Lecture 20

Section 10.7 Improper Integrals

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What are Improper Integrals?

$$\int_1^{\infty} \frac{1}{x^2}\,dx = ?, \qquad \int_0^1 \frac{1}{x^2}\,dx = ?$$

Known: $\displaystyle\int_a^b f(x)\,dx = \int_a^b \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_a^b = \frac{1}{a} - \frac{1}{b}, \quad 0 < a < b,$

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- unbounded intervals :
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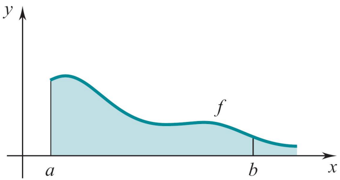

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We define

$$\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$$

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The improper integral diverges if the limit doesn't exist.

If $\lim_{b \to \infty} f(x) \neq 0$, then $\int_a^{\infty} f(x)\,dx$ diverges.

If f is continuous on $(-\infty, b]$, $\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$.

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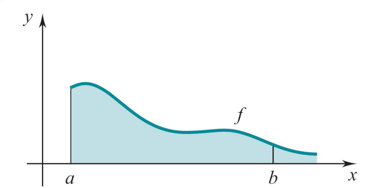

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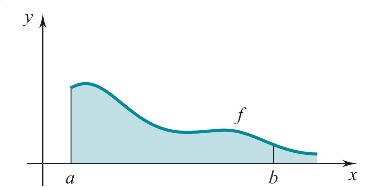

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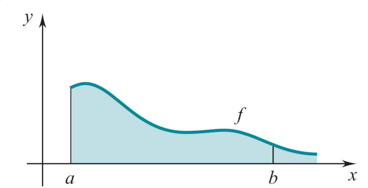

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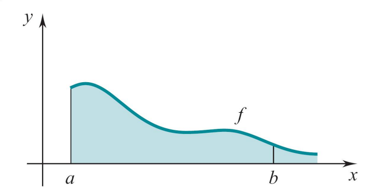

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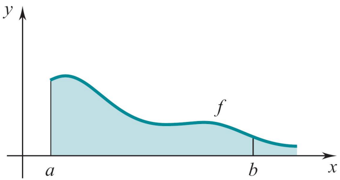

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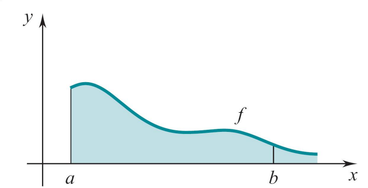

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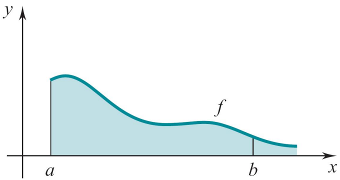

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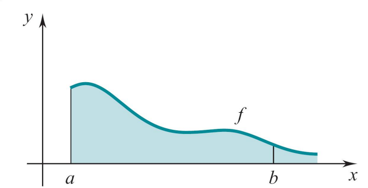

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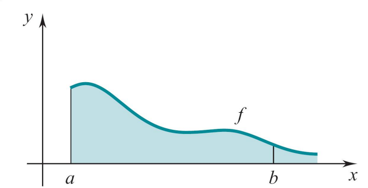

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$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

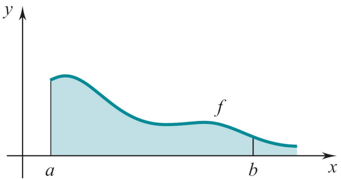
The improper integral converges if the limit exists.
The improper integral diverges if the limit doesn't exist.

If $\lim_{b\to\infty} f(x) \neq 0$, then $\int_a^{\infty} f(x)\,dx$ diverges.

If f is continuous on $(-\infty, b]$, $\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$.

