Lecture 22

Section 11.2 The Integral Test; Comparison Tests

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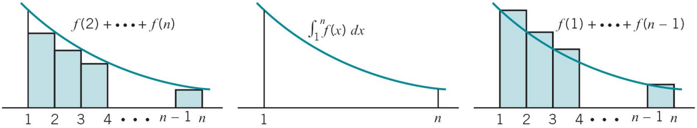

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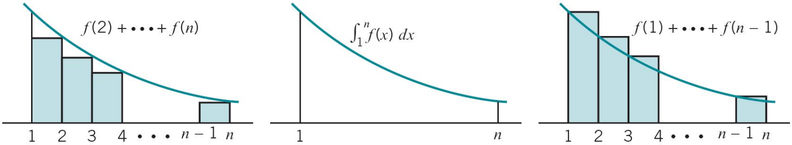

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Since f decreases on $[1, \infty)$ $\displaystyle\sum_{k=2}^{n} a_k < \int_1^n f(x)dx < \sum_{k=1}^{n-1} a_k$.

Since f is positive on $[1, \infty)$, $\displaystyle\sum_{k=2}^{\infty} a_k \le \int_1^{\infty} f(x)dx \le \sum_{k=1}^{\infty} a_k$.

Therefore, either both converge or both diverge.

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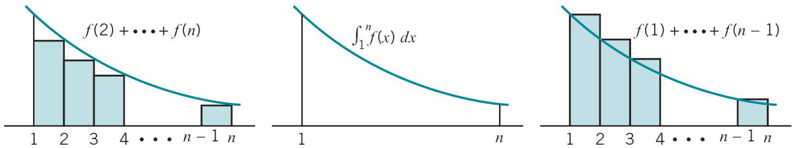

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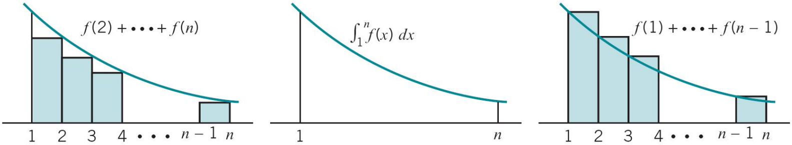

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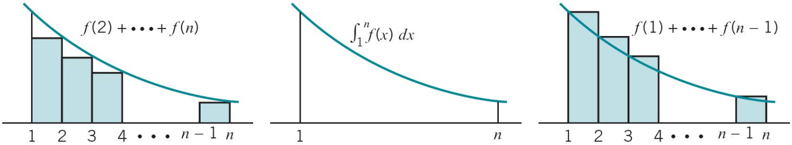

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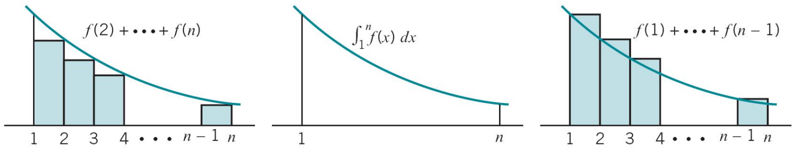

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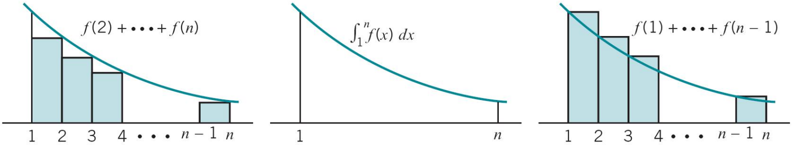

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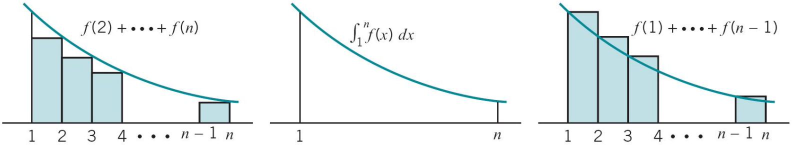

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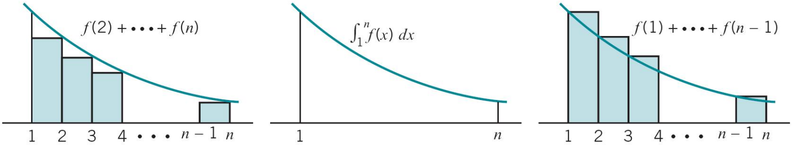

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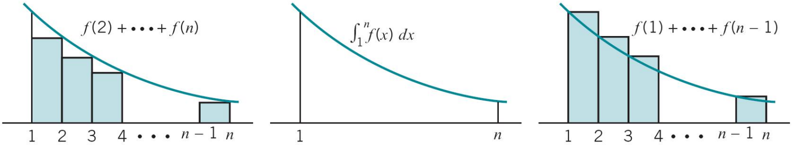

