Lecture 12

Section 9.3 Polar Coordinates
Section 9.4 Graphing in Polar Coordinates

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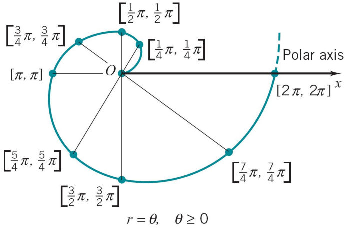
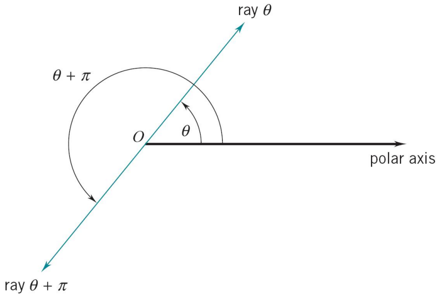


spiral of Archimedes

Polar Coordinate System



The purpose of the polar coordinates is to represent curves that have symmetry about a point or spiral about a point.

Frame of Reference

In the polar coordinate system, the frame of reference is a point O that we call the pole and a ray that emanates from it that we call the polar axis.

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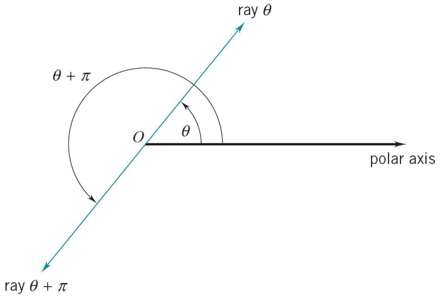

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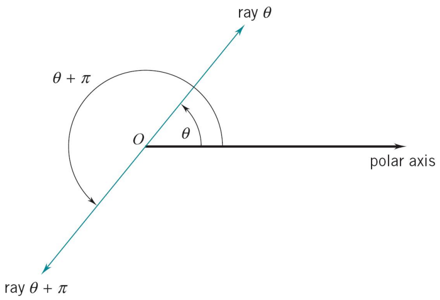

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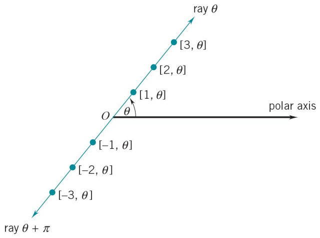
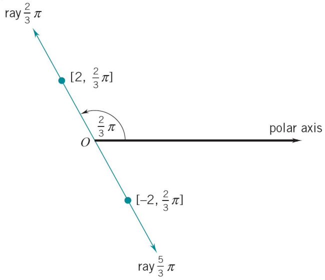


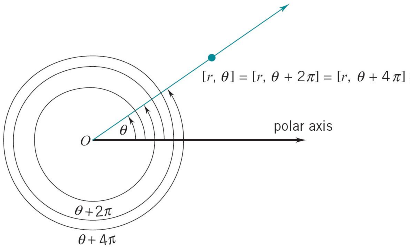


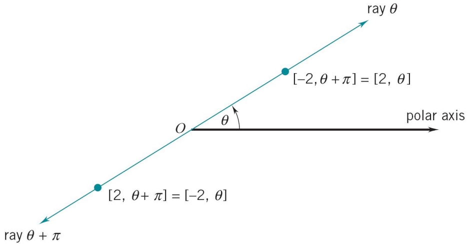
Definition

A point is given polar coordinates $[r, \theta]$ iff it lies at a distance $|r|$ from the pole

a long the ray θ, if $r \geq 0$, and along the ray $\theta + \pi$, if $r < 0$.

Points in Polar Coordinates





Points in Polar Coordinates

- $O = [0, \theta]$ for all θ.
- $[r, \theta] = [r, \theta + 2n\pi]$ for all integers n.
- $[r, -\theta] = [r, \theta + \pi]$.

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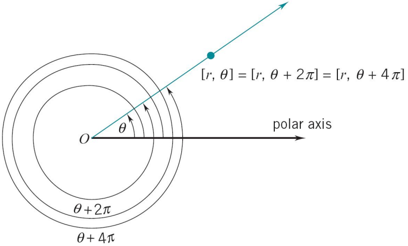

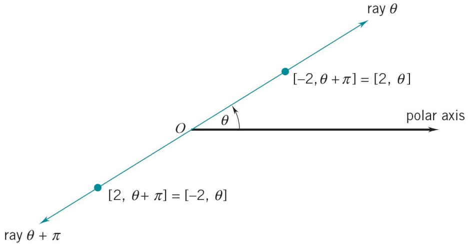

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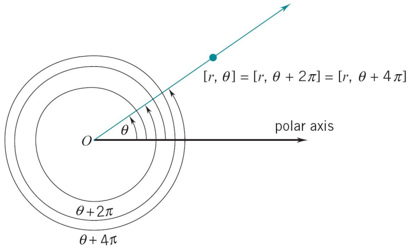

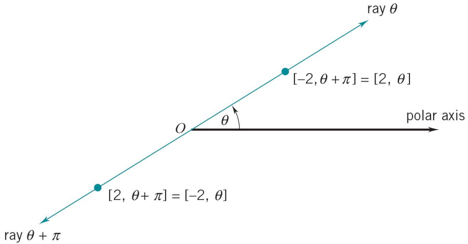

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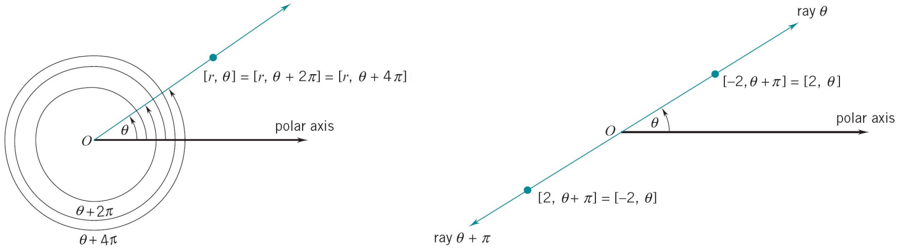

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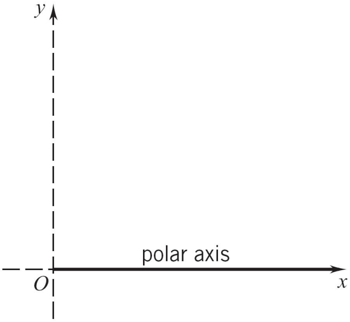

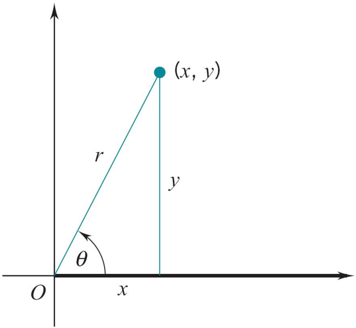

Relation to Rectangular Coordinates

- $x = r\cos\theta,\ y = r\sin\theta. \quad \Rightarrow \quad x^2 + y^2 = r^2, \quad \tan\theta = \dfrac{y}{x}$
- $r = \sqrt{x^2 + y^2},\ \theta = \tan^{-1}\dfrac{y}{x}.$