If f is cont. on $(-\infty, \infty)$, $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx$

Integrals Over Unbounded Intervals



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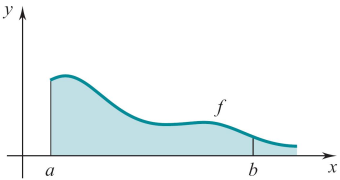

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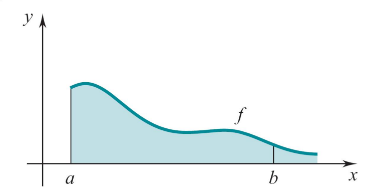

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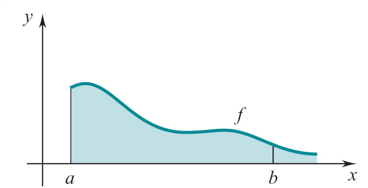

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Fundamental Theorem of Integral Calculus

Let $f(x) = F'(x)$ for $\forall x \geq a$. Then

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx = \lim_{b\to\infty}\left(F(x)\big|_a^b\right) = \lim_{b\to\infty} F(b) - F(a)$$

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$$\int_1^\infty \frac{1}{x^3}\,dx = -\frac{2}{x^2}\Big|_1^\infty = \lim_{x\to\infty}\left(-\frac{2}{x^2}\right) - (-2) = 2$$

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- Note that we did not define $\int_{-\infty}^{\infty} f(x)\,dx$ as $\lim_{b\to\infty} \int_{-b}^{b} f(x)\,dx$.

$$\int_{-\infty}^{\infty} f(x)\,dx = L \quad \Rightarrow \quad \lim_{b\to\infty} \int_{-b}^{b} f(x)\,dx = L$$

$$\lim_{b\to\infty} \int_{-b}^{b} f(x)\,dx = L \quad \not\Rightarrow \quad \int_{-\infty}^{\infty} f(x)\,dx = L$$

- For every odd function f, we have $\lim_{b\to\infty} \int_{-b}^{b} f(x)\,dx = 0$;

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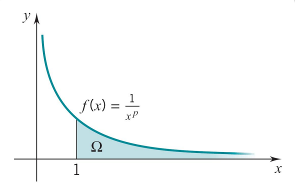
$\int_1^\infty \frac{1}{x^\alpha}\,dx,\ \alpha > 0$



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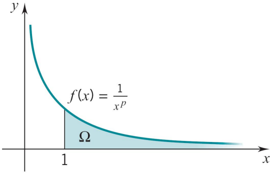

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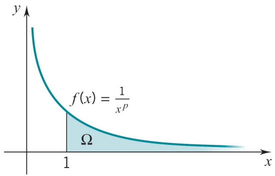

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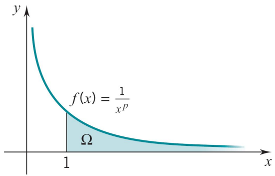

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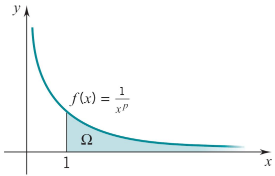

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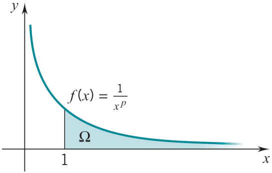

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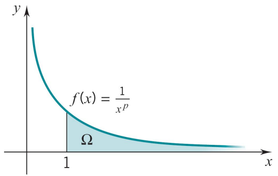

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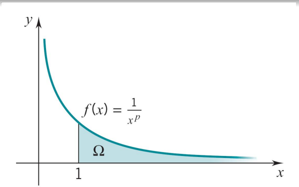

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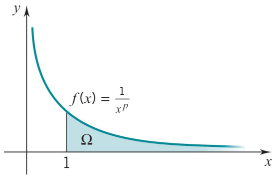

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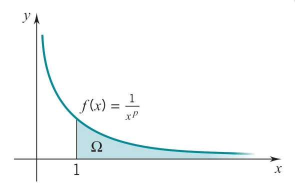

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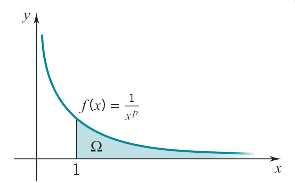

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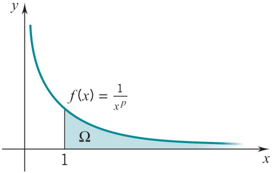

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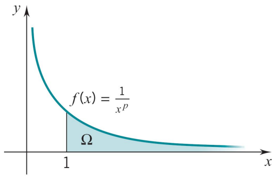

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Quiz

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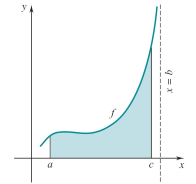
1. Find $\lim_{x \to \infty} \frac{x^2}{e^x}$

   (a) 0, (b) 1, (c) ∞.

