis between $x = 1$ and $x = 4$.



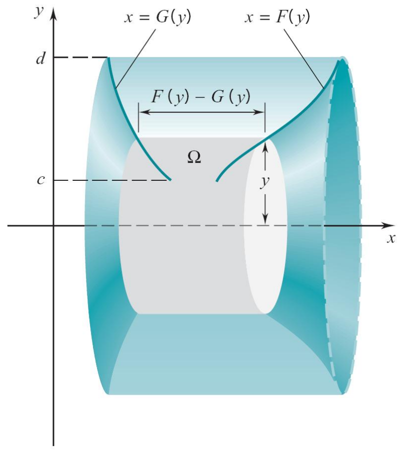
Solid of Revolution About the y-Axis: Shell



$$V = \int_a^b 2\pi\, x\,[f(x) - g(x)]\, dx = \lim_{\|P\|\to 0} \sum 2\pi\, x_i^*\,[f(x_i^*) - g(x_i^*)]\Delta x_i.$$

The integrand $2\pi\, x\,[f(x) - g(x)]$ is the lateral area of the cylinder.

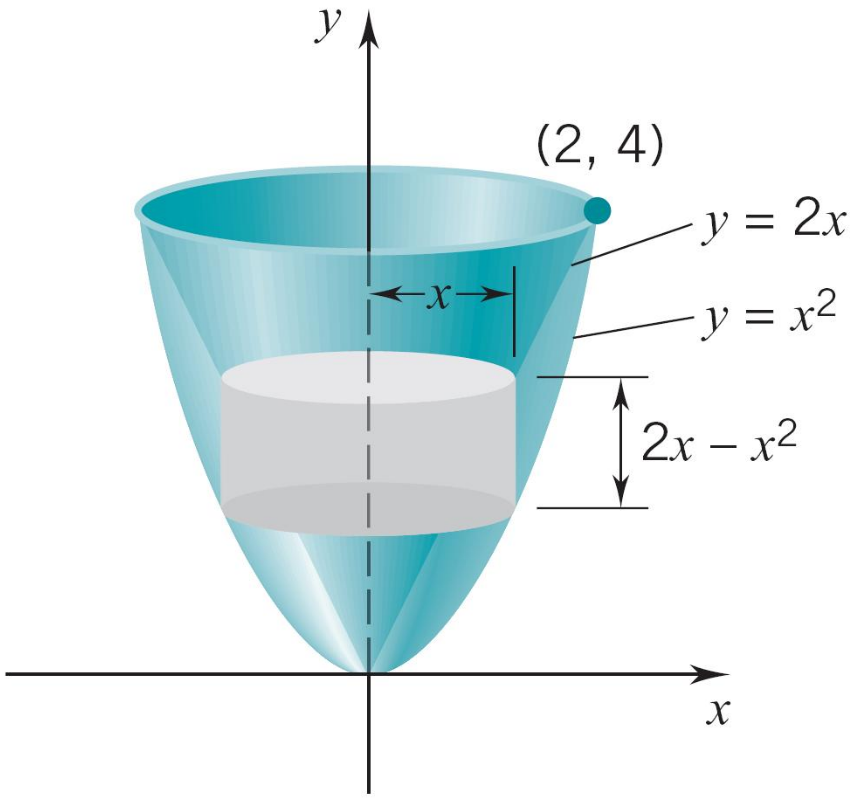
Solid of Revolution About the x-Axis: Shell



$$V = \int_c^d 2\pi\, y\, [F(y) - G(y)]\, dy = \lim_{\|P\| \to 0} \sum 2\pi\, y_i^*\, [F(y_i^*) - G(y_i^*)] \Delta y_i.$$