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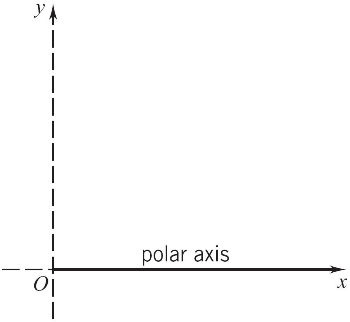

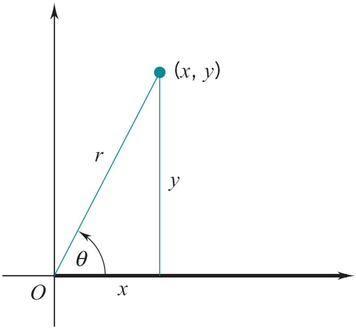

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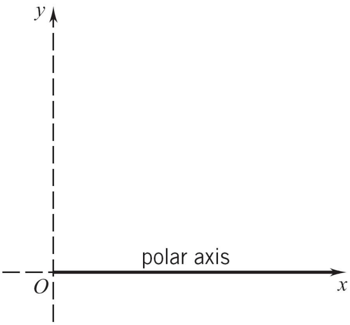

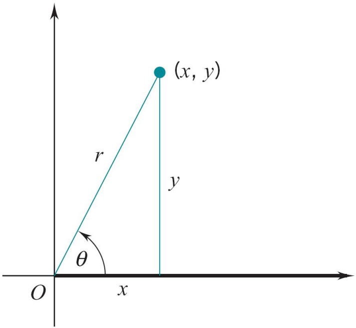

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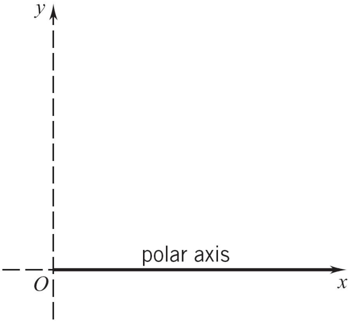

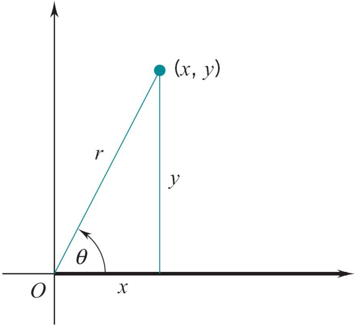

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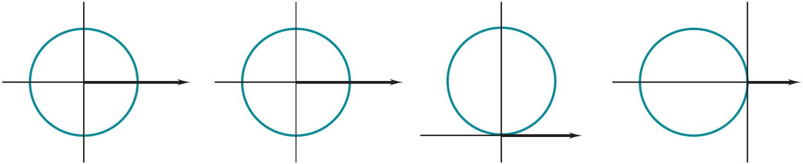
Circles in Polar Coordinates



$r = 2$ $\qquad$ $r = -2$ $\qquad$ $r = 4\sin\theta$ $\qquad$ $r = -4\cos\theta$

Circles in Polar Coordinates

In rectangular coordinates	In polar coordinates
$x^2 + y^2 = a^2$	$r = a$
$x^2 + (y - a)^2 = a^2$	$r = 2a\sin\theta$
$(x - a)^2 + y^2 = a^2$	$r = 2a\cos\theta$

$$x^2 + y^2 = a^2 \quad \Rightarrow \quad r^2 = a^2$$

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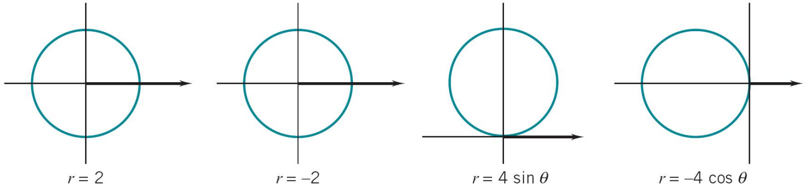

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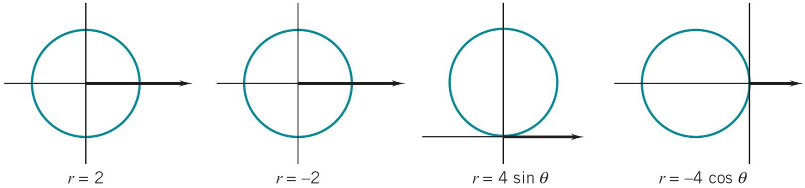

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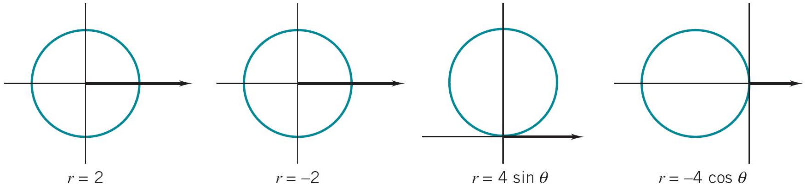

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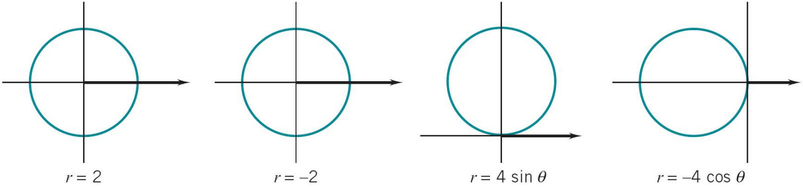

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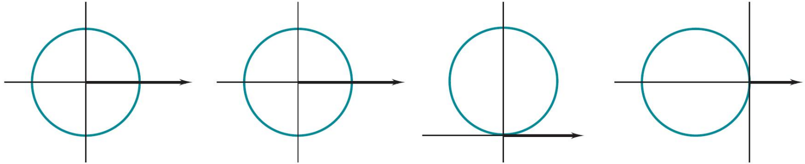

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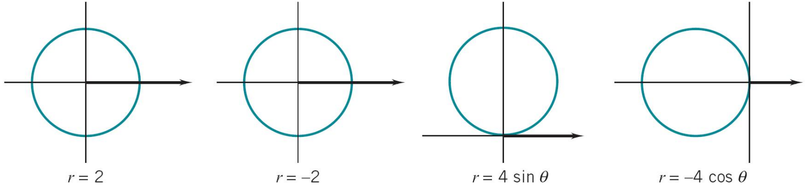

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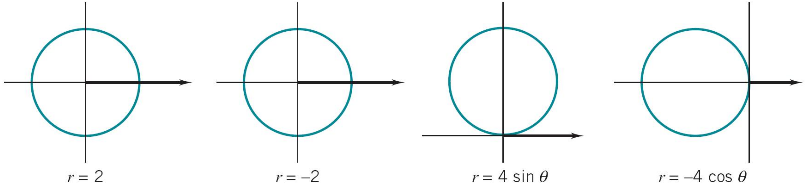