2. Find $\lim_{x \to \infty} \frac{x}{\ln x}$

   (a) 0, (b) 1, (c) ∞.

Integrals of Unbounded Functions



Let f be continuous on $[a, b)$, but $f(x) \to \pm\infty$ as $x \to b^-$. We define

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx$$

The improper integral converges if the limit exists.
The improper integral diverges if the limit doesn't exist.

If f is continuous on $(a, b]$, but $f(x) \to \pm\infty$ as $x \to a^+$,

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If f is cont. on $[a, b]$, except at some point c in (a, b) where $f(x) \to \pm\infty$ as $x \to c^-$ or $x \to c^+$,

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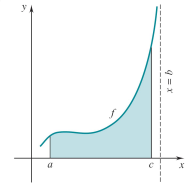

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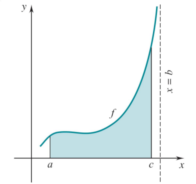

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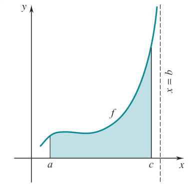

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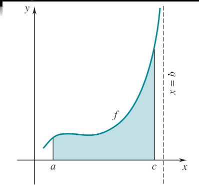

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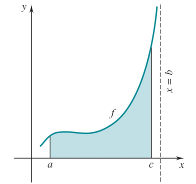

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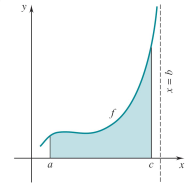
Integrals of Unbounded Functions



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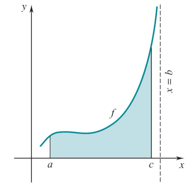

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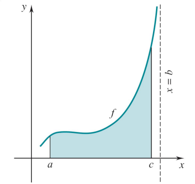

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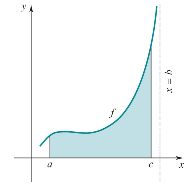

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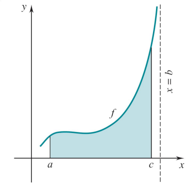

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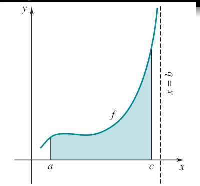

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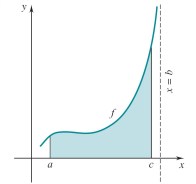

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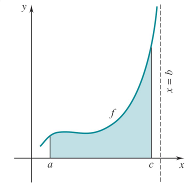

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Let $f(x) = F'(x)$ for $\forall x \in [a, b)$ and $f(x) \to \pm\infty$ as $x \to b^-$. Then

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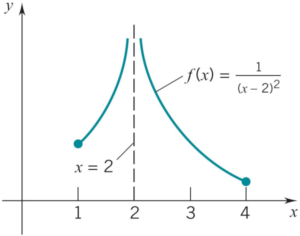

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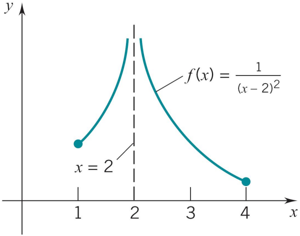

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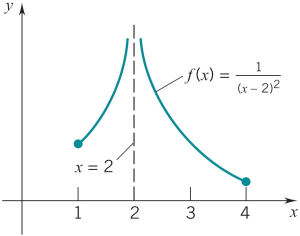

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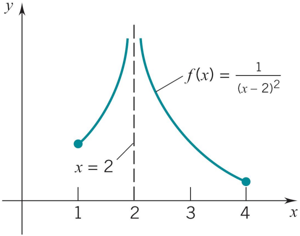

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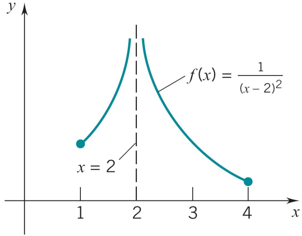

