Review for Test 3

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Some Important Limits 1-4

$$\lim_{n\to\infty} \frac{1}{n^\alpha} = 0, \quad \alpha > 0.$$

$$\lim_{n\to\infty} x^{\frac{1}{n}} = 1, \quad x > 0.$$

$$\lim_{n\to\infty} x^n = 0 \quad \text{if } |x| < 1.$$

(The limit does not exist if $|x| > 1$ or $x = -1$.)

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L'Hôpital's Rule and Some Important Limits

L'Hôpital's Rule

Suppose that $\lim_{x \to \star} \frac{f(x)}{g(x)}$ is a $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Then $\lim_{x \to \star} \frac{f(x)}{g(x)} = \lim_{x \to \star} \frac{f'(x)}{g'(x)}$

L'Hôpital's rule does not apply in cases where the numerator or the denominator has a finite non-zero limit!!!

$$\lim_{x \to 0^+} x^\alpha \ln x = 0, \quad \alpha > 0.$$

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Exponential Forms: 1^∞, 0^0, and ∞^0

If $\lim_{x\to\star} \ln f(x) = L$, then $\lim_{x\to\star} f(x) = e^L$.

$$\lim_{x\to 0^+} x^x \quad (0^0) \quad = e^0 = 1$$

$$\lim_{x\to\infty} x^{1/x} \quad (\infty^0) \quad = e^0 = 1$$

$$\lim_{x\to 0^+} (1+x)^{1/x} \quad (1^\infty) \quad = e^1 = e$$

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Peculiar Indeterminate Forms

Sometimes the application of L'Hôpital's rule leads to peculair situations.

$$\lim_{x\to\infty} \frac{2x}{\sqrt{1+x^2}} = \lim_{x\to\infty} \frac{2\sqrt{1+x^2}}{x} = \lim_{x\to\infty} \frac{2x}{\sqrt{1+x^2}}$$

Actually it is easy to calculate by algebraic methods.

$$\lim_{x\to\infty} \frac{2x}{\sqrt{1+x^2}} = \lim_{x\to\infty} \frac{2x}{x} \frac{1}{\sqrt{1+\left(\frac{1}{x}\right)^2}}$$

$$= 2 \lim_{x\to\infty} \frac{1}{\sqrt{1+\left(\frac{1}{x}\right)^2}} == 2 \lim_{u\to 0^+} \frac{1}{\sqrt{1+u^2}} = 2$$

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Problem 1

Compute $\lim_{n\to\infty}(2n)^{\left(\frac{3}{n}\right)}$

a) 0

b) $\frac{2}{3}$

c) Doesn't exist

d) 1

e) $\frac{3}{2}$

f) None of the above.

$$\lim_{u\to\infty} u^{\frac{1}{u}} = 1.$$

Set $u = 2n$.

$$\lim_{n\to\infty}(2n)^{\frac{1}{2n}} = \lim_{u\to\infty} u^{\frac{1}{u}} = 1.$$

Since $f(t) = t^6$ is continuous at 1,

$$\lim_{n\to\infty}(2n)^{\left(\frac{3}{n}\right)} = \lim_{n\to\infty}\left[(2n)^{\left(\frac{1}{2n}\right)}\right]^6 = 1$$

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Problem 2

Compute $\lim_{n\to\infty}\left(\frac{\ln n^3}{\sqrt{n}}\right)$

a) 1

b) 2

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d) -1

e) 0

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$$\lim_{n\to\infty}\frac{\ln n}{n^{\alpha}} = 0, \quad \alpha > 0.$$

$$\lim_{n\to\infty}\left(\frac{\ln n^3}{\sqrt{n}}\right) = 3\lim_{n\to\infty}\frac{\ln n}{n^{1/2}} = 0.$$

$$\lim_{x\to\infty}\left(\frac{\ln x^3}{\sqrt{x}}\right) = \lim_{x\to\infty}\left(\frac{\frac{1}{x^3}3x^2}{\frac{1}{2}x^{-1/2}}\right) = \lim_{x\to\infty}\frac{6}{x^{1/2}} = 0$$

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Problem 3

Compute

$$\lim_{x\to\infty}\left(\frac{\sqrt{x^2+3x+1}}{5x-1}\right)$$

a) 1

b) $\frac{2}{5}$

c) -2

d) $\frac{1}{5}$

e) $-\frac{1}{3}$

f) None of the above.