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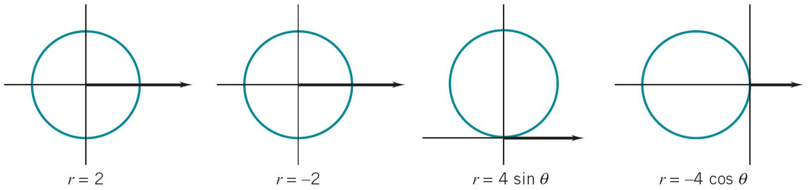

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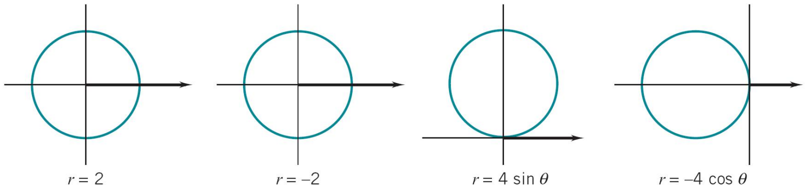

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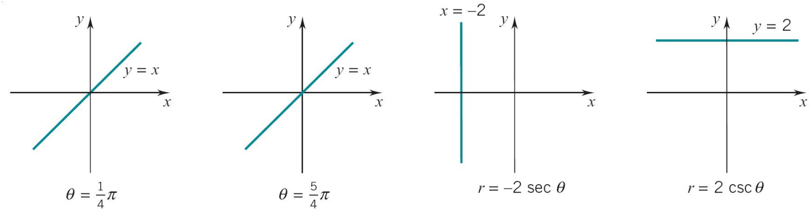

Lines in Polar Coordinates

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$x = a$	$r = a \sec\theta$
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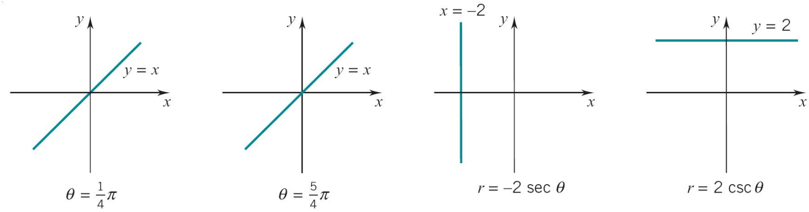

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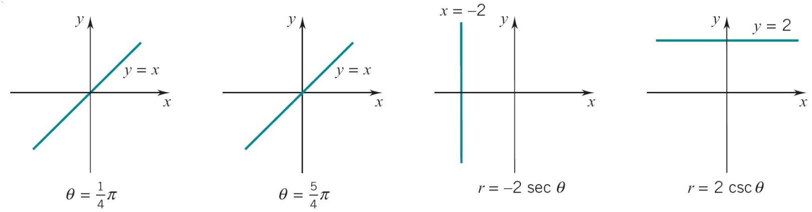

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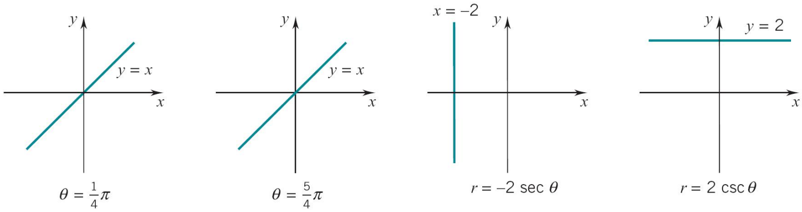

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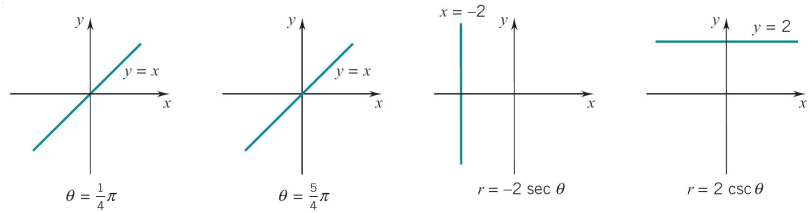

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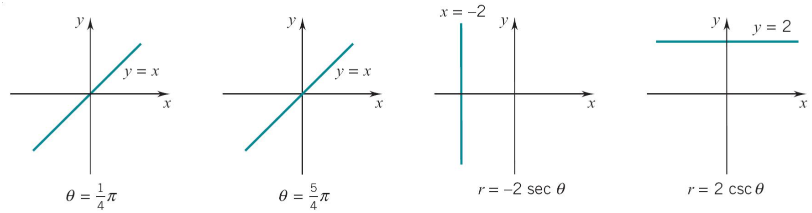

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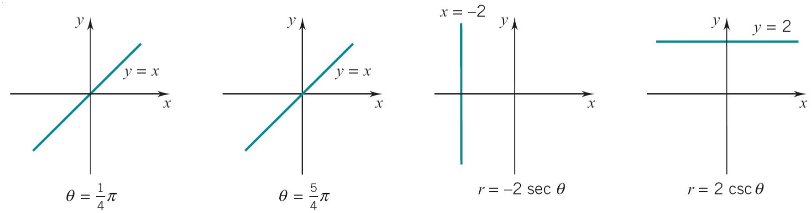

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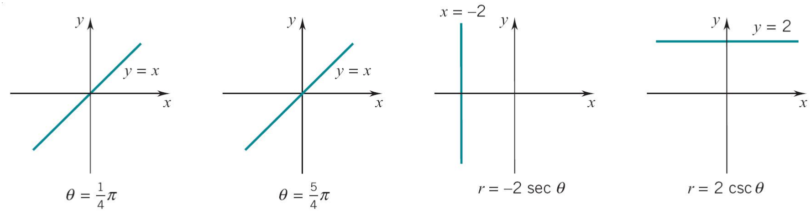

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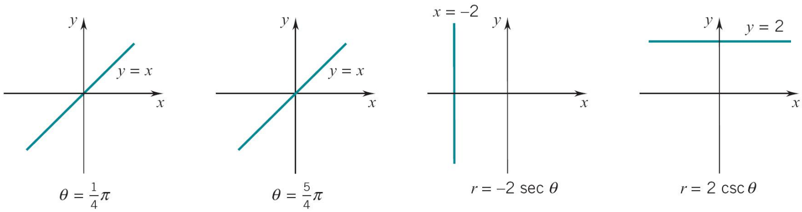

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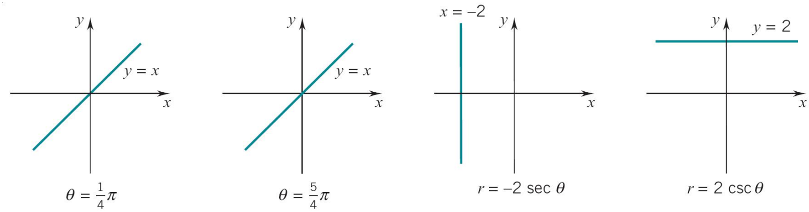