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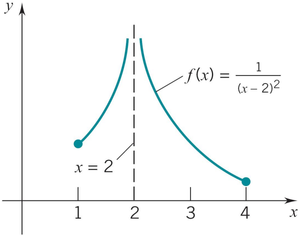

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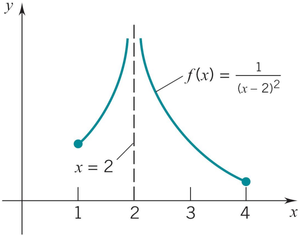

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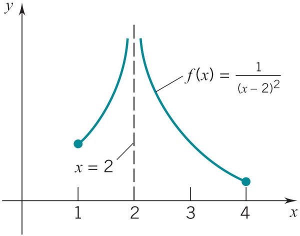

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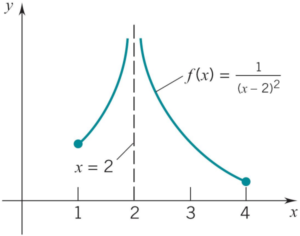

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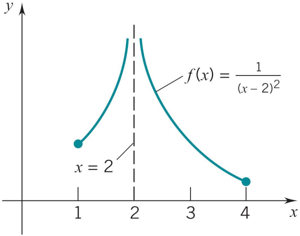

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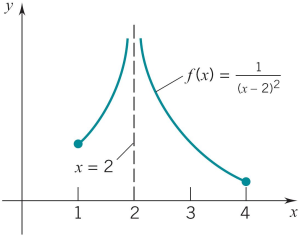

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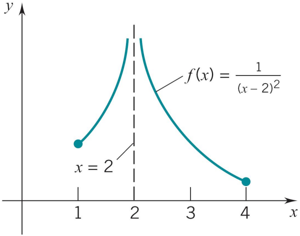

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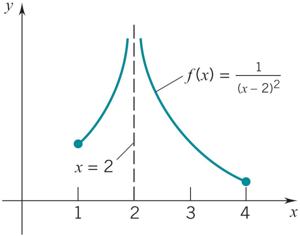
$$\int_1^2 \frac{1}{(x-2)^2}\,dx = -\frac{1}{x-2}\Big|_1^2 = \lim_{x\to 2^-}\left(-\frac{1}{x-2}\right) - 1 = \infty$$



Note that $\frac{1}{(x-2)^2} \to \infty$ as $x \to 2^-$ or $x \to 2^+$. To evaluate $\int_1^4 \frac{1}{(x-2)^2}\,dx$ we need to calculate the improper integrals $\int_1^2 \frac{1}{(x-2)^2}\,dx$ and $\int_2^4 \frac{1}{(x-2)^2}\,dx$.

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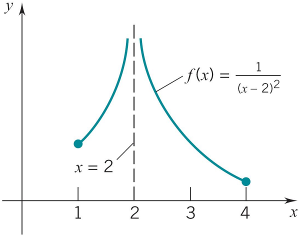

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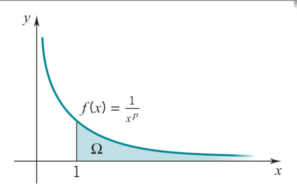
$\int_0^1 \frac{1}{x^\alpha}\,dx,\ \alpha > 0$



$$\int_0^1 \frac{1}{x^\alpha}\,dx = \begin{cases} \dfrac{1}{1-\alpha}, & \text{if } \alpha < 1, \\ \infty, & \text{if } \alpha \geq 1. \end{cases}$$

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$$\int_0^1 \frac{1}{x^\alpha}\,dx = \frac{1}{1-\alpha} x^{1-\alpha}\Big|_0^1 = \frac{1}{1-\alpha} - \frac{1}{1-\alpha}\lim_{x\to 0^+} x^{1-\alpha}$$
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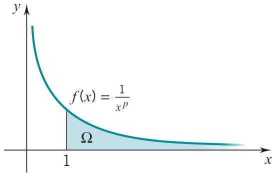

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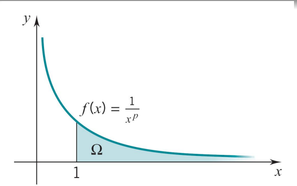

