Lecture 19

Section 10.5 Indeterminate Form $(0/0)$
Section 10.6 Other Indeterminate Forms (∞/∞), $(0 \cdot \infty)$, ...

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What is the Indeterminate Form (0/0)?

Example

$$\lim_{x\to 2}\frac{x-2}{x^2-4}=\lim_{x\to 2}\frac{x-2}{(x-2)(x+2)}=\lim_{x\to 2}\frac{1}{x+2}=\frac{1}{2+2}=\frac{1}{4}$$

$$\text{(L'Hôpital's Rule)}=\lim_{x\to 2}\frac{f'(x)}{g'(x)}=\lim_{x\to 2}\frac{1}{2x}=\frac{1}{2\cdot 2}=\frac{1}{4}\quad\text{(Amazing!!)}$$

As $x\to 2$, $f(x)=x-2\to 0$ and $g(x)=x^2-4\to 0$, but the quotient

$$\frac{f(x)}{g(x)}=\frac{x-2}{x^2-4}\to\frac{0}{0}.$$

We can not determine a value for "$\frac{0}{0}$" without some extra work!

If $\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$, then $\lim_{x\to a}\frac{f(x)}{g(x)}$ might equal any number or even fail to exist!

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Indeterminate Forms

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- It is indeterminate because, if $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ might equal any number or even fail to exist!
- Specific cases: $\lim_{x\to 0} \frac{\sin x}{x}$, $\lim_{x\to\infty} \frac{e^{-x}}{\sin(1/x)}$, $\cdots$.
- The forms $\frac{\infty}{\infty}$ and $0\cdot\infty$ are also indeterminate.
- They are actually equivalent to $\frac{0}{0}$, since any specific case of one can be recast as a specific case of another.
- For example,

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- They are actually equivalent to $\frac{0}{0}$, since any specific case of one can be recast as a specific case of another.
- For example,

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{x}{1/\ln x}$$

$$0\cdot\infty \qquad \frac{\infty}{\infty} \qquad \frac{0}{0}$$

Indeterminate Forms

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- The most basic indeterminate form is $\frac{0}{0}$.
- It is indeterminate because, if $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ might equal any number or even fail to exist!
- Specific cases: $\lim_{x\to 0} \frac{\sin x}{x}$, $\lim_{x\to\infty} \frac{e^{-x}}{\sin(1/x)}$, $\cdots$.
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Quiz

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1. Give the limit for $\left\{1 - \dfrac{n}{n+2}\right\}_{n=1}^{\infty}$

   (a) 0, (b) 1, (c) 2.

2. Give the limit for $\left\{1 + \dfrac{n}{n+2}\right\}_{n=1}^{\infty}$

   (a) 0, (b) 1, (c) 2.

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Examples

$$\lim_{x\to 2} \frac{x-2}{x^2-4} \left(\frac{0}{0}\right) = \lim_{x\to 2} \frac{1}{2x} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

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What is wrong with the following argument?

$$\lim_{x\to 0^+} \frac{x^2}{\sin x} \left(\frac{0}{0}\right) = \lim_{x\to 0^+} \frac{2x}{\cos x} = \lim_{x\to 0^+} \frac{2}{-\sin x} = \frac{2}{-0} = -\infty$$

Remark

L'Hôpital's rule does not apply in cases where the numerator or the denominator has a finite non-zero limit!!!

For example,

$$\lim_{x\to 0^+} \frac{2x}{\cos x} = \frac{0}{1} = 0,$$

but a blind application of L'Hôpital's rule leads incorrectly to

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L'Hôpital's rule does not apply in cases where the numerator or the denominator has a finite non-zero limit!!!

For example,

$$\lim_{x\to 0^+} \frac{2x}{\cos x} = \frac{0}{1} = 0,$$

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Quiz

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3. Give the limit for $\left\{ n^{\frac{1}{n}} \right\}_{n=1}^{\infty}$

   (a) 1, (b) 2, (c) e.

4. Give the limit for $\left\{ \left(1 + \frac{1}{n} \right)^{n} \right\}_{n=1}^{\infty}$

   (a) 1, (b) 2, (c) e.

Form $\frac{0}{0}$

Examples

$$\lim_{x\to 0}\frac{\sin x - x}{x^3}\ \left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{\cos x - 1}{3x^2}\ \left(\frac{0}{0}\right)\ \text{still!}$$

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Form $\frac{0}{0}$

Examples

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Limit Properties of ln x

$$\lim_{x \to 0^+} x^\alpha \ln x = 0, \quad \alpha > 0.$$

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$$\lim_{x\to 0^+} x^\alpha \ln x \ (0\cdot\infty) = \lim_{x\to 0^+} \frac{\ln x}{x^{-\alpha}} \left(\frac{\infty}{\infty}\right)$$
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- $\ln x$ tends to infinity slower than any positive power of x.
- Any positive power of $\ln x$ tends to infinity slower than x.

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Exponential Forms: 1^∞, 0^0, and ∞^0

Since $\ln x^p = p \ln x$, the log can be used to convert each of exponential forms 1^∞, 0^0, and ∞^0 to $0 \cdot \infty$:

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Form $\infty - \infty$

The form $\infty - \infty$ can be handled by converting it to a quotient.

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  - Indeterminate Form (0/0)
  - Other Indeterminate Forms
- L'Hôpital's Rule
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- More Examples
  - Form $\frac{0}{0}$
  - Form $\frac{\infty}{\infty}$
  - Limit Properties of $\ln x$ and e^x
  - Exponential Forms: 1^∞, 0^0, and ∞^0
  - Form $\infty - \infty$