The application of L'Hôpital's rule leads to peculair situations. Actually it is easy to calculate by algebraic methods.

$$\lim_{x\to\infty}\left(\frac{\sqrt{x^2+3x+1}}{5x-1}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x}{5x-1}\sqrt{1+3\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2}\right)$$

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Fundamental Theorem of Integral Calculus

Let $f(x) = F'(x)$ for $\forall x \geq a$. Then

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx = \lim_{b\to\infty}\left(F(x)\Big|_a^b\right) = \lim_{b\to\infty} F(b) - F(a)$$

Let $f(x) = F'(x)$ for $\forall x \leq b$. Then

$$\int_{-\infty}^b f(x)\,dx = F(x)\Big|_{-\infty}^b = F(b) - \lim_{x\to-\infty} F(x)$$

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$$\int_1^\infty \frac{1}{x^3}\,dx = -\frac{2}{x^2}\Big|_1^\infty = \lim_{x\to\infty}\left(-\frac{2}{x^2}\right) - (-2) = 2$$

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Fundamental Theorem of Integral Calculus

Let $f(x) = F'(x)$ for $\forall x \in [a, b)$ and $f(x) \to \pm\infty$ as $x \to b^-$. Then

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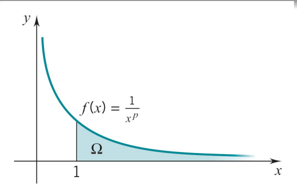
$$\int_0^1 \frac{1}{x}\,dx = \ln x\big|_0^1 = \ln 1 - \lim_{x\to 0^+}(\ln x) = \infty$$

$$\int_0^{\frac{\pi}{2}} \tan x\,dx = \ln\sec x\big|_0^{\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}^-}(\ln\sec x) - \ln 1 = \infty$$

$\int_0^1 \frac{1}{x^\alpha}\,dx,\ \alpha > 0$



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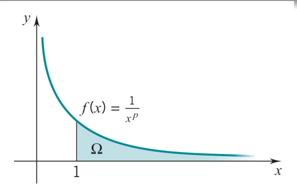

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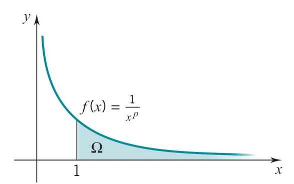

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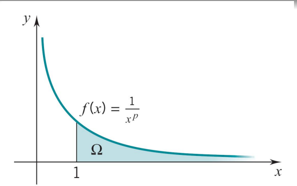

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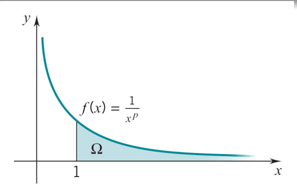

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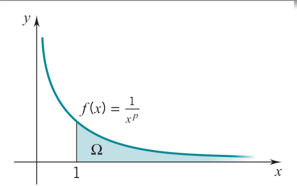

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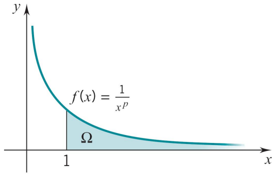

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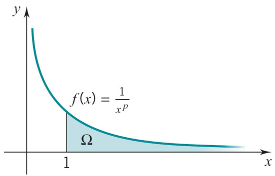

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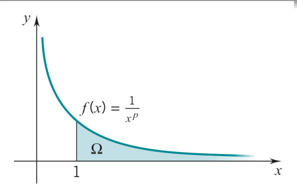

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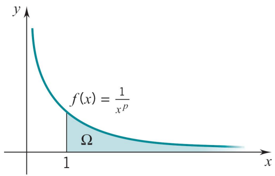

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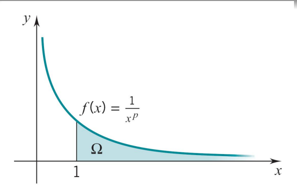

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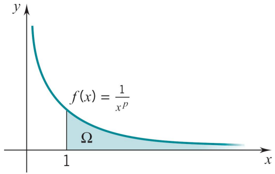