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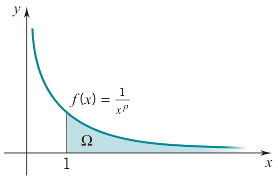

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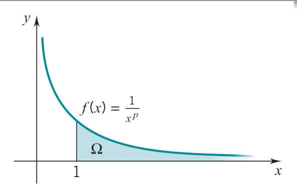

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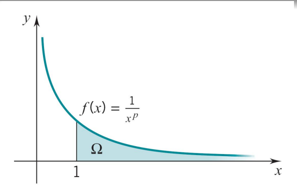

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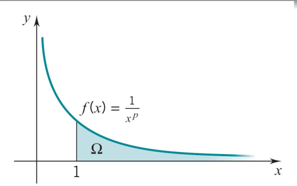

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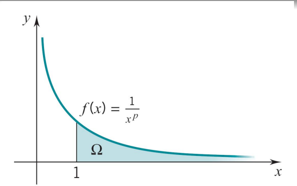

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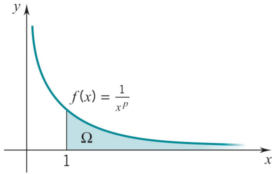

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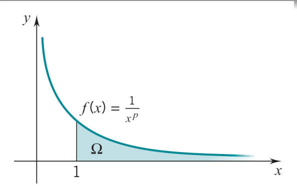

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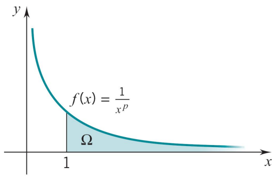

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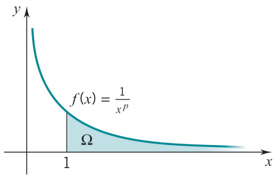

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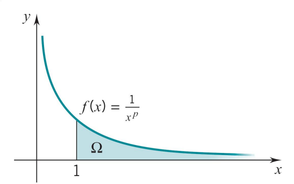

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Quiz

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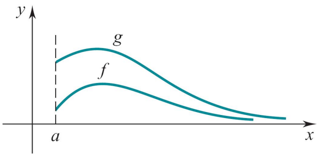
3. Find $\lim_{x\to\infty} x \sin \frac{1}{x}$

   (a) 0, (b) 1, (c) ∞.

4. Find $\lim_{x\to 0^+} \frac{x}{e^x}$

   (a) 0, (b) 1, (c) ∞.

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges,
- If $\int_I f(x)\,dx$ diverges,

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^3}}\,dx$ converges since

$$0 \le \frac{1}{\sqrt{1+x^3}} < \frac{1}{\sqrt{x^3}}, \ \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{\sqrt{x^3}}\,dx \ \text{converges.}$$

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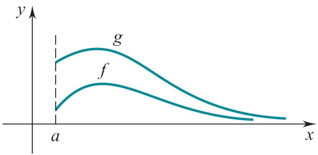

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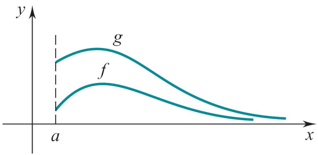

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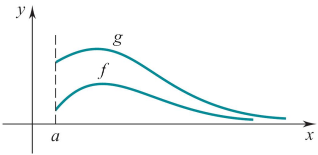

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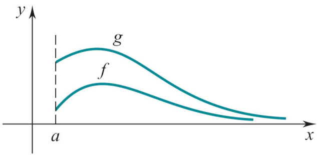

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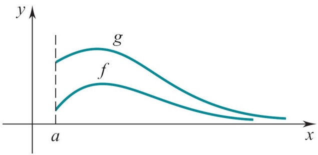

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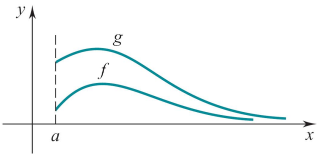

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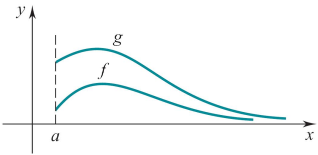

