# Lecture 8

## Section 8.3 Powers and Products of Trigonometric Functions

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$$\int \sin^m x \cos^n x \, dx, \quad \int \tan^n x \, dx, \quad \int \sec^n x \, dx$$

# $\int \sin^m x \cos x\,dx$: $u(=\sin x)$-Substitution

$$\int \sin^m x \cos x\,dx$$

Set $u = \sin x$, $du = \cos x\,dx$

- For $m \neq -1$,

$$\int \sin^m x \cos x\,dx = \int u^m\,du$$

- For $m = -1$,

$$\int \sin^{-1} x \cos x\,dx$$

# $\int \sin^m x \cos x \, dx$: $u (= \sin x)$-Substitution

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$$\int \sin x \cos^n x \, dx$$

Set $u = \cos x$, $du = -\sin x \, dx$

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$$\int \sin x \cos^n x \, dx = -\int u^n \, du$$

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$$\int \sin^m x \cos^n x \, dx, \; n > 1 \text{ odd}$$

Set $u = \sin x, \; du = \cos x \, dx$. Note $\cos^2 x = 1 - \sin^2 x$

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&= \int u^m \left(1 - u^2\right)^{\frac{n-1}{2}} du = \int \text{polynomial}
\end{aligned}$$

Write $\cos^n x = \cos^{n-1} x \cos x$. Write $\cos^{n-1} x = \left(\cos^2 x\right)^{\frac{n-1}{2}}$.
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## Example

$$\int \sin^2 x \cos^5 x \, dx = \int u^2 (1 - u^2)^{\frac{5-1}{2}} \, du$$

$$= \int (u^2 - 2u^4 + u^6)^2 \, du$$

$$= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$$

$$= \sin^3 x \left( \frac{1}{3} - \frac{2}{5}\sin^2 x + \frac{1}{7}\sin^4 x \right) + C$$

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$$\begin{aligned}\int \sin^2 x \cos^5 x\, dx &= \int u^2 (1-u^2)^{\frac{5-1}{2}}\, du \\ &= \int (u^2 - 2u^4 + u^6)^2\, du \\ &= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C \\ &= \sin^3 x\left(\frac{1}{3} - \frac{2}{5}\sin^2 x + \frac{1}{7}\sin^4 x\right) + C\end{aligned}$$

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\int \sin^m x \cos^n x \, dx &= \int \left(\sin^{m-1} x \sin x\right) \cos^n x \, dx \\
&= \int \left(\sin^2 x\right)^{\frac{m-1}{2}} \cos^n x \sin x \, dx \\
&= -\int \left(1 - \cos^2 x\right)^{\frac{m-1}{2}} \cos^n x \, d\cos x \\
&= -\int \left(1 - u^2\right)^{\frac{m-1}{2}} u^n \, du = \int \text{ polynomial}
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Write $\sin^m x = \sin^{m-1} x \cos x$. Write $\sin^{m-1} x = \left(\sin^2 x\right)^{\frac{m-1}{2}}$.
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# $\int \sin^m x \cos^n x \, dx, \; m > 1$ odd: : $u(= \cos x)$-Substitution

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Set $u = \cos x, \; du = -\sin x \, dx$. Note $\sin^2 x = 1 - \cos^2 x$

$$\begin{aligned}
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## Example

$$\int \sin^3 x \cos^2 x \, dx = -\int (1 - u^2)^{\frac{3-1}{2}} u^2 \, du$$
$$= \int (u^4 - u^2) \, du$$
$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$
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## Example

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# Power Reduction

## Power Reduction Formulas

$$\sin x \cos x = \frac{1}{2}\sin 2x, \ \ \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x, \ \ \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

## Examples

$$\int \cos^2 x \, dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

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## Examples

$$\begin{aligned}\int \sin^2 x \cos^2 x \, dx &= \int \left(\sin x \cos x\right)^2 dx \\ &= \frac{1}{4}\int \sin^2 2x \, dx \\ &= \frac{1}{4}\int \left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx \\ &= \frac{1}{8}x - \frac{1}{32}\sin 4x + C\end{aligned}$$

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## Examples

$$\int \sin^2 x \cos^2 x \, dx = \int \Big( \sin x \cos x \Big)^2 \, dx$$

$$= \frac{1}{4} \int \sin^2 2x \, dx$$

$$= \frac{1}{4} \int \Big( \frac{1}{2} - \frac{1}{2} \cos 4x \Big) \, dx$$

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## Examples

$$\begin{aligned}
\int \sin^2 x \cos^2 x \, dx &= \int \Big(\sin x \cos x\Big)^2 dx \\
&= \frac{1}{4}\int \sin^2 2x \, dx \\
&= \frac{1}{4}\int \left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx \\
&= \frac{1}{8}x - \frac{1}{32}\sin 4x + C
\end{aligned}$$