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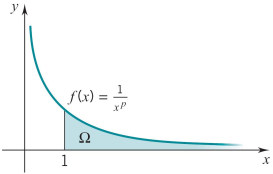

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Problem 1

- Explain why the definite integral below is an improper integral
$$\int_0^1 \frac{1}{(1-x)^3}\,dx$$
- Use proper limit notation to rewrite this improper integral as a limit of proper definite integrals.
- Solve the limit problem formed to determine whether the improper integral converges or diverges. If the improper integral converges, give its value.

$$\int_0^1 \frac{1}{(1-x)^3}\,dx = \int_0^1 \frac{1}{t^3}\,dt = \lim_{c\to 0^+} \int_c^1 \frac{1}{t^3}\,dt$$
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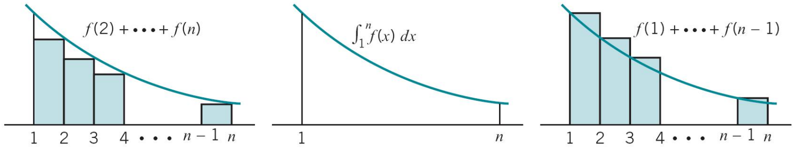
The Integral Test



Let $a_k = f(k)$, where f is continuous, decreasing and positive on $[1, \infty)$, then

$$\sum_{k=1}^{\infty} a_k \text{ converges} \quad \text{iff} \quad \int_1^{\infty} f(x)dx \text{ converges}$$

(The p-Sereis)

$$\sum_{k=1}^{\infty} \frac{1}{k^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots \text{ converges} \quad \text{iff} \quad p > 1.$$

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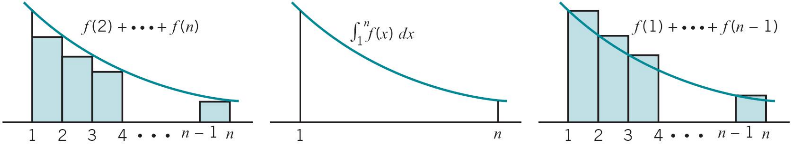

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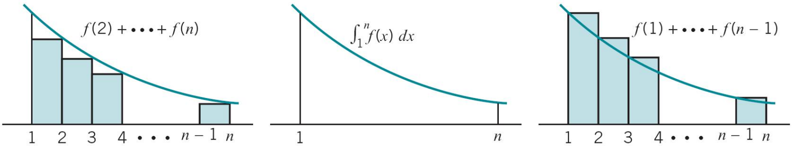

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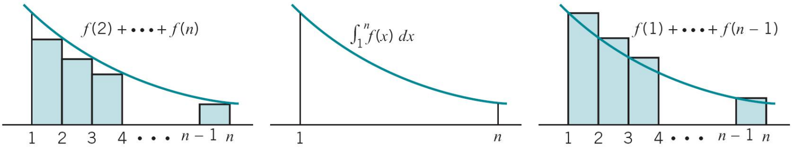

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Example

Show that $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ diverges.

The related improper integral is

$$\int_2^{\infty} \frac{1}{x \ln x}\,dx = \int_{\ln 2}^{\infty} \frac{1}{u}\,du = \ln u\Big|_{\ln 2}^{\infty} = \lim_{x\to\infty} \ln x - \ln\ln 2 = \infty.$$

Since this impproper integral diverges, so does the infinite series.

Show that $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges.

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$$\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \int_{\ln 2}^{\infty} \frac{1}{u^2}\,du = -u^{-1}\Big|_{\ln 2}^{\infty} = \frac{1}{\ln 2} - \lim_{x\to\infty} x^{-1} = \frac{1}{\ln 2}$$

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$$\int_{2}^{\infty} \frac{1}{x \ln x}\, dx = \int_{\ln 2}^{\infty} \frac{1}{u}\, du = \ln u \Big|_{\ln 2}^{\infty} = \lim_{x\to\infty} \ln x - \ln \ln 2 = \infty.$$

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Basic Series that Converge

Geometric series: $\sum x^k$, if $|x| < 1$

p-series: $\sum \frac{1}{k^p}$, if $p > 1$

Basic Series that Diverge

Any series $\sum a_k$ for which $\lim_{k\to\infty} a_k \neq 0$

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Convergence Tests (1)

Basic Test for Convergence

Keep in Mind that,

if $a_k \nrightarrow 0$, then the series $\sum a_k$ diverges;

therefore there is no reason to apply any special convergence test.