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$$\sum_{k=1}^{\infty} \frac{1}{k^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots \text{ converges} \quad \text{iff} \quad p > 1.$$

Proof.

$$\sum_{k=1}^{\infty} \frac{1}{k^p} \text{ converges} \quad \text{iff} \quad \int_1^{\infty} \frac{1}{x^p}\, dx \text{ converges.}$$

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Example ($p = 1$): Harmonic Series

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Harmonic Series ($p = 1$)

When $p = 1$, this is the harmonic series

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots,$$

and the related improper integral is

$$\int_1^{\infty} \frac{1}{x}\,dx = \ln x\Big|_1^{\infty} = \lim_{x\to\infty} \ln x = \infty.$$

Since this impproper integral diverges, so does the harmonic series.

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and the related improper integral is

$$\int_1^{\infty} \frac{1}{x^2}\,dx = -x^{-1}\Big|_1^{\infty} = 1 - \lim_{x\to\infty} x^{-1} = 1.$$

By the integral test,

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \sum_{k=2}^{\infty} \frac{1}{k^2} \le 1 + \int_1^{\infty} \frac{1}{x^2}\,dx = 2$$

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Example

Show that $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ diverges.

The related improper integral is

$$\int_2^\infty \frac{1}{x \ln x}\, dx = \int_{\ln 2}^\infty \frac{1}{u}\, du = \ln u \Big|_{\ln 2}^\infty = \lim_{x\to\infty} \ln x - \ln \ln 2 = \infty.$$

Since this impproper integral diverges, so does the infinite series.

Show that $\sum_{k=2}^{\infty} \frac{1}{k (\ln k)^2}$ converges.

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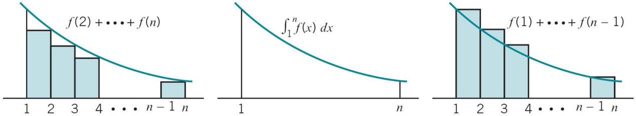
The Integral Test for the nth Remainder $\sum_{k=n+1}^{\infty} a_k$, $n \geq 1$



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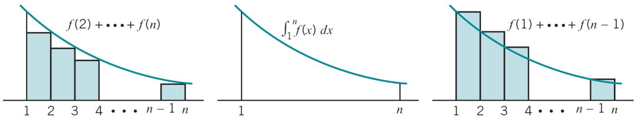

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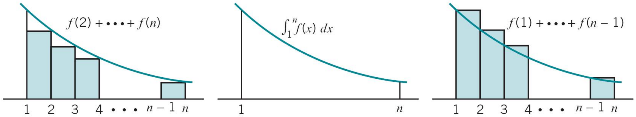

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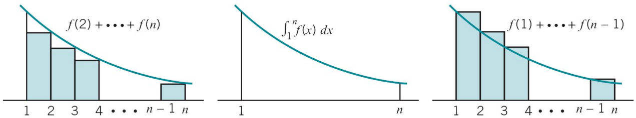

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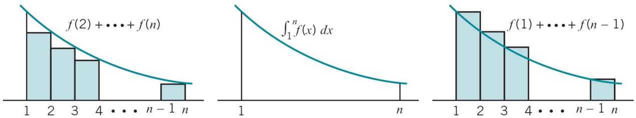

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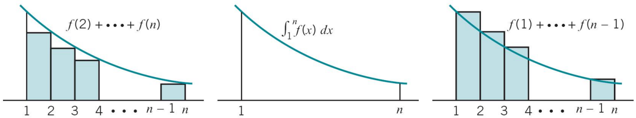

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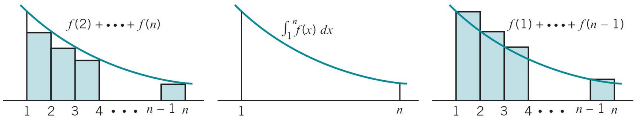

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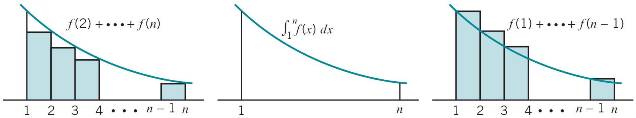

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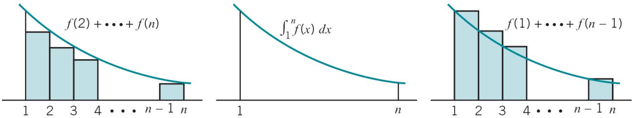

