

# Lecture 19

**Section 10.5 Indeterminate Form (0/0)**

**Section 10.6 Other Indeterminate Forms ( $\infty/\infty$ ), ( $0 \cdot \infty$ ),**

...

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# What is the Indeterminate Form (0/0)?

## Example

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x+2)} = \lim_{x \rightarrow 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$$

$$(\text{L'Hôpital's Rule}) = \lim_{x \rightarrow 2} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 2} \frac{1}{2x} = \frac{1}{2 \cdot 2} = \frac{1}{4} \quad (\text{Amazing!})$$

As  $x \rightarrow 2$ ,  $f(x) = x - 2 \rightarrow 0$  and  $g(x) = x^2 - 4 \rightarrow 0$ , but the quotient

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We can not determine a value for  $\frac{0}{0}$  without some extra work!

If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ , then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  might equal any number or even fail to exist!

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- Specific cases:  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ ,  $\lim_{x \rightarrow \infty} \frac{e^{-x}}{\sin(1/x)}$ , ...
- The forms  $\frac{\infty}{\infty}$  and  $0 \cdot \infty$  are also **indeterminate**.
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- It is **indeterminate** because, if  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ , then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  might equal any number or even fail to exist!
- Specific cases:  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ ,  $\lim_{x \rightarrow \infty} \frac{e^{-x}}{\sin(1/x)}$ , ... .
- The forms  $\frac{\infty}{\infty}$  and  $0 \cdot \infty$  are also **indeterminate**.
- They are actually equivalent to  $\frac{0}{0}$ , since any specific case of one can be recast as a specific case of another.
- For example,

$$\lim_{x \rightarrow 0^+} x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \lim_{x \rightarrow 0^+} \frac{x}{1/\ln x}$$

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# Quiz

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1. Give the limit for  $\left\{ 1 - \frac{n}{n+2} \right\}_{n=1}^{\infty}$

- (a) 0, (b) 1, (c) 2.

2. Give the limit for  $\left\{ 1 + \frac{n}{n+2} \right\}_{n=1}^{\infty}$

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$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4} \left( \frac{0}{0} \right) = \lim_{x \rightarrow 2} \frac{1}{2x} = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \left( \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x \sin x} \left( \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x + x \cos x} \left( \frac{0}{0} \right) \text{ still}$$

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# L'Hôpital's Rule

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# What is Wrong with This?

What is **wrong** with the following argument?

$$\lim_{x \rightarrow 0^+} \frac{x^2}{\sin x} = \left( \frac{0}{0} \right) = \lim_{x \rightarrow 0^+} \frac{2x}{\cos x} = \lim_{x \rightarrow 0^+} \frac{2}{-\sin x} = \frac{2}{-0} = -\infty$$

## Remark

L'Hôpital's rule does not apply in cases where the numerator or the denominator has a finite non-zero limit!!!

For example,

$$\lim_{x \rightarrow 0^+} \frac{2x}{\cos x} = \frac{0}{1} = 0,$$

but a blind application of L'Hôpital's rule leads incorrectly to

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# Quiz

## Quiz

3. Give the limit for  $\left\{ n^{\frac{1}{n}} \right\}_{n=1}^{\infty}$

- (a) 1, (b) 2, (c) e.

4. Give the limit for  $\left\{ \left( 1 + \frac{1}{n} \right)^n \right\}_{n=1}^{\infty}$

- (a) 1, (b) 2, (c) e.



Form  $\frac{0}{0}$ 

## Examples

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \left( \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} \left( \frac{0}{0} \right) \text{ still!}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{6x} \left( \frac{0}{0} \right) \text{ still!} = \lim_{x \rightarrow 0} \frac{-\cos x}{6} = \frac{-1}{6}$$

$$\lim_{x \rightarrow 0} \frac{2 \cos x - 2 + x^2}{x^4} \left( \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{-2 \sin x + 2x}{4x^3} \left( \frac{0}{0} \right) \text{ still!}$$

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# Form $\frac{0}{0}$

## Examples

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \left( \begin{array}{c} 0 \\ \hline 0 \end{array} \right) = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} \left( \begin{array}{c} 0 \\ \hline 0 \end{array} \right) \text{ still!}$$



Form  $\frac{0}{0}$ 

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# Form $\frac{0}{0}$

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Form  $\frac{0}{0}$ 

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Form  $\frac{0}{0}$

## Examples

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \left( \begin{matrix} 0 \\ 0 \end{matrix} \right) = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} \left( \begin{matrix} 0 \\ 0 \end{matrix} \right) \text{ still!}$$

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Form  $\frac{0}{0}$

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$$= \lim_{x \rightarrow 0} \frac{-\sin x}{6x} \left( \begin{array}{l} \text{red} \\ \frac{0}{0} \end{array} \right) \text{ still! } = \lim_{x \rightarrow 0} \frac{-\cos x}{6} = \frac{-1}{6}$$



# Form $\frac{0}{0}$

## Examples

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Form  $\frac{0}{0}$ 

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Form  $\frac{0}{0}$

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Form  $\frac{0}{0}$

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# Form $\frac{0}{0}$

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$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \left( \begin{matrix} 0 \\ 0 \end{matrix} \right) = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} \left( \begin{matrix} 0 \\ 0 \end{matrix} \right) \text{ still!}$$

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$$\lim_{x \rightarrow 0} \frac{2 \cos x - 2 + x^2}{x^4} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \lim_{x \rightarrow 0} \frac{-2 \sin x + 2x}{4x^3} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ still!}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \cos x + 2}{12x^2} \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \text{ still!}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x}{24x} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ still! } = \lim_{x \rightarrow 0} \frac{2 \cos x}{24} = \frac{2}{24} = \frac{1}{12}$$