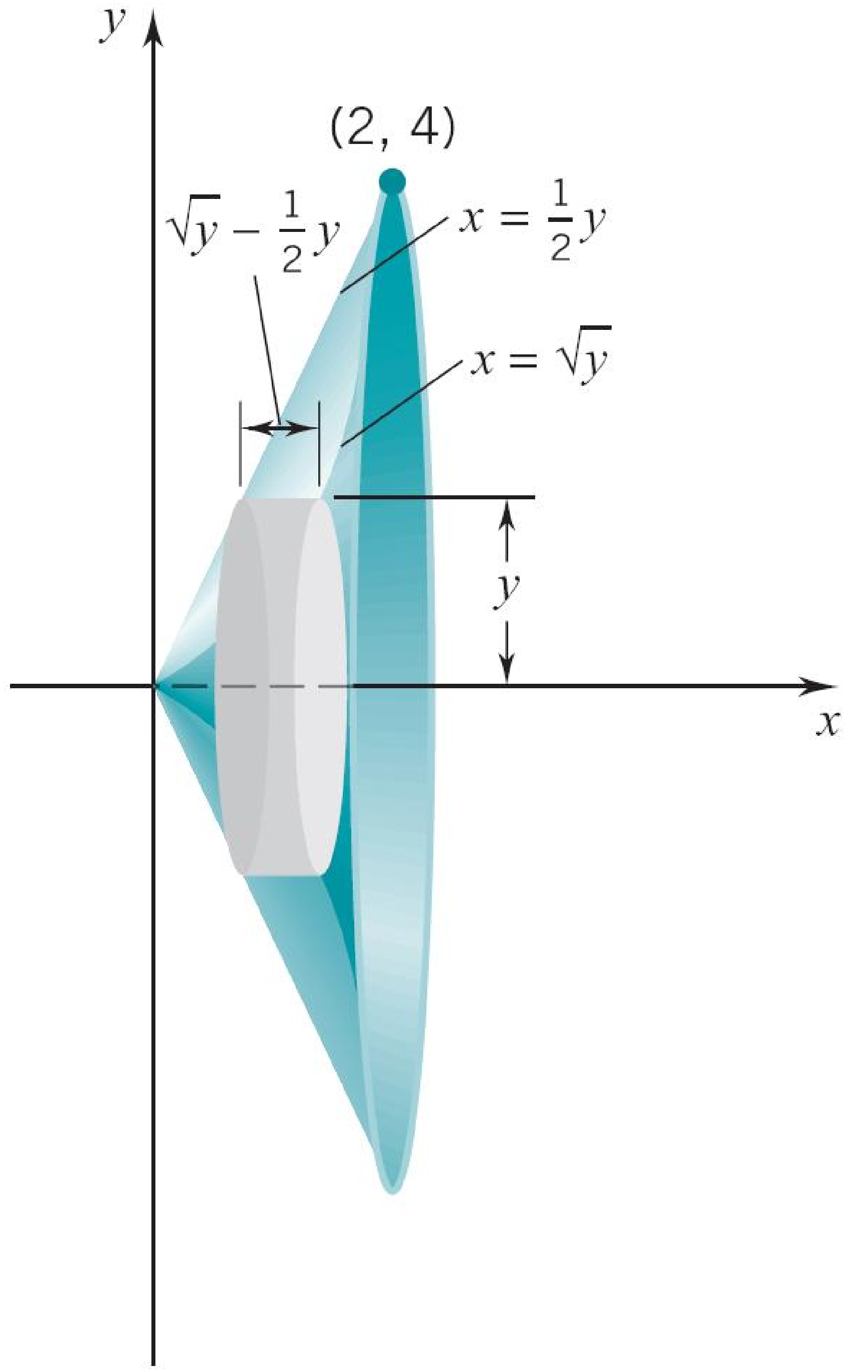
The integrand $2\pi\, y\, [F(y) - G(y)]$ is the lateral area of the cylinder.

Example



Example 7. Find the volume of the solid generated by revolving about the y-axis the region between $y = x^2$ and $y = 2x$.

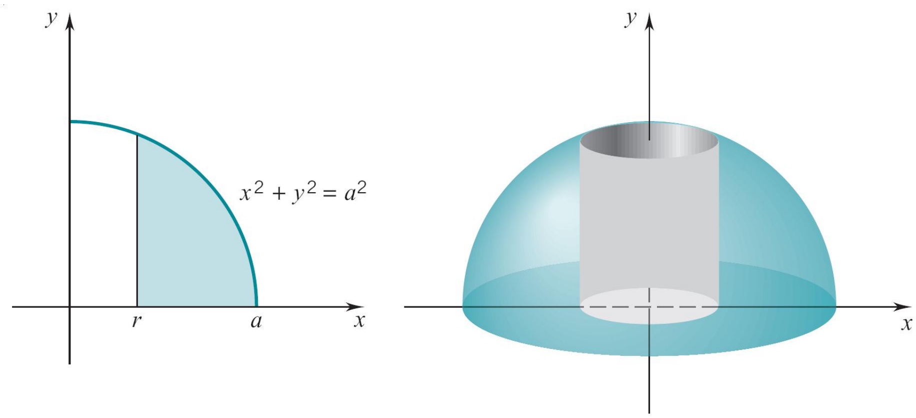
Example



y
(2, 4)
$\sqrt{y} - \frac{1}{2}y$
$x = \frac{1}{2}y$
$x = \sqrt{y}$
y
x

Example 8. Find the volume of the solid generated by revolving about the x-axis the region between $y = x^2$ and $y = 2x$.

Example
Example 9. A round hole of radius r is drilled through the center of a hemisphere of radius a. Find the volume of the potion of the hemisphere that remains.



Example
Example 10. The region Ω between $y = \sqrt{x}$ and $y = x^2$, $0 \leq x \leq 1$, is revolved about the line $x = -2$. Find the volume of the solid that is generated.