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Use a partial sum to approximate $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ with an error < 0.01

Estimating the nth remainder of the series

$$R_n = \sum_{k=1}^{\infty} \frac{1}{k^2+1} - \sum_{k=1}^{n} \frac{1}{k^2+1} < \int_n^{\infty} \frac{1}{x^2+1} dx$$

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Therefore

$$1.066\ldots < \sum_{k=1}^{\infty} \frac{1}{k^2+1} < 1.076\ldots$$

Example

Use a partial sum to approximate $\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ with an error < 0.01

Estimating the nth remainder of the series

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$$\sum_{k=1}^{\infty} a_k \text{ converges} \quad \text{iff} \quad \sum_{k=j}^{\infty} a_k \text{ converges, } \forall j \geq 1.$$

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

Geometric series: $\sum x^k$, if $|x| < 1$

p-series: $\sum \frac{1}{k^p}$, if $p > 1$

Basic Series that Diverge

Any series $\sum a_k$ for which $\lim_{k\to\infty} a_k \neq 0$

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Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

$$\text{If } \sum b_k \text{ converges, then so does } \sum a_k.$$

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Comparison with p-Series

$$\sum a_k \text{ converges if } \quad 0 \le a_k \le \frac{c}{k^p}, \quad p > 1.$$

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Examples

$$\sum \frac{1}{2k^3+1} \text{ converges by comparison with } \sum \frac{1}{k^3}$$

$$\frac{1}{2k^3+1} < \frac{1}{k^3} \quad \text{and} \quad \sum \frac{1}{k^3} \text{ converges.}$$

$$\sum \frac{k^3}{k^5+4k^4+7} \text{ converges by comparison with } \sum \frac{1}{k^2}$$

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$$\frac{1}{k^3-k^2} < \frac{1}{k^3-k^3/2} = \frac{2}{k^3},\ k \geq 2, \quad \text{and} \quad \sum \frac{2}{k^3} \text{ converges.}$$

Examples

$\sum \frac{1}{3k+1}$ diverges by comparison with $\sum \frac{1}{3(k+1)}$

$$\frac{1}{3k+1} > \frac{1}{3(k+1)} \quad \text{and} \quad \frac{1}{3}\sum \frac{1}{k+1} \text{ diverges.}$$

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$$\frac{\ln(k+6)}{k+6} \to 0 \text{ as } k \to \infty, \quad \ln(k+6) < k+6 \text{ for } k \text{ large}$$

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Limit Comparison Test

Basic Comparison Test

Suppose that $a_k > 0$ and $b_k > 0$ for sufficiently large k, and that $\lim_{k\to\infty} \frac{a_k}{b_k} = L$ for some $L > 0$.

$$\sum a_k \text{ converges} \quad \text{iff} \quad \sum b_k \text{ converges.}$$

$\lim_{k\to\infty} \frac{a_k}{b_k} = L$ means that $a_k \approx L b_k$ for large k

If $\lim_{k\to\infty} \frac{a_k}{b_k} = 0$, and

$$\text{if } \sum b_k \text{ converges, then} \quad \sum a_k \text{ converges.}$$

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Example

$\sum \frac{1}{k^3 - 1}$ converges by comparison with $\sum \frac{1}{k^3}$.

For large k, $\frac{1}{k^3 - 1}$ differs little from $\frac{1}{k^3}$.

$$\frac{1}{k^3 - 1} \div \frac{1}{k^3} = \frac{k^3}{k^3 - 1} \to 1 \text{ as } k \to \infty$$

and

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$$\sum \frac{3k^2+2k+1}{k^3+1} \text{ diverges by comparison with } \sum \frac{3}{k}$$

For large k, $\dfrac{3k^2+2k+1}{k^3+1}$ differs little from $\dfrac{3k^2}{k^3} = \dfrac{3}{k}$.

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$$\sum \frac{5\sqrt{k}+100}{2k^2\sqrt{k}-9\sqrt{k}}$$ converges by comparison with $$\sum \frac{5}{2k^2}$$

For large k, $\dfrac{5\sqrt{k}+100}{2k^2\sqrt{k}-9\sqrt{k}}$ differs little from $\dfrac{5\sqrt{k}}{2k^2\sqrt{k}} = \dfrac{5}{2k^2}$.

$$\frac{5\sqrt{k}+100}{2k^2\sqrt{k}-9\sqrt{k}} \div \frac{5}{2k^2} = \frac{10k^2\sqrt{k}+200k^2}{10k^2\sqrt{k}-45\sqrt{k}} \to 1 \text{ as } k \to \infty$$

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Outline

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  - Applying the Integral Test
  - Comparison Tests