Lecture 23 Section 11.3 The Root Test; The Ratio Test

Jiwen He

Department of Mathematics, University of Houston

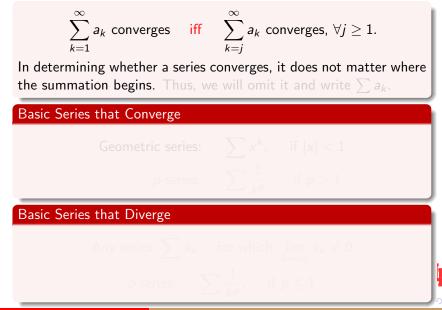
jiwenhe@math.uh.edu http://math.uh.edu/~jiwenhe/Math1432



Jiwen He, University of Houston





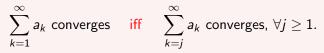


Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008

2 / 14



In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge



Basic Series that Diverge

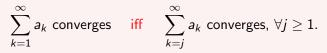
series $\sum a_k$ for which lim $a_k
eq 0$

Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

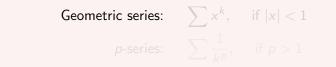
April 8, 2008

2 / 14



In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge



Basic Series that Diverge

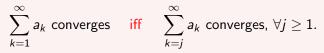
eries $\sum a_k$ for which lim $a_k
eq 0$

Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008

2 / 14



In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge



Basic Series that Diverge

ny series $\sum a_k$ for which $\lim_{k \to \infty} a_k
eq 0$

Jiwen He, University of Houston

$$\sum_{k=1}^{\infty} a_k$$
 converges iff $\sum_{k=j}^{\infty} a_k$ converges, $\forall j \ge 1$.

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge



Basic Series that Diverge

ny series $\sum a_k$ for which $\lim_{k o\infty}a_k
eq$

Jiwen He, University of Houston

$$\sum_{k=1}^{\infty} a_k$$
 converges iff $\sum_{k=j}^{\infty} a_k$ converges, $\forall j \ge 1$.

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

Basic Series that Diverge



Jiwen He, University of Houston

$$\sum_{k=1}^{\infty} a_k ext{ converges} \quad ext{iff} \quad \sum_{k=j}^{\infty} a_k ext{ converges}, \ orall j \geq 1.$$

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

Geometric series:
$$\sum x^k$$
, if $|x| < 1$
p-series: $\sum \frac{1}{k^p}$, if $p > 1$

Basic Series that Diverge

Any series
$$\sum a_k$$
 for which $\lim_{k \to \infty} a_k \neq 0$

Jiwen He, University of Houston

$$\sum_{k=1}^{\infty} a_k ext{ converges} \quad ext{iff} \quad \sum_{k=j}^{\infty} a_k ext{ converges}, \ orall j \geq 1.$$

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

Geometric series:
$$\sum x^k$$
, if $|x| < 1$
p-series: $\sum \frac{1}{k^p}$, if $p > 1$

Basic Series that Diverge

Any series
$$\sum a_k$$
 for which $\lim_{k \to \infty} a_k \neq 0$

Jiwen He, University of Houston

$$\sum_{k=1}^{\infty} a_k ext{ converges} \quad ext{iff} \quad \sum_{k=j}^{\infty} a_k ext{ converges}, \ orall j \geq 1.$$

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

Basic Series that Diverge

Any series
$$\sum a_k$$
 for which $\lim_{k \to \infty} a_k \neq 0$
p-series: $\sum \frac{1}{kp}$, if $p \leq 1$

Jiwen He, University of Houston

$$\sum_{k=1}^{\infty} a_k$$
 converges iff $\sum_{k=j}^{\infty} a_k$ converges, $\forall j \ge 1$.

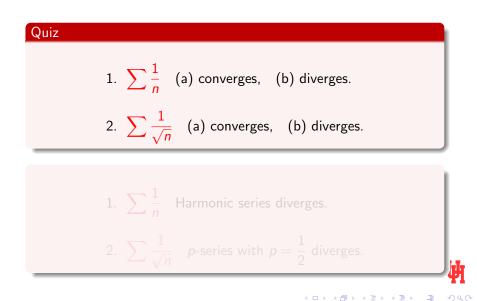
In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

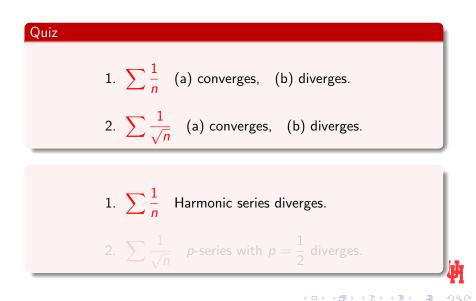
Basic Series that Diverge

Any series
$$\sum a_k$$
 for which $\lim_{k\to\infty} a_k \neq 0$
p-series: $\sum \frac{1}{k^p}$, if $p \leq 1$

Jiwen He, University of Houston

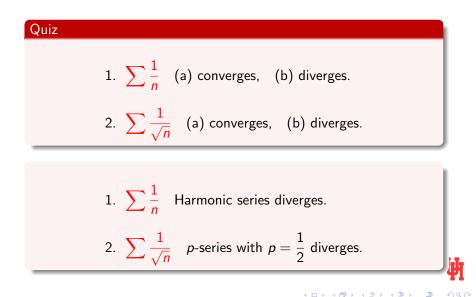


Jiwen He, University of Houston



Jiwen He, University of Houston

April 8, 2008 3 / 1



Jiwen He, University of Houston

April 8, 2008 3 / 1

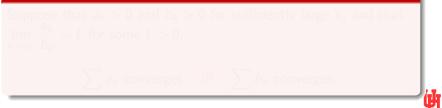
Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