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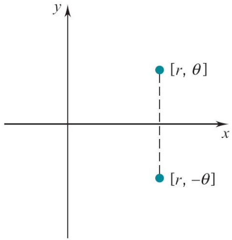
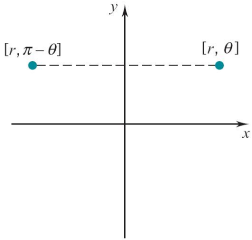
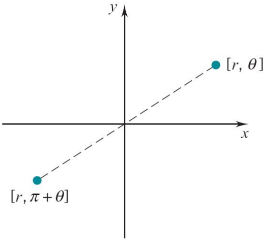
Symmetry



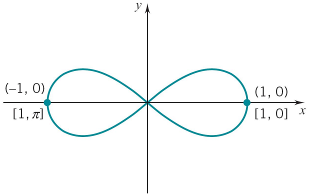
symmetry about the x-axis



symmetry about the y-axis



symmetry about the origin



$r^2 = \cos 2\theta$
lemniscate

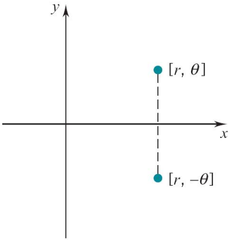
Lemniscate (ribbon) $r^2 = \cos 2\theta$

$\cos[2(-\theta)] = \cos(-2\theta) = \cos 2\theta$

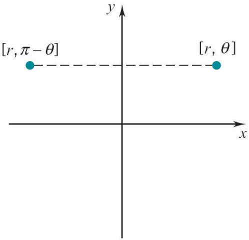
$\Rightarrow$ if $[r, \theta] \in$ graph, then $[r, -\theta] \in$ graph

$\Rightarrow$ symmetric about the x-axis.

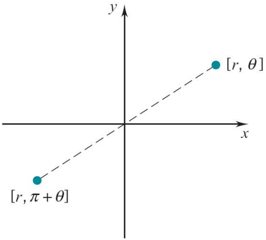
Symmetry



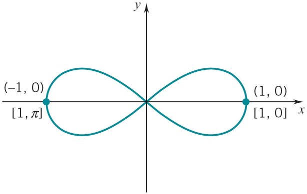
symmetry about the x-axis



symmetry about the y-axis



symmetry about the origin



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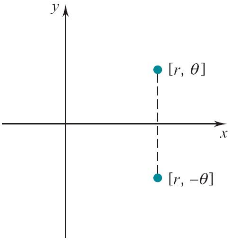
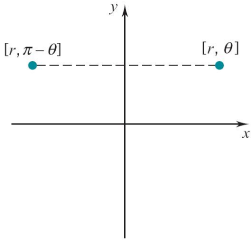
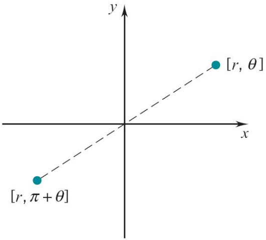


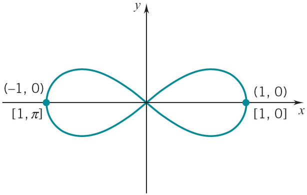
symmetry about the x-axis



symmetry about the y-axis



symmetry about the origin



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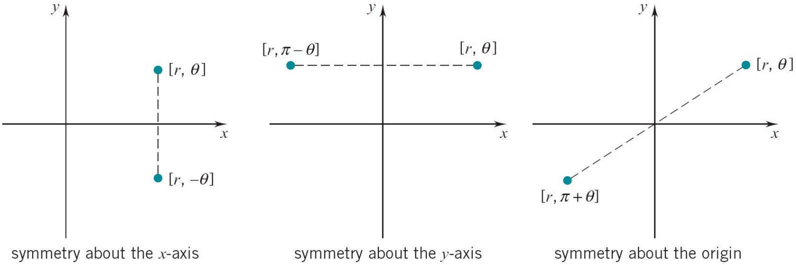

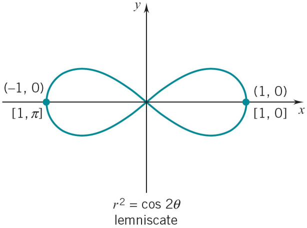

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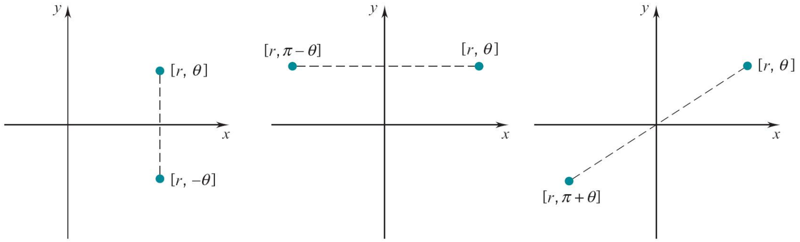
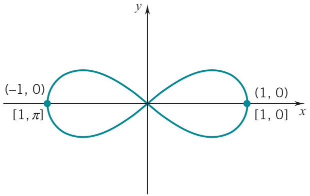
Symmetry



symmetry about the x-axis　　symmetry about the y-axis　　symmetry about the origin



$r^2 = \cos 2\theta$
lemniscate

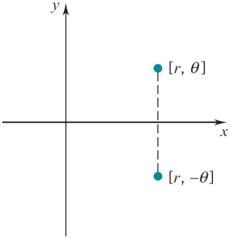
Lemniscate (ribbon) $r^2 = \cos 2\theta$

$$\cos[2(\pi - \theta)] = \cos(2\pi - 2\theta) = \cos 2\theta$$

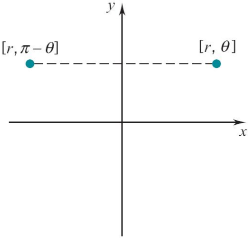
$\Rightarrow$ if $[r, \theta] \in$ graph, then $[r, \pi - \theta] \in$ graph

$\Rightarrow$ symmetric about the y-axis.

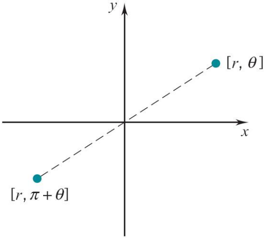
Symmetry



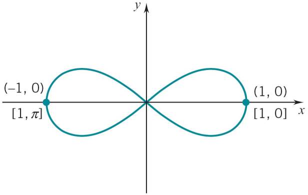
symmetry about the x-axis



symmetry about the y-axis



symmetry about the origin



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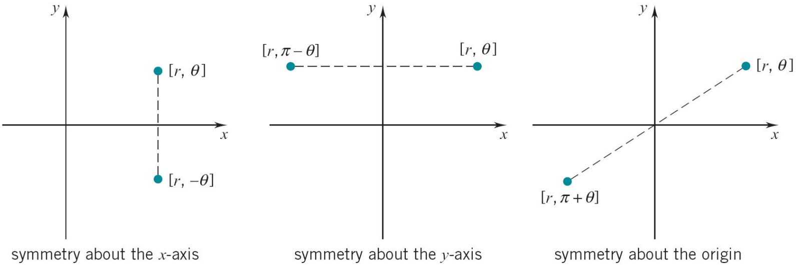
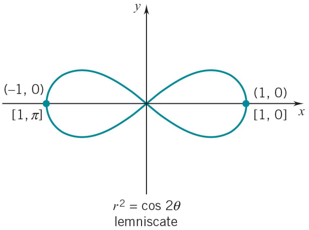
Symmetry





