Lecture 21

Section 11.1 Infinite Series

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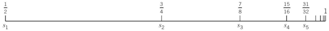

$$\sum_{k=1}^{\infty} \frac{1}{2^k} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = 1$$

What is an Infinite Series?

Let the sequence $a_k = \frac{1}{2^k}$: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots$. Form the partial sums

$$s_1 = a_1 = \sum_{k=1}^{1} a_k = \frac{1}{2}$$

$$s_2 = a_1 + a_2 = \sum_{k=1}^{2} a_k = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$s_3 = a_1 + a_2 + a_3 = \sum_{k=1}^{3} a_k = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$

$$s_4 = a_1 + a_2 + a_3 + a_4 = \sum_{k=1}^{4} a_k = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$$

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$$s_n = \sum_{k=1}^{n} a_k = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + + \frac{1}{16} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^{n+1}}$$

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Quiz

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1. $$\int_0^1 \frac{1}{x^2}\,dx =$$

(a) 1, (b) 2, (c) ∞.

2. $$\int_1^\infty \frac{1}{x^2}\,dx =$$

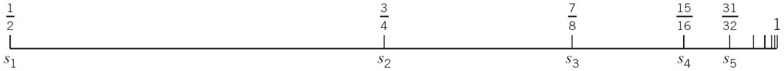
(a) 1, (b) 2, (c) ∞.

Infinite Series: Definition

Given a sequence $\{a_k\}_{k=1}^{\infty}$, the infinite series is defined by the limit of the sequence of partial sums:

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

The series converges if the limit exists; otherwise, it diverges.



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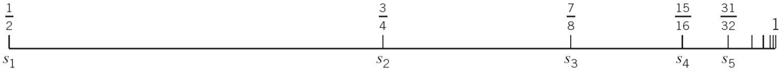

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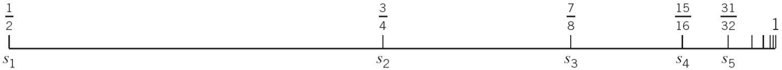

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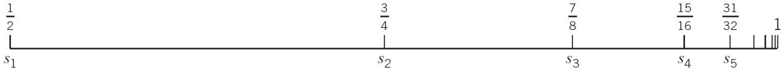

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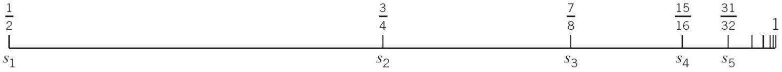

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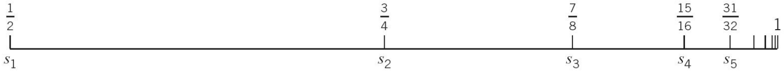

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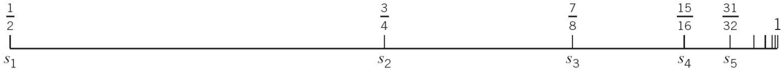
Given a sequence $\{a_k\}_{k=1}^{\infty}$, the infinite series is defined by the limit of the sequence of partial sums:

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

The series converges if the limit exists; otherwise, it diverges.



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If $\sum_{k=1}^{\infty} a_k$ converges, then $a_k \to 0$ as $k \to \infty$

If $a_k \not\to 0$ as $k \to \infty$, then $\sum_{k=1}^{\infty} a_k$ diverges.

Example

$$\frac{k}{k+1} \to 1 \neq 0 \text{ as } k \to \infty, \text{ then } \sum_{k=1}^{\infty} \frac{k}{k+1} = \frac{1}{2} + \frac{2}{3} + \cdots \text{ diverges.}$$

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Basic Properties

- If $\sum_{k=1}^{\infty} a_k$ converges and c is a constant, then $\sum_{k=1}^{\infty} c\,a_k$ converges, and
$$\sum_{k=1}^{\infty} c\,a_k = c\sum_{k=1}^{\infty} a_k.$$
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Quiz

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3. $\displaystyle\int_0^1 \frac{1}{x}\,dx =$

(a) 1, (b) 2, (c) ∞.

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Geometric Series

Geometric Series: $\sum_{k=0}^{\infty} x^k$

$$\sum_{k=0}^{\infty} x^k = \begin{cases} \dfrac{1}{1-x}, & \text{if } |x| < 1, \\ \text{diverges}, & \text{if } |x| \geq 1. \end{cases}$$

Proof.

- We have a "closed" formula for the nth partial sum:

$$s_n = 1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}, \qquad \text{if } x \neq 1$$

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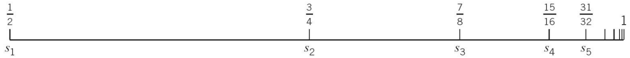
Examples



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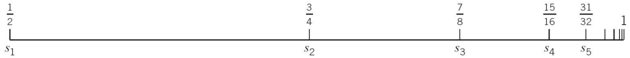

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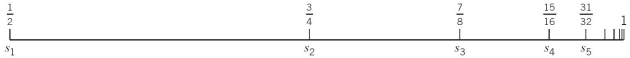

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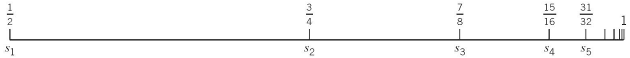

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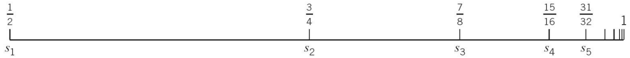

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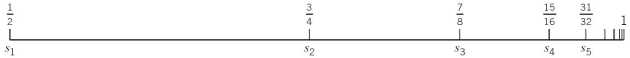