# Reduction by Parts

## Reduction Formulas

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

## Proof.

Set $u = \sin^{n-1} x$, $dv = \sin x\,dx$, $du = (n-1)\sin^{n-2} x\cos x\,dx$, $v = -\cos x$.

$$\int \sin^n x\,dx = \int \sin^{n-1} \sin x, dx$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x\,dx$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx$$

$$-(n-1)\int \sin^n x\,dx$$

Rearrange and solve the equation for $\int \sin^n x\,dx$.

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\end{aligned}$$

Rearrange and solve the equation for $\int \sin^n x\,dx$.

# Reduction by Parts

## Reduction Formulas

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

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$$\begin{aligned}\int \sin^n x\,dx &= \int \sin^{n-1} \sin x, dx\\ &= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x\,dx\\ &= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx\\ &= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx\\ &\quad - (n-1)\int \sin^n x\,dx\end{aligned}$$

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Rearrange and solve the equation for $\int \sin^n x\,dx$.

# Reduction by Parts

## Reduction Formulas

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$
$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

## Examples

$$\begin{aligned}\int \sin^2 x\,dx &= -\frac{1}{2}\sin x\cos x + \frac{1}{2}\int dx\\ &= -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + C\end{aligned}$$

$$\begin{aligned}\int \sin^5 x\,dx &= -\frac{1}{5}\sin^4 x\cos x + \frac{4}{5}\int \sin^3 x\,dx\\ &= -\frac{1}{5}\sin^4 x\cos x + \frac{4}{5}\left(-\frac{1}{3}\sin^2 x\cos x + \frac{2}{3}\int \sin x\,dx\right)\\ &= -\frac{1}{5}\sin^4 x\cos x - \frac{4}{15}\sin^2 x\cos x - \frac{8}{15}\cos x + C\end{aligned}$$

# Reduction by Parts

## Reduction Formulas

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$
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## Examples

$$\int \sin^2 x\,dx = -\frac{1}{2}\sin x\cos x + \frac{1}{2}\int dx = -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + C$$

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# Reduction by Parts

## Reduction Formulas

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

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## Examples

$$\begin{aligned}\int \sin^2 x\,dx &= -\frac{1}{2}\sin x\cos x + \frac{1}{2}\int dx\\ &= -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + C\end{aligned}$$

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## Examples

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# Reduction by Parts

## Reduction Formulas

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$$\begin{aligned}\int \sin^2 x\,dx &= -\frac{1}{2}\sin x\cos x + \frac{1}{2}\int dx \\ &= -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + C\end{aligned}$$

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$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$
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# Products of Sine and Cosine

## Product-to-Sum Formulas

$$\sin A \cos B = \frac{1}{2}(\sin(A - B) + \sin(A + B))$$
$$\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$$
$$\cos A \cos B = \frac{1}{2}(\cos(A - B) + \cos(A + B))$$

## Example

$$\int \sin 5x \sin 2x \, dx = \int \frac{1}{2}(\cos(5x - 2x) - \cos(5x + 2x)) \, dx$$
$$= \frac{1}{2}\int (\cos 3x - \cos 7x) \, dx$$
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# Summary

## Summary

$$\int \sin^m x \cos^n x \, dx$$

- Odd power of sine: substitute $u = \cos x$, $du = -\sin x \, dx$
- Odd power of cosine: substitute $u = \sin x$, $du = \cos x \, dx$
- Both powers even: Reduce powers

$$\sin x \cos x = \frac{1}{2} \sin 2x, \quad \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x, \quad \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$$

or integration by parts

- Remember the formulas

$$\cos^2 x + \sin^2 x = 1$$
$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
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# Powers of Tangent

## Reduction Formulas

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

## Proof.

Idea: pair a power of $\tan x$ with $\sec^2 x$ to have $\int u^k\,du$ with $u = \tan x$, $du = \sec^2 x\,dx$. Note $\tan^2 x = \sec^2 x - 1$.

$$\begin{aligned}\tan^n x &= \tan^{n-2} x \tan^2 x = \tan^{n-2} x(\sec^2 x - 1)\\ &= \tan^{n-2} x \sec^2 x - \tan^{n-2} x\end{aligned}$$

$$\begin{aligned}\int \tan^n x\,dx &= \int \tan^{n-2} x \sec^2 x\,dx - \int \tan^{n-2} x\,dx\\ &= \int u^{n-2}\,du - \int \tan^{n-2} x\,dx\\ &= \frac{1}{n-1}u^{n-1} - \int \tan^{n-2} x\,dx\\ &= \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx\end{aligned}$$

# Powers of Tangent

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## Proof.

