

Math 1432 - Exam III  
Morgan, Spring 2003

Name: \_\_\_\_\_  
Social Sec.: \_\_\_\_\_

Answer problems 1-12 in the spaces provided below.

Questions (5 points each)	Answers
1. Give the exact value of $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{2^n}$ .	
2. Give the exact value of $\sum_{n=2}^{\infty} \frac{1}{n^2 + n}$ .	
3. Suppose $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n-1}}{(2n)!}$ . Give the 16 <sup>th</sup> derivative of $f$ at $x = 0$ .	
4. Give the 5 <sup>th</sup> degree Taylor polynomial for $f(x) = \cosh(x)$ centered at 0.	
5. Give the Taylor series expansion for $f(x) = \cos(x)$ centered at 0.	
6. Give the value for the improper integral $\int_{-1}^1 \frac{1}{x^2 - 2x + 1} dx$ .	
7. Does $\sum_{n=2}^{\infty} \frac{n^2 + 3n - 2}{4n^4 + n - 1}$ converge or diverge?	
8. Does $\sum_{n=1}^{\infty} \frac{n^2 + 3n - 2}{\sqrt{4n^7 + n - 1}}$ converge or diverge?	
9. Does $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)}$ converge absolutely, converge conditionally, or diverge?	
10. Does $\sum_{n=0}^{\infty} \frac{(-1)^n n!}{3^n}$ converge absolutely, converge conditionally, or diverge?	

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11. $f(1) = -2, f'(1) = 1, f''(1) = -3$ . Give the 2 <sup>nd</sup> degree Taylor polynomial for $f$ centered at 1.	
12. Rewrite $f(x) = -3x^3 + 5x^2 - x + 1$ in powers of $(x + 2)$ .	

**Write the solutions to Problems 13-17 in your blue book. Show all of your work.**

13. (5 points) Give the interval of convergence for the power series  $\sum_{n=1}^{\infty} \frac{x^n}{3n^2 + 1}$ .

14. (10 points) Give a power series representation for each of  $f(x) = \frac{1}{1+x}$ ,  $g(x) = \ln(1+x)$ , and  $h(x) = \frac{1}{(1+x)^2}$ , and give the radius of convergence in each case.

15. (10 points) Give a value of  $n$  so that the Taylor polynomial of degree  $n$  for  $f(x) = \sin(2x)$  centered at 0 can be used to approximate  $f(x)$  within  $10^{-4}$  on the interval  $\left[-\frac{1}{2}, \frac{1}{2}\right]$ .

16. (15 points) Determine whether the following series converge absolutely, converge conditionally or diverge.

a.  $\sum_{n=2}^{\infty} \frac{(-1)^n 3^n n!}{n^n}$

b.  $\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{3^n + 1}$

c.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{5n^2 + 2n - 1}$

d.  $\sum_{n=2}^{\infty} \frac{\cos(\pi n) n^n}{n!}$

e.  $\sum_{n=2}^{\infty} \left( \frac{n}{n+1} \right)^n$

**Blue Version**