Lemniscate (ribbon) $r^2 = \cos 2\theta$

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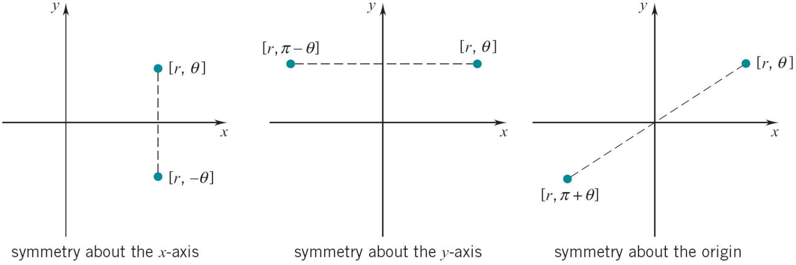
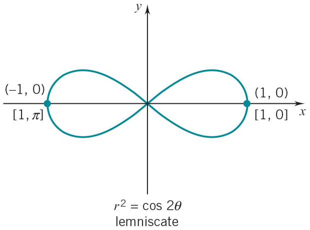




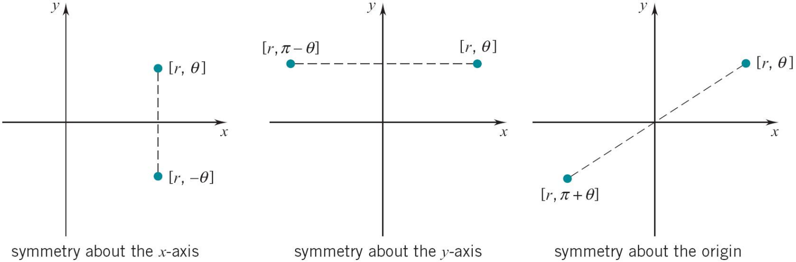
Lemniscate (ribbon) $r^2 = \cos 2\theta$

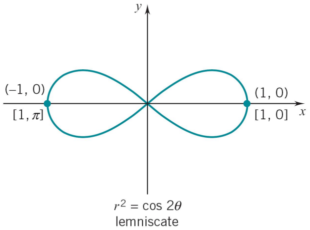
$\cos[2(\pi + \theta)] = \cos(2\pi + 2\theta) = \cos 2\theta$

$\Rightarrow$ if $[r, \theta] \in$ graph, then $[r, \pi + \theta] \in$ graph

$\Rightarrow$ symmetric about the origin.

Symmetry





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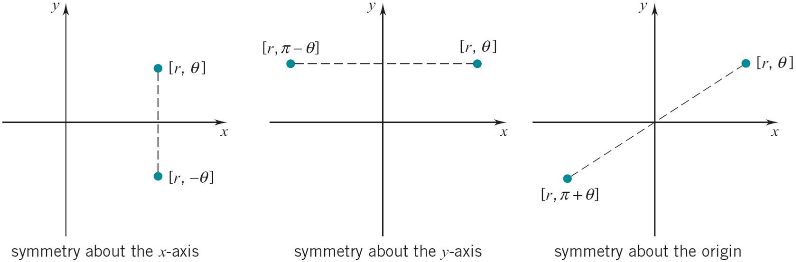
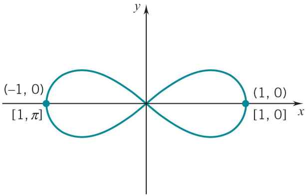
Symmetry





$r^2 = \cos 2\theta$
lemniscate

Lemniscate (ribbon) $r^2 = \cos 2\theta$

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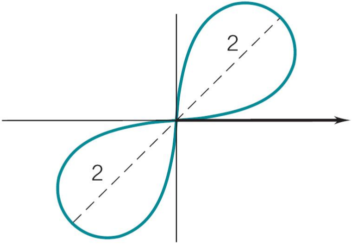
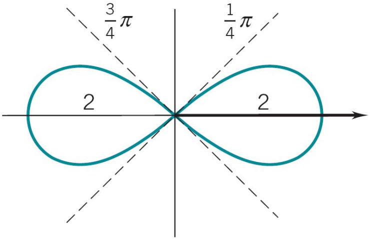
$\Rightarrow$ if $[r, \theta] \in$ graph, then $[r, \pi + \theta] \in$ graph

$\Rightarrow$ symmetric about the origin.

Lemniscates (Ribbons) $r^2 = a \sin 2\theta$, $r^2 = a \cos 2\theta$



$r^2 = 4 \sin 2\theta$



$r^2 = 4 \cos 2\theta$

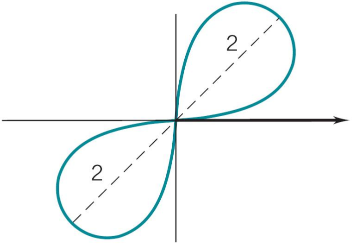
Lemniscate $r^2 = a \sin 2\theta$

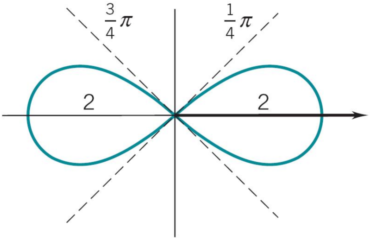
$\sin[2(\pi + \theta)] = \sin(2\pi + 2\theta) = \sin 2\theta$

$\Rightarrow$ if $[r, \theta] \in$ graph, then $[r, \pi + \theta] \in$ graph

$\Rightarrow$ symmetric about the origin.

Lemniscates (Ribbons) $r^2 = a\sin 2\theta$, $r^2 = a\cos 2\theta$





Lemniscate $r^2 = a\sin 2\theta$

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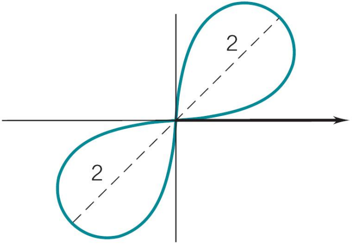
Lemniscates (Ribbons) $r^2 = a\sin 2\theta$, $r^2 = a\cos 2\theta$



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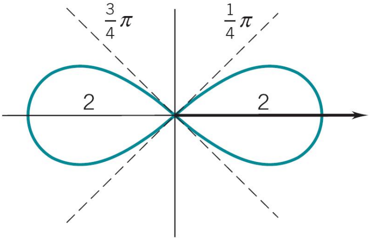

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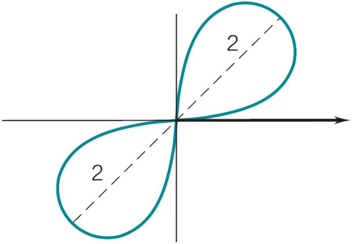
Lemniscates (Ribbons) $r^2 = a\sin 2\theta$, $r^2 = a\cos 2\theta$



