Lecture 7

Section 8.2 Integration by Parts

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$$d(uv) = u\,dv + v\,du, \qquad \int u\,dv = uv - \int v\,du$$

Integration by Parts: Undoing the Product Rule

Product Rule

- The product rule:
$$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$$
- In terms of differentials:
$$d(uv) = u\,dv + v\,du$$
- Rearrange:
$$u\,dv = d(uv) - v\,du$$
- Integrate:
$$\int u\,dv = uv - \int v\,du$$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

- Reduction: poly. x^k can be reduced by diff.
- Cycling: $\sin x$, $\cos x$, e^x, $\sinh x$, $\cosh x$, $\cdots$, retain their form after diff. or int.
- Change of Form: $\ln x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cdots$ completely change their form after diff.

Success depends on choosing u and dv so that

$$\int v\,du \quad \text{is easier that} \quad \int u\,dv$$

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Reduction use the fact that poly. x^k can be reduced by diff.

Examples

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$
$$= x\sin x + \cos x + C$$

$u = x$, $dv = \cos x\,dx$, thus $du = dx$, $v = \sin x$

$\int v\,du = \int \sin x\,dx$ is easier than $\int u\,dv = \int x\cos x\,dx$

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Examples

$$\int xe^{-\frac{x}{2}}\,dx = x\left(-2e^{-\frac{x}{2}}\right) - \int \left(-2e^{-\frac{x}{2}}\right) dx$$

$$= -2xe^{-\frac{x}{2}} - 4e^{-\frac{x}{2}} + C$$

$u = x$, $dv = e^{-\frac{x}{2}}\,dx$, thus $du = dx$, $v = -2e^{-\frac{x}{2}}$

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$$\int x^2 \sin x\,dx = x^2(-\cos x) - \int (-\cos x) 2x\,dx$$
$$= -x^2 \cos x + 2\int x \cos x\,dx$$
$$= -x^2 \cos x + 2(x \sin x + \cos x) + C$$

$u = x^2$, $dv = \sin x\,dx$, thus $du = 2x dx$, $v = -\cos x$

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$$\int x^5 \cos x^3\,dx = \frac{1}{3}x^3 \sin x^3 - \int x^2 \sin x^3\,dx$$
$$= \frac{1}{3}x^3 \sin x^3 + \frac{1}{3}\cos x^3 + C$$

$u = x^3$, $dv = x^2 \cos x^3\,dx$, thus $du = 3x^2 dx$, $v = \frac{1}{3}\sin x^3$

$$\int x^2 \sin x^3\,dx = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos x^3 + C$$

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Reduction Formulas

$$\int u\,dv = uv - \int v\,du$$

If $F'(x) = f(x)$, then

$$\int x^n f(x)\,dx = x^n F(x) - n\int x^{n-1}F(x)dx$$

(Set $u = x^n$, $dv = f(x)\,dx$, $du = nx^{n-1}\,dx$, $v = F(x)$)

- With $f(x) = e^{ax}$ and $F(x) = \frac{1}{a}e^{ax}$,

$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx$$

- With $f(x) = \cos(kx)$ and $F(x) = \frac{1}{k}\sin(kx)$,
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$$\int x^n \cos(kx)\,dx = \frac{1}{k}x^n \sin(kx) - \frac{n}{k}\int x^{n-1}\sin(kx)dx$$
- With $f(x) = \sin(kx)$ and $F(x) = -\frac{1}{k}\cos(kx)$,
$$\int x^n \sin(kx)\,dx = -\frac{1}{k}x^n \cos(kx) + \frac{n}{k}\int x^{n-1}\cos(kx)dx$$

Reduction Formulas

$$\int u\,dv = uv - \int v\,du$$

If $F'(x) = f(x)$, then

$$\int x^n f(x)\,dx = x^n F(x) - n\int x^{n-1}F(x)dx$$

(Set $u = x^n$, $dv = f(x)\,dx$, $du = nx^{n-1}\,dx$, $v = F(x)$)