Examples

$\sum x^k$ with $|x| \geq 1$ (e.g, $\sum (-1)^k$) diverge since $x^k \nrightarrow 0$.

$\sum \dfrac{k}{k+1}$ diverges since $\frac{k}{k+1} \to 1 \neq 0$.

$\sum \left(1 - \dfrac{1}{k}\right)^k$ diverges since $a_k = \left(1 - \frac{1}{k}\right)^k \to e^{-1} \neq 0$.

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Convergence Tests (2)

Comparison Tests

Rational terms are most easily handled by basic comparison or limit comparison with p-series $\sum 1/k^p$

Basic Comparison Test

$\sum \frac{1}{2k^3+1}$ converges by comparison with $\sum \frac{1}{k^3}$

$\sum \frac{k^3}{k^5+4k^4+7}$ converges by comparison with $\sum \frac{1}{k^2}$

$\sum \frac{1}{k^3-k^2}$ converges by comparison with $\sum \frac{2}{k^3}$

$\sum \frac{1}{3k+1}$ diverges by comparison with $\sum \frac{1}{3(k+1)}$

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$\sum \frac{3k^2 + 2k + 1}{k^3 + 1}$ diverges by comparison with $\sum \frac{3}{k}$

$\sum \frac{5\sqrt{k} + 100}{2k^2\sqrt{k} - 9\sqrt{k}}$ converges by comparison with $\sum \frac{5}{2k^2}$

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Convergence Tests (3)

Root Test and Ratio Test

The root test is used only if powers are involved.

Root Test

$\sum \frac{k^2}{2^k}$ converges: $(a_k)^{1/k} = \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1$

$\sum \frac{1}{(\ln k)^k}$ converges: $(a_k)^{1/k} = \frac{1}{\ln k} \to 0$

$\sum \left(1 - \frac{1}{k}\right)^{k^2}$ converges: $(a_k)^{1/k} = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1}$

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Root Test and Ratio Test

The root test is used only if powers are involved.

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Absolute Convergence

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A series $\sum a_k$ is said to converge absolutely if $\sum |a_k|$ converges.

Alternating p-Series with $p > 1$

$\sum \frac{(-1)^k}{k^p}$, $p > 1$, converge absolutely because $\sum \frac{1}{k^p}$ converges.

Conditional Convergence

A series $\sum a_k$ is said to converge conditionally if $\sum a_k$ converges while $\sum |a_k|$ diverges.

Alternating p-Series with $0 < p \leq 1$

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Examples

Alternating Series Test

Let $\{a_k\}$ be a decreasing sequence of positive numbers.

$$\text{If } a_k \to 0, \quad \text{then } \sum (-1)^k a_k \text{ converges.}$$

$\sum \frac{(-1)^k}{2k+1}$, converge since $f(x) = \frac{1}{2x+1}$ is decreasing, i.e., $f'(x) = -\frac{2}{(2x+1)^2} > 0$ for $\forall x > 0$, and $\lim_{x\to\infty} f(x) = 0$.

$\sum \frac{(-1)^k k}{k^2+10}$, converge since $f(x) = \frac{x}{x^2+10}$ is decreasing, i.e., $f'(x) = -\frac{x^2-10}{(x^2+10)^2} > 0$, for $\forall x > \sqrt{10}$, and $\lim_{x\to\infty} f(x) = 0$.

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$\sum \frac{(-1)^k}{2k+1}$, converge since $f(x) = \frac{1}{2x+1}$ is decreasing, i.e.,
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b) converges absolutely

c) diverges

d) None of the above.

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Outline

- Sections 10.4–10.6
  - Important Limits
  - Indeterminate Forms
  - Review Problems
- Section 10.7 Improper Integrals
  - Improper Integrals
  - Review Problems
- Sections 11.1–11.3
  - The Integral Test
  - Convergence Tests
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  - Review Problems