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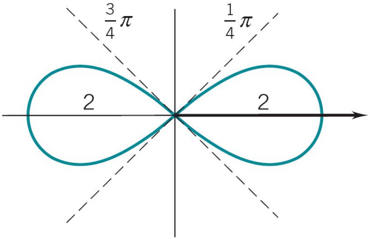

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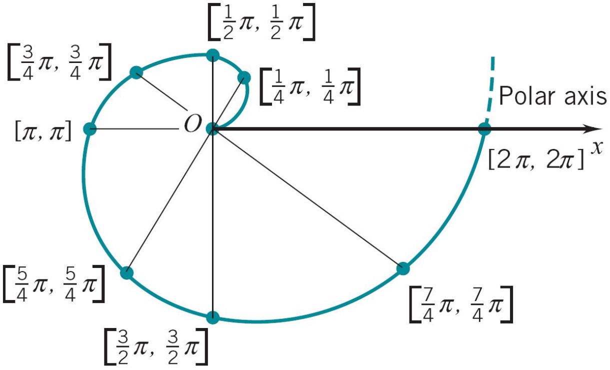
$\Rightarrow$ symmetric about the origin.

Quiz

Quiz

1. $r = \sec\theta$ is sym. about (a) x-axis, (b) y-axis, (c) origin.
2. $r = 2\sin\theta$ is a (a) line, (b) circle, (c) lemniscate.

Spiral of Archimedes $r = \theta$, $\theta \geq 0$

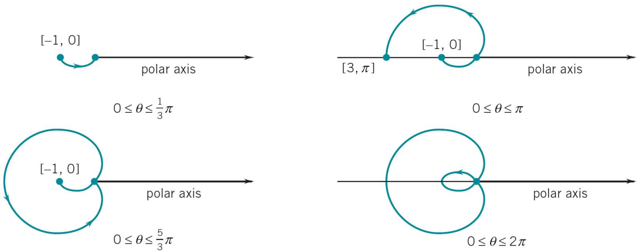


$r = \theta, \quad \theta \geq 0$

spiral of Archimedes

The curve is a nonending spiral. Here it is shown in detail from $\theta = 0$ to $\theta = 2\pi$.

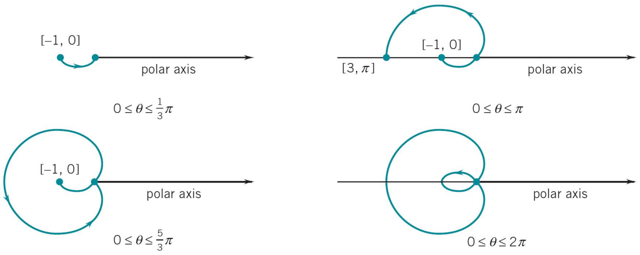
Limaçon (Snail): $r = 1 - 2\cos\theta$



θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	2π
r	-1	-0.41	0	1	2	2.41	3	2.41	2	1	0	-0.41	-1

- $r = 0$ at $\theta = \frac{1}{3}\pi, \frac{5}{3}\pi$; $|r|$ is a local maximum at $\theta = 0, \pi, 2\pi$.
- Sketch in 4 stages: $[0, \frac{1}{3}\pi]$, $[\frac{1}{3}\pi, \pi]$, $[\pi, \frac{5}{3}\pi]$, $[\frac{5}{3}\pi, 2\pi]$.
- $\cos(-\theta) = \cos\theta \Rightarrow$ if $[r, \theta] \in$ graph, then $[r, -\theta] \in$ graph $\Rightarrow$ symmetric about the x-axis.

Limaçon (Snail): $r = 1 - 2\cos\theta$



θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	2π
r	-1	-0.41	0	1	2	2.41	3	2.41	2	1	0	-0.41	-1

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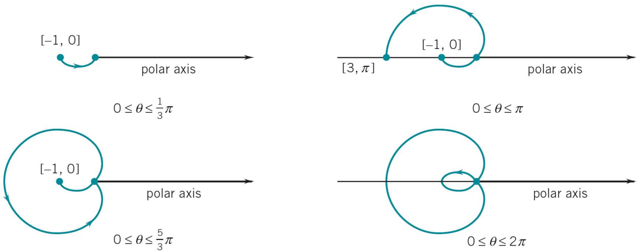
Limaçon (Snail): $r = 1 - 2\cos\theta$



θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	2π
r	-1	-0.41	0	1	2	2.41	3	2.41	2	1	0	-0.41	-1

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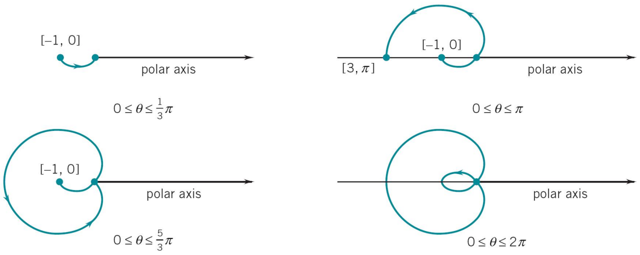
Limaçon (Snail): $r = 1 - 2\cos\theta$



θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	2π
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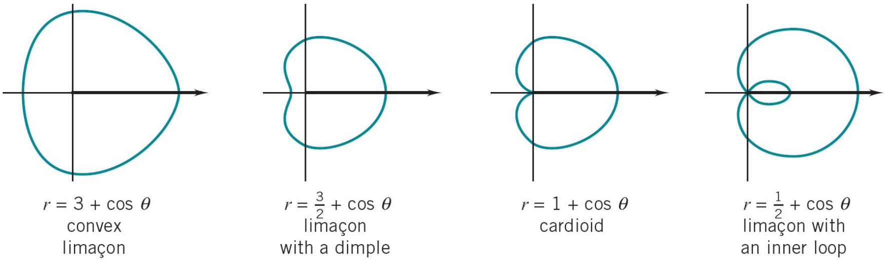
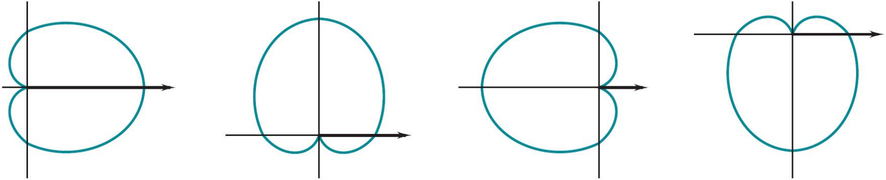
- $\cos(-\theta) = \cos\theta \Rightarrow$ if $[r, \theta] \in$ graph, then $[r, -\theta] \in$ graph $\Rightarrow$ symmetric about the x-axis.

Limaçons (Snails): $r = a + b\cos\theta$



The general shape of the curve depends on the relative magnitudes of $|a|$ and $|b|$.