- With $f(x) = e^{ax}$ and $F(x) = \frac{1}{a}e^{ax}$,
$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx$$
- With $f(x) = \cos(kx)$ and $F(x) = \frac{1}{k}\sin(kx)$,
$$\int x^n \cos(kx)\,dx = \frac{1}{k}x^n\sin(kx) - \frac{n}{k}\int x^{n-1}\sin(kx)dx$$
- With $f(x) = \sin(kx)$ and $F(x) = -\frac{1}{k}\cos(kx)$,
$$\int x^n \sin(kx)\,dx = -\frac{1}{k}x^n\cos(kx) + \frac{n}{k}\int x^{n-1}\cos(kx)dx$$

Reduction Formulas

$$\int u\,dv = uv - \int v\,du$$

If $F'(x) = f(x)$, then

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- With $f(x) = \cos(kx)$ and $F(x) = \frac{1}{k}\sin(kx)$,
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$$\int x^n \sin(kx)\,dx = -\frac{1}{k}x^n \cos(kx) + \frac{n}{k}\int x^{n-1}\cos(kx)dx$$

Reduction Formulas

$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx.$$

Examples

$$\begin{aligned}
\int x^3 e^{2x}\,dx &= \frac{1}{2}x^3e^{2x} - \frac{3}{2}\int x^2 e^{2x}\,dx \\
&= \frac{1}{2}x^3e^{2x} - \frac{3}{2}\left(\frac{1}{2}x^2e^{2x} - \int xe^{2x}\,dx\right) \\
&= \frac{1}{2}x^3e^{2x} - \frac{3}{2}\left(\frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\,dx\right)\right) \\
&= \frac{1}{2}x^3e^{2x} - \frac{3}{2}\left(\frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)\right) + C \\
&= \frac{1}{8}e^{2x}\left(4x^3 - 6x^2 + 6x - 3\right) + C
\end{aligned}$$

Reduction Formulas

$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx.$$

Examples

$$\begin{aligned}
\int x^3 e^{2x}\,dx &= \frac{1}{2}x^3 e^{2x} - \frac{3}{2}\int x^2 e^{2x}\,dx \\
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&= \frac{1}{2}x^3 e^{2x} - \frac{3}{2}\left(\frac{1}{2}x^2 e^{2x} - \left(\frac{1}{2}x e^{2x} - \frac{1}{2}\int e^{2x}\,dx\right)\right) \\
&= \frac{1}{2}x^3 e^{2x} - \frac{3}{2}\left(\frac{1}{2}x^2 e^{2x} - \left(\frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}\right)\right) + C \\
&= \frac{1}{8}e^{2x}\left(4x^3 - 6x^2 + 6x - 3\right) + C
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Reduction Formulas

$$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx.$$

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&= \frac{1}{2}x^3e^{2x} - \frac{3}{2}\left(\frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)\right) + C \\
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Reduction Formulas

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\end{aligned}$$

Integration by Parts: Cycling

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Cycling use the fact that functions such as $\sin x$, $\cos x$, e^x, $\sinh x$, $\cosh x$, $\cdots$, retain their form after differentiation or integration.

Example

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$
$$= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$u = e^x$, $dv = \cos x\,dx$, thus $du = e^x dx$, $v = \sin x$

$\int e^x \sin x\,dx$

$u = e^x$, $dv = \sin x\,dx$, thus $du = e^x dx$, $v = -\cos x$

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$u = e^x$, $dv = \cos x\,dx$, thus $du = e^x dx$, $v = \sin x$

$\int e^x \sin x\,dx$ is of the same form as $\int e^x \cos x\,dx$

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$$= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

$u = e^x$, $dv = \cos x\,dx$, thus $du = e^x dx$, $v = \sin x$

$$\int e^x \sin x\,dx = e^x(-\cos x) - \int e^x(-\cos x)\,dx$$

$u = e^x$, $dv = \sin x\,dx$, thus $du = e^x dx$, $v = -\cos x$

Integration by Parts: Cycling

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

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Example

$$\begin{aligned}\int e^x \cos x\,dx &= e^x \sin x - \int e^x \sin x\,dx \\ &= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx\end{aligned}$$

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$\int e^x \sin x\,dx$ is cycling after two integrations by parts.