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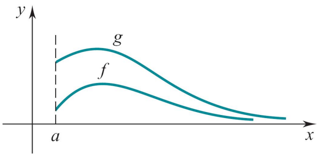
$$0 \le \frac{1}{\sqrt{1+x^3}} < \frac{1}{\sqrt{x^3}}, \; \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{\sqrt{x^3}}\,dx \; \text{converges.}$$

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

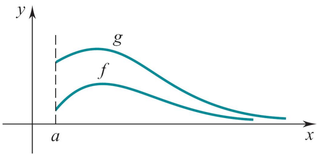
$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges, then so does $\int_I f(x)\,dx$.
- If $\int_I f(x)\,dx$ diverges, then so does $\int_I g(x)\,dx$.

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^3}}\,dx$ converges since

$$0 \le \frac{1}{\sqrt{1+x^3}} < \frac{1}{\sqrt{x^3}}, \ \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{\sqrt{x^3}}\,dx \ \text{converges.}$$

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

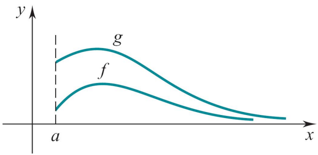
$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges, then so does $\int_I f(x)\,dx$.
- If $\int_I f(x)\,dx$ diverges, then so does $\int_I g(x)\,dx$.

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^3}}\,dx$ converges since

$$0 \le \frac{1}{\sqrt{1+x^3}} < \frac{1}{\sqrt{x^3}}, \quad \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{\sqrt{x^3}}\,dx \text{ converges.}$$

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

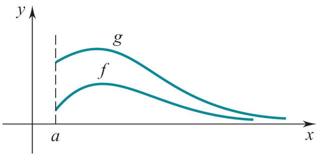
$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges, then so does $\int_I f(x)\,dx$.
- If $\int_I f(x)\,dx$ diverges, then so does $\int_I g(x)\,dx$.

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^3}}\,dx$ converges since

$$0 \le \frac{1}{\sqrt{1+x^3}} < \frac{1}{\sqrt{x^3}}, \ \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{\sqrt{x^3}}\,dx \ \text{ converges.}$$

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

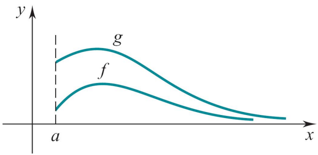
$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges, then so does $\int_I f(x)\,dx$.
- If $\int_I f(x)\,dx$ diverges, then so does $\int_I g(x)\,dx$.

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^2}}\,dx$ diverges since

$$0 \le \frac{1}{1+x} < \frac{1}{\sqrt{1+x^2}}, \ \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{1+x}\,dx \ \text{diverges.}$$

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

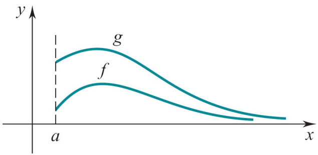
$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges, then so does $\int_I f(x)\,dx$.
- If $\int_I f(x)\,dx$ diverges, then so does $\int_I g(x)\,dx$.

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^2}}\,dx$ diverges since

$$0 \le \frac{1}{1+x} < \frac{1}{\sqrt{1+x^2}}, \ \forall x \ge 1$$ and $\int_1^\infty \frac{1}{1+x}\,dx$ diverges.

Comparison Test for Convergence



Let $\int_I$ be improper integration over an interval I, and suppose that f and g are continuous on I such that

$$0 \le f(x) \le g(x), \quad \forall x \in I$$

- If $\int_I g(x)\,dx$ converges, then so does $\int_I f(x)\,dx$.
- If $\int_I f(x)\,dx$ diverges, then so does $\int_I g(x)\,dx$.

The improper integral $\int_1^\infty \frac{1}{\sqrt{1+x^2}}\,dx$ diverges since

$$0 \le \frac{1}{1+x} < \frac{1}{\sqrt{1+x^2}}, \ \forall x \ge 1 \quad \text{and} \quad \int_1^\infty \frac{1}{1+x}\,dx \ \text{diverges.}$$

Outline

- Improper Integrals
  - Integrals Over Unbounded Intervals
  - Integrals of Unbounded Functions
- Comparison Test for Convergence
  - Comparison Test for Convergence