Cardioids (Heart-Shaped): $r = 1 \pm \cos\theta$, $r = 1 \pm \sin\theta$



$r = 1 + \cos\theta$ $r = 1 + \sin\theta$ $r = 1 - \cos\theta$ $r = 1 - \sin\theta$

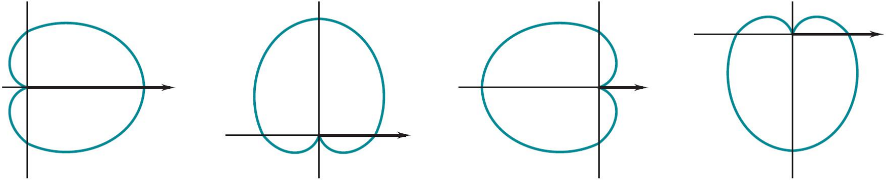
Each change

$$\cos\theta \to \sin\theta \to -\cos\theta \to -\sin\theta$$

represents a counterclockwise rotation by $\frac{1}{2}\pi$ radians.

- Rotation by $\frac{1}{2}\pi$: $r = 1 + \cos(\theta - \frac{1}{2}\pi) = 1 + \sin\theta$.
- Rotation by $\frac{1}{2}\pi$: $r = 1 + \sin(\theta - \frac{1}{2}\pi) = 1 - \cos\theta$.
- Rotation by $\frac{1}{2}\pi$: $r = 1 - \cos(\theta - \frac{1}{2}\pi) = 1 - \sin\theta$.

Cardioids (Heart-Shaped): $r = 1 \pm \cos\theta$, $r = 1 \pm \sin\theta$



$r = 1 + \cos\theta$ $r = 1 + \sin\theta$ $r = 1 - \cos\theta$ $r = 1 - \sin\theta$

Each change

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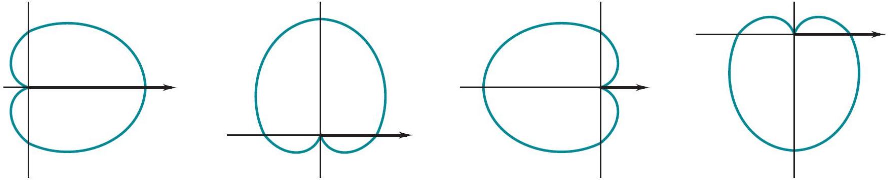
represents a counterclockwise rotation by $\frac{1}{2}\pi$ radians.

- Rotation by $\frac{1}{2}\pi$: $r = 1 + \cos(\theta - \frac{1}{2}\pi) = 1 + \sin\theta$.
- Rotation by $\frac{1}{2}\pi$: $r = 1 + \sin(\theta - \frac{1}{2}\pi) = 1 - \cos\theta$.
- Rotation by $\frac{1}{2}\pi$: $r = 1 - \cos(\theta - \frac{1}{2}\pi) = 1 - \sin\theta$.

Cardioids (Heart-Shaped): $r = 1 \pm \cos\theta$, $r = 1 \pm \sin\theta$



$r = 1 + \cos\theta$ $r = 1 + \sin\theta$ $r = 1 - \cos\theta$ $r = 1 - \sin\theta$

Each change

$$\cos\theta \to \sin\theta \to -\cos\theta \to -\sin\theta$$

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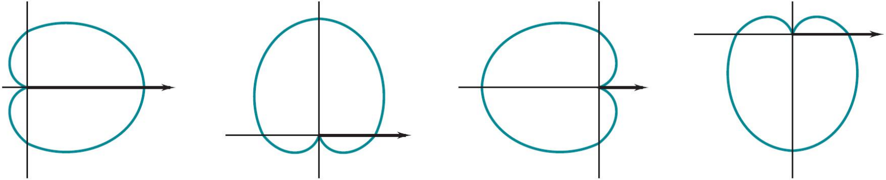

$r = 1 + \cos\theta$ $\quad$ $r = 1 + \sin\theta$ $\quad$ $r = 1 - \cos\theta$ $\quad$ $r = 1 - \sin\theta$

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Quiz

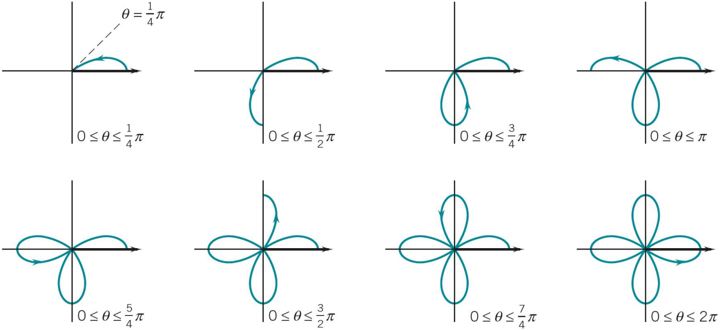
Quiz

3. $r = \sin\theta$ is the rotation by $\frac{\pi}{2}$ of:

   (a) $r = \cos\theta$, (b) $r = -\sin\theta$, (c) $r = -\cos\theta$.

4. *Today* is (a) Feb. 19, (b) Feb. 20, (c) Feb. 21.

Petal Curve: $r = \cos 2\theta$



- $r = 0$ at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$;
  $|r|$ is a local maximum at $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.
- Sketch the curve in 8 stages.
- $\cos[2(-\theta)] = \cos 2\theta$, $\cos[2(\pi \pm \theta)] = \cos 2\theta$
  $\Rightarrow$ symmetric about the x-axis, the y-axis, and the origin.

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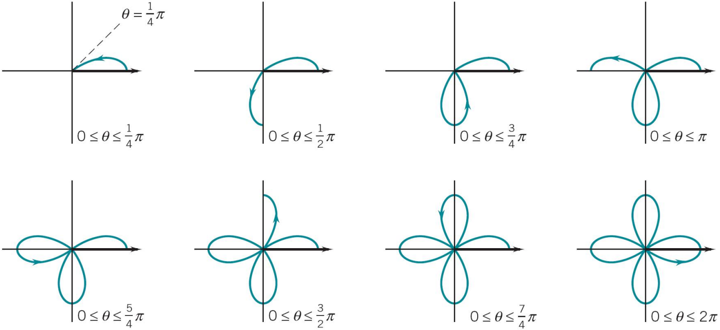

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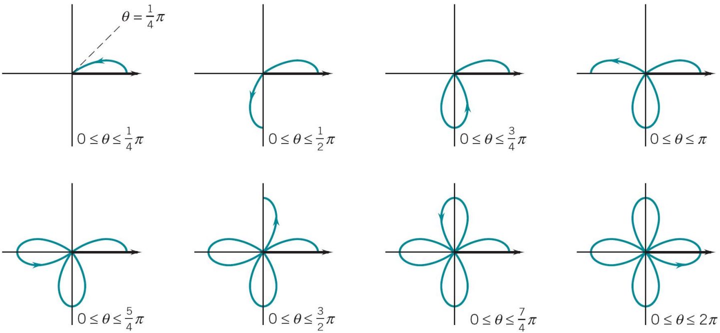