Integration by Parts: Cycling

Integration by Parts

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Example

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$
$$= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$$

"Solve" the equation for $\int e^x \cos x\,dx$:

$$2\int e^x \cos x\,dx = e^x(\sin x + \cos x) + C$$
$$\int e^x \cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x) + C$$

Integration by Parts: Cycling

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Integration by Parts: Change of Form

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Change of form use the fact that functions $\ln x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cdots$ completely change their form after differentiation.

Examples

$$\int \ln x\,dx = x\ln x - \int x x^{-1}\,dx$$
$$= x\ln x - \int dx = x\ln x - x + C$$

$u = \ln x$, $dv = dx$, thus $du = x^{-1}dx$, $v = x$

$\int v\,du = \int dx$ is easier than $\int u\,dv = \int \ln x\,dx$

Integration by Parts: Change of Form

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$$\int u\,dv = uv - \int v\,du$$

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Integration by Parts: Change of Form

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$$\int u\,dv = uv - \int v\,du$$

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Examples

$$\begin{aligned}\int \ln x\,dx &= x\ln x - \int x x^{-1}\,dx \\ &= x\ln x - \int dx = x\ln x - x + C\end{aligned}$$

$u = \ln x$, $dv = dx$, thus $du = x^{-1}dx$, $v = x$

$\int v\,du = \int dx$ is easier than $\int u\,dv = \int \ln x\,dx$

Integration by Parts: Change of Form

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Change of form use the fact that functions $\ln x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cdots$ completely change their form after differentiation.

Examples

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C$$

$u = \tan^{-1} x$, $dv = dx$, thus $du = \frac{1}{1+x^2}\,dx$, $v = x$

Set $u = 1 + x^2$, $du = 2x\,dx$,

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

Integration by Parts: Change of Form

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Change of form use the fact that functions $\ln x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cdots$ completely change their form after differentiation.

Examples

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$$
$$= x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C$$

$u = \tan^{-1} x$, $dv = dx$, thus $du = \frac{1}{1+x^2}\,dx$, $v = x$

Set $u = 1+x^2$, $du = 2x\,dx$,
$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

Integration by Parts: Change of Form

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Change of form use the fact that functions $\ln x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cdots$ completely change their form after differentiation.

Examples

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C$$

$u = \tan^{-1} x$, $dv = dx$, thus $du = \frac{1}{1+x^2}\,dx$, $v = x$

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$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

Integration by Parts: Change of Form

Integration by Parts

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Change of form use the fact that functions $\ln x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cdots$ completely change their form after differentiation.

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$u = \tan^{-1} x$, $dv = x^2\,dx$, thus $du = \frac{1}{1+x^2}\,dx$, $v = \frac{1}{3}x^3$

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Parts on Definite Integrals

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$$\int_a^b u\,dv = (uv)\Big|_a^b - \int_a^b v\,du$$

Example

$$\int_0^2 x^3\sqrt{4-x^2}\,dx = -\frac{1}{3}x^2(4-x^2)^{\frac{3}{2}}\Big|_0^2 - \frac{1}{3}\int_0^2 (4-x^2)^{\frac{3}{2}}(-2x)\,dx$$

$$= -\frac{1}{3}\cdot\left(-\frac{64}{5}\right) = \frac{64}{15}$$

$u = x^2$, $dv = x\sqrt{4-x^2}\,dx$, thus $du = 2x\,dx$, $v = -\frac{1}{3}(4-x^2)^{\frac{3}{2}}$

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Outline

- Integration by Parts
  - Undoing the Product Rule
  - Reduction
  - Cycling
  - Change of Form
- Parts on Definite Integrals