Idea: pair a power of $\tan x$ with $\sec^2 x$ to have $\int u^k\,du$ with $u = \tan x$, $du = \sec^2 x\,dx$. Note $\tan^2 x = \sec^2 x - 1$.

$$\begin{aligned}\tan^n x &= \tan^{n-2} x \tan^2 x = \tan^{n-2} x(\sec^2 x - 1)\\ &= \tan^{n-2} x \sec^2 x - \tan^{n-2} x\end{aligned}$$

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# Powers of Tangent

## Reduction Formulas

$$\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx$$

## Proof.

Idea: pair a power of $\tan x$ with $\sec^2 x$ to have $\int u^k \, du$ with $u = \tan x$, $du = \sec^2 x \, dx$. Note $\tan^2 x = \sec^2 x - 1$.

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# Powers of Tangent

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# Powers of Tangent

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$$= \int u^{n-2}\,du - \int \tan^{n-2}x\,dx$$
$$= \frac{1}{n-1}u^{n-1} - \int \tan^{n-2}x\,dx$$
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# Powers of Tangent

## Reduction Formulas

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&= \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x\,dx
\end{aligned}$$

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\end{aligned}$$

# Powers of Tangent

## Reduction Formulas

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$

## Example

$$\begin{aligned}\int \tan^6 x\,dx &= \frac{1}{5}\tan^5 x - \int \tan^4 x\,dx\\ &= \frac{1}{5}\tan^5 x - \left(\frac{1}{3}\tan^3 x - \int \tan^2 x\,dx\right)\\ &= \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \left(\tan x - \int dx\right)\\ &= \frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x + C\end{aligned}$$

# Powers of Tangent

## Reduction Formulas

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# Powers of Secant

## Reduction Formulas

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

## Proof.

Idea: integration by parts with $u = \sec^{n-2} x$, $dv = \sec^2 x\,dx$, $du = (n-2)\sec^{n-2} x \tan x\,dx$, $v = \tan x$.

$$\begin{aligned}\int \sec^n x\,dx &= \int \sec^{n-2} x \sec^2 x\,dx \\ &= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x \tan^2 x\,dx \\ &= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x(\sec^2 x - 1)\,dx \\ &= \sec^{n-2} x \tan x - (n-2)\int \sec^n x\,dx \\ &\quad + (n-2)\int \sec^{n-2} x\,dx\end{aligned}$$

# Powers of Secant

## Reduction Formulas

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# Powers of Secant

## Reduction Formulas

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

## Example

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\int \sec x\,dx$$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln\left|\sec x + \tan x\right| + C$$

# Powers of Secant

## Reduction Formulas

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

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# Powers of Secant

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## Example

$$\begin{aligned}\int \sec^3 x\,dx &= \frac{1}{2}\sec x \tan x + \frac{1}{2}\int \sec x\,dx \\ &= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C\end{aligned}$$

# Powers of Tangent and Secant

## Summary

$$\int \tan^m x \sec^n x \, dx$$

- Even power of secant: substitute $u = \tan x$, $du = \sec^2 x \, dx$
- Odd power of tangent: substitute $u = \sec x$, $du = \sec x \tan x \, dx$
- Odd power of secant and even power of tangent: use $\tan^2 x = \sec^2 x - 1$ and integrate in terms of $\sec x$

# Powers of Tangent and Secant

## Summary

$$\int \tan^m x \sec^n x \, dx$$

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# Powers of Tangent and Secant

## Summary

$$\int \tan^m x \sec^n x \, dx$$

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# Powers of Tangent and Secant

## Summary

$$\int \tan^m x \sec^n x \, dx$$

- Even power of secant: substitute $u = \tan x$, $du = \sec^2 x \, dx$
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# Outline

- Sine and Cosine
  - Odd Powers
  - Even Powers
  - Powers of Sine or Cosine
  - Products of Sine and Cosine
  - Summary
- Other Trigonometric Powers
  - Powers of Tangent or Secant
  - Powers of Tangent and Secant