Form  $\frac{0}{0}$

## Examples

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ still!}$$

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Form  $\frac{0}{0}$

## Examples

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## Form $\frac{\infty}{\infty}$

## Examples



Form  $\frac{\infty}{\infty}$

## Examples

$$\lim_{x \rightarrow \infty} \frac{x^2}{e^x} \left( \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{2x}{e^x} \left( \frac{\infty}{\infty} \right) \text{ still!}$$



Form  $\frac{\infty}{\infty}$

## Examples

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## Examples

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Form  $\frac{\infty}{\infty}$ 

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$$= \lim_{x \rightarrow \infty} \frac{\ln 3 + (2/3)^x \ln 2}{1 + (2/3)^x} = \frac{\ln 3 + 0}{1 + 0} = \ln 3$$



Form  $\frac{\infty}{\infty}$ 

## Examples

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Form  $\frac{\infty}{\infty}$ 

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Form  $\frac{\infty}{\infty}$ 

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Form  $\frac{\infty}{\infty}$ 

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Form  $\frac{\infty}{\infty}$

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# Limit Properties of $\ln x$

$$\lim_{x \rightarrow 0^+} x^\alpha \ln x = 0, \quad \alpha > 0.$$

**Proof.**

$$\begin{aligned} \lim_{x \rightarrow 0^+} x^\alpha \ln x \quad (0 \cdot \infty) &= \lim_{x \rightarrow 0^+} \frac{\ln x}{x^{-\alpha}} \quad \left( \frac{\infty}{\infty} \right) \\ &= \lim_{x \rightarrow 0^+} \frac{1/x}{-\alpha x^{-\alpha-1}} = \lim_{x \rightarrow 0^+} \frac{x^\alpha}{-\alpha} = 0 \end{aligned}$$

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$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^\alpha} \quad \left( \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{1/x}{\alpha x^{\alpha-1}} = \lim_{x \rightarrow \infty} \frac{1}{\alpha x^{\alpha-1}} = 0$$



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Proof.

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Remark

- $\ln x$  tends to infinity **slower than** any positive power of  $x$ .
- Any positive power of  $\ln x$  tends to infinity **slower than**  $x$ .



# Limit Properties of $\ln x$

$$\lim_{x \rightarrow 0^+} x^\alpha \ln x = 0, \quad \alpha > 0.$$

Proof.

$$\begin{aligned}\lim_{x \rightarrow 0^+} x^\alpha \ln x \quad (0 \cdot \infty) &= \lim_{x \rightarrow 0^+} \frac{\ln x}{x^{-\alpha}} \quad \left( \frac{\infty}{\infty} \right) \\ &= \lim_{x \rightarrow 0^+} \frac{1/x}{-\alpha x^{-\alpha-1}} = \lim_{x \rightarrow 0^+} \frac{x^\alpha}{-\alpha} = 0\end{aligned}$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^\alpha} = 0, \quad \alpha > 0.$$

Remark

- $\ln x$  tends to infinity **slower than** any positive power of  $x$ .
- Any positive power of  $\ln x$  tends to infinity **slower than**  $x$ .



# Limit Properties of $e^x$

$$\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = 0.$$

Proof.

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \frac{x^n}{e^x} & \left( \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{nx^{n-1}}{e^x} \left( \frac{\infty}{\infty} \right) \\
 & = \lim_{x \rightarrow \infty} \frac{n(n-1)x^{n-2}}{e^x} \left( \frac{\infty}{\infty} \right) \\
 & = \dots \\
 & = \lim_{x \rightarrow \infty} \frac{n!}{e^x} = \frac{n!}{\infty} = 0
 \end{aligned}$$

Conclusion:

$e^x$  tends to infinity faster than any positive power of  $x$ .



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$$\ln(1^\infty) = \infty \cdot 0, \quad \ln(0^0) = \infty \cdot 0, \quad \ln(\infty^0) = 0 \cdot \infty.$$

If  $\lim_{x \rightarrow a} \ln f(x) = L$ , then  $\lim_{x \rightarrow a} f(x) = e^L$ .

## Example

$$\lim_{x \rightarrow 0^+} x^x (0^0) = e^0 = 1$$

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## Examples

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$$\lim_{x \rightarrow 0^+} (1+x)^{1/x} \quad (1^\infty) = e^1 = e$$

$$\lim_{x \rightarrow \infty} \ln x^{1/x} \quad (\ln \infty^0) = \lim_{x \rightarrow \infty} \frac{\ln x}{x} \quad \left( \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0$$

$$\lim_{x \rightarrow 0^+} \ln(1+x)^{1/x} \quad (\ln 1^\infty) = \lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} \quad \left( \frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{1+x} = 1$$

- Set  $x = n$ , we have the familiar result: as  $n \rightarrow \infty$ ,  $n^{\frac{1}{n}} \rightarrow 1$ .
- Set  $x = 1/n$ , we have: as  $n \rightarrow \infty$ ,  $(1 + \frac{1}{n})^n \rightarrow e$ .



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# Outline

- Indeterminate Forms
  - Indeterminate Form (0/0)
  - Other Indeterminate Forms
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- More Examples
  - Form  $\frac{0}{0}$
  - Form  $\frac{\infty}{\infty}$
  - Limit Properties of  $\ln x$  and  $e^x$
  - Exponential Forms:  $1^\infty$ ,  $0^0$ , and  $\infty^0$
  - Form  $\infty - \infty$