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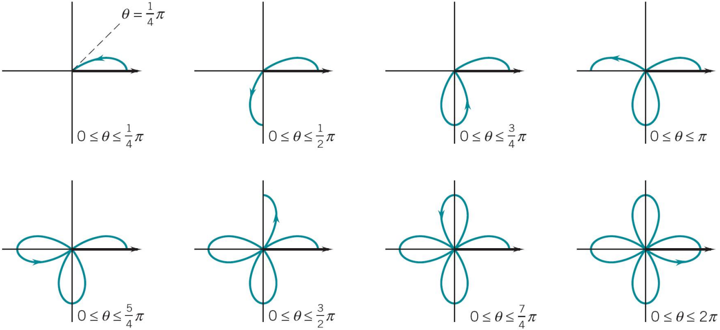

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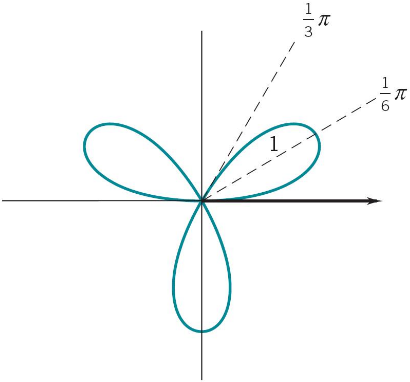
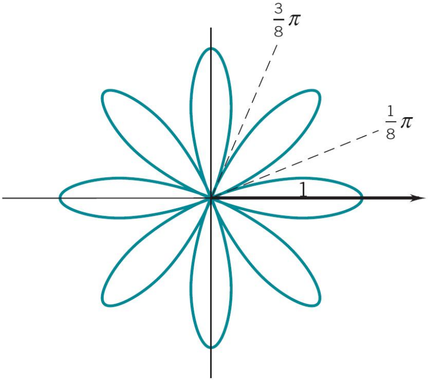
Petal Curves: $r = a\cos n\theta$, $r = a\sin n\theta$



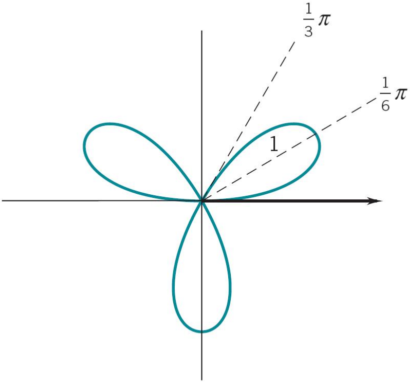
$r = \sin 3\theta$



$r = \cos 4\theta$

- If n is odd, there are n petals.
- If n is even, there are $2n$ petals.

Petal Curves: $r = a \cos n\theta$, $r = a \sin n\theta$



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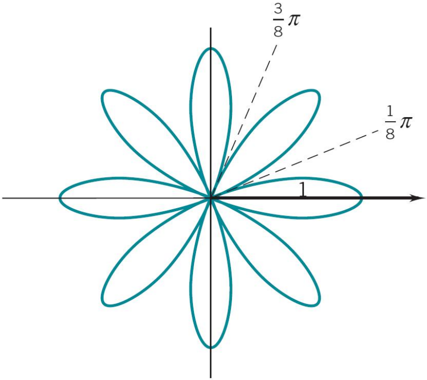

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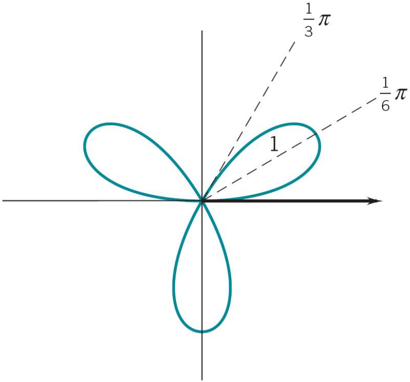

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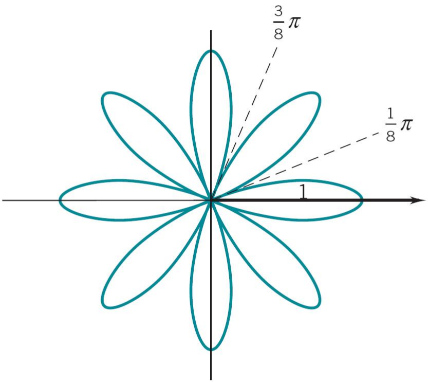

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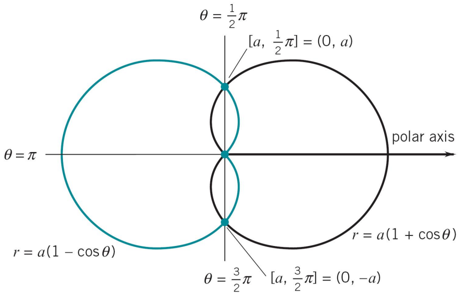
Intersections: $r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta)$



- $r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta) \Rightarrow r = a$ and $\cos\theta = 0$
  $\Rightarrow r = a$ and $\theta = \frac{\pi}{2} + n\pi \Rightarrow [a, \frac{\pi}{2} + n\pi] \in$ intersection
  $\Rightarrow n$ even, $[a, \frac{\pi}{2} + n\pi] = [a, \frac{\pi}{2}]$; n odd, $[a, \frac{\pi}{2} + n\pi] = [a, \frac{3\pi}{2}]$
- Two intersection points: $[a, \frac{\pi}{2}] = (0, a)$ and $[a, \frac{3\pi}{2}] = (0, -a)$.
- The intersection third point: the origin; but the two cardioids pass through the origin at different times (θ).

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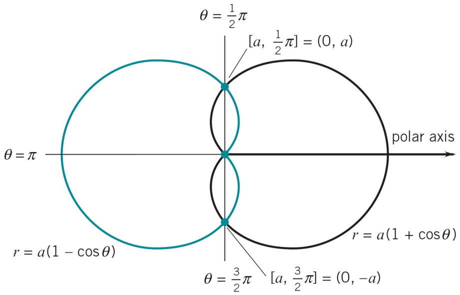

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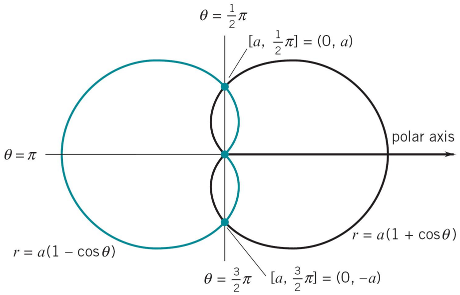

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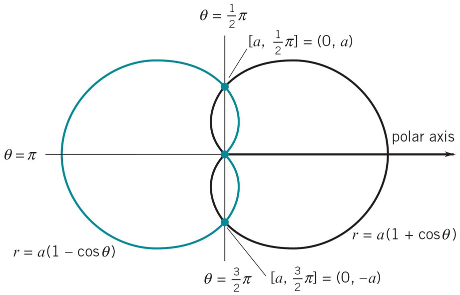

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Outline

- Polar Coordinates
  - Polar Coordinates
  - Relation to Rectangular Coordinates
  - Symmetry
- Graphing in Polar Coordinates
  - Spiral
  - Limaçons
  - Flowers
  - Intersections