If
$$\sum b_k$$
 converges, then so does $\sum a_k$

 $^{z}\sum a_{k}$ diverges, then so does $\sum b_{k}$

Limit Comparison Test





A (10) < A (10) </p>

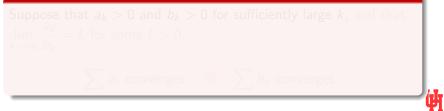
Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.



f $\sum a_k$ diverges, then so does $\sum b_k$

Limit Comparison Test



Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

f
$$\sum b_k$$
 converges, then so does $\sum a_k$

 $\sum a_k$ diverges, then so does $\sum \sum$

Limit Comparison Test



A (10) F (10)

Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

If
$$\sum b_k$$
 converges, then so does $\sum a_k$.
If $\sum a_k$ diverges, then so does $\sum b_k$.

Limit Comparison Test

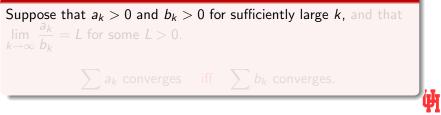


Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

If
$$\sum b_k$$
 converges, then so does $\sum a_k$.
If $\sum a_k$ diverges, then so does $\sum b_k$.

Limit Comparison Test



Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

If
$$\sum b_k$$
 converges, then so does $\sum a_k$.
If $\sum a_k$ diverges, then so does $\sum b_k$.

Limit Comparison Test

Suppose that $a_k > 0$ and $b_k > 0$ for sufficiently large k, and that $\lim_{k \to \infty} \frac{a_k}{b_k} = L \text{ for some } L > 0.$ $\sum a_k \text{ converges} \quad \text{iff} \quad \sum b_k \text{ converges.}$

Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

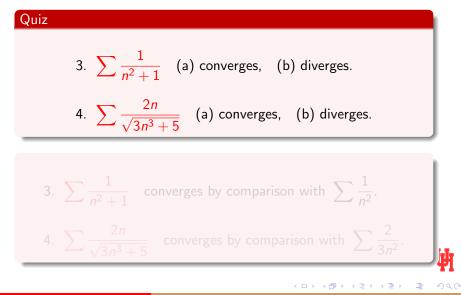
If
$$\sum b_k$$
 converges, then so does $\sum a_k$.
If $\sum a_k$ diverges, then so does $\sum b_k$.

Limit Comparison Test

Suppose that
$$a_k > 0$$
 and $b_k > 0$ for sufficiently large k , and that

$$\lim_{k \to \infty} \frac{a_k}{b_k} = L \text{ for some } L > 0.$$

$$\sum a_k \text{ converges iff } \sum b_k \text{ converges.}$$



Quiz 3. $\sum \frac{1}{n^2 + 1}$ (a) converges, (b) diverges. 4. $\sum \frac{2n}{\sqrt{3n^3 + 5}}$ (a) converges, (b) diverges.



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008 5 / 1

イロト 不得下 イヨト イヨト 二日

Quiz

3.
$$\sum \frac{1}{n^2 + 1}$$
 (a) converges, (b) diverges.
4. $\sum \frac{2n}{\sqrt{3n^3 + 5}}$ (a) converges, (b) diverges.

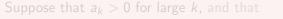
3.
$$\sum \frac{1}{n^2 + 1}$$
 converges by comparison with $\sum \frac{1}{n^2}$.
4. $\sum \frac{2n}{\sqrt{3n^3 + 5}}$ converges by comparison with $\sum \frac{2}{3n^2}$.

・ロト ・ 日 ト ・ ヨ ト ・ ヨ ト

Comparison Tests Root Test Ratio Test Root Test

The Root Test: Comparison with Geometric Series

Root Test



$$\lim_{\to\infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

Comparison with Geometric Series



Comparison Tests Root Test Ratio Test Root Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

Comparison with Geometric Series



Root Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If $\rho = 1$, then the test is inconclusive.

Comparison with Geometric Series

• If $\sum a_k$ is a geometric series, e.g., $\sum \rho^k$, $\rho > 0$, then $(a_k)^{\frac{1}{k}}$ is constant, i.e., ρ . If $\rho < 1$, then $\sum a_k$ converges. If $\rho \ge 1$, then $\sum a_k$ diverges.

• If $\lim_{k o\infty}(a_k)^{ar{k}}=
ho<1$, then for large $k,~a_k<\mu^k$ with

1. By the basic comparison test, $\sum a_k$ converge



Root Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If ho=1, then the test is inconclusive.

Comparison with Geometric Series

- If ∑ a_k is a geometric series, e.g., ∑ ρ^k, ρ > 0, then (a_k)[†] is constant, i.e., ρ. If ρ < 1, then ∑ a_k converges. If ρ ≥ 1, then ∑ a_k diverges.
- If $\lim_{k \to \infty} (a_k)^{rac{1}{k}} =
 ho < 1$, then for large $k, \; a_k < \mu^k$ with
 - k < 1. By the basic comparison test, $\sum a_k$ converges



oot Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If ho = 1, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a geometric series, e.g., ∑ ρ^k, ρ > 0, then (a_k)[‡] is constant, i.e., ρ. If ρ < 1, then ∑ a_k converges. If ρ ≥ 1, then ∑ a_k diverges.

• If $\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho < 1$, then for large $k, a_k < \mu^k$ with

 $p < \mu < 1.$ By the basic comparison test, $\sum a_k$ converges.