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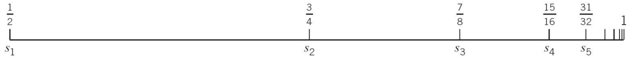

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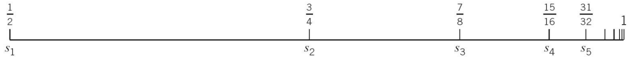

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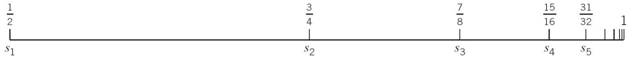

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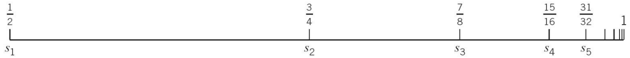

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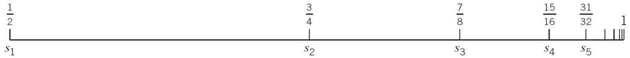
Examples



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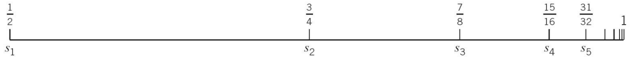

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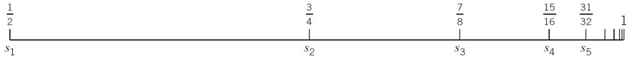

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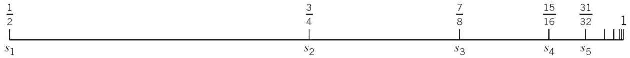

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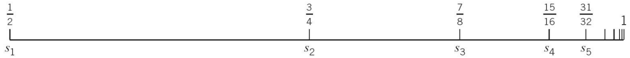

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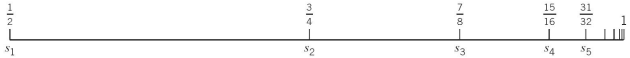

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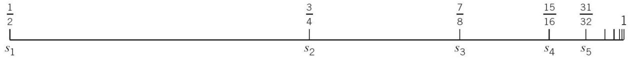

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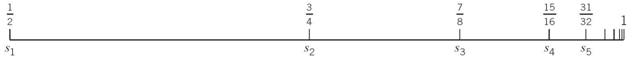

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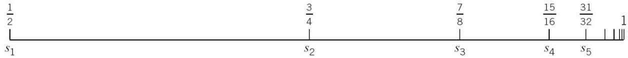

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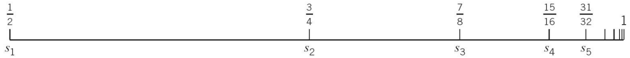

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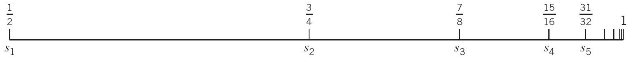

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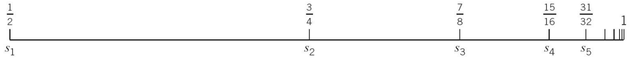

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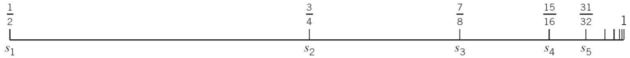

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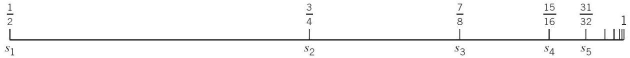

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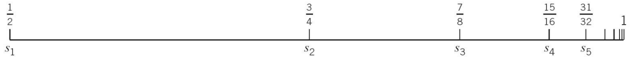

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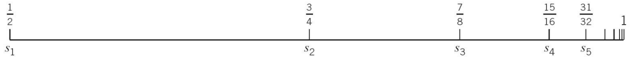

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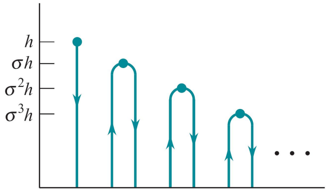

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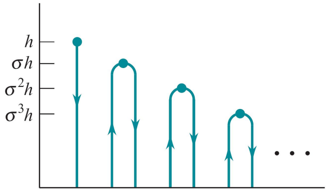

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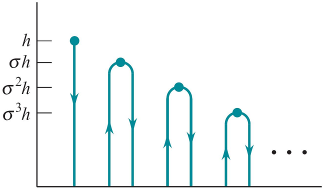

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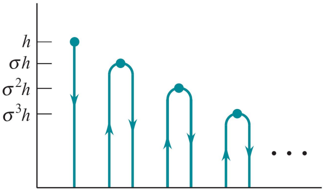

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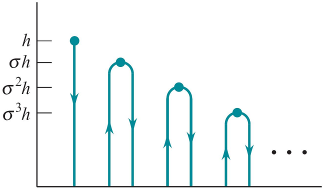

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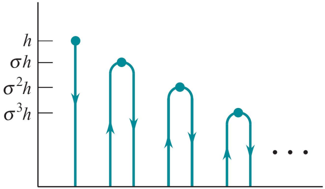

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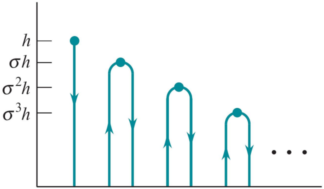

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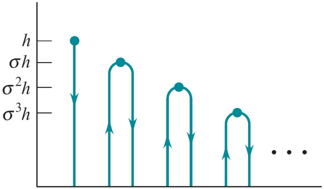

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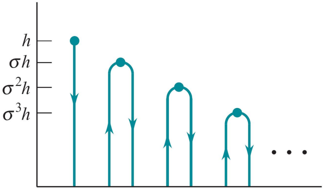

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Outline

- Infinite Series
  - Basic Properties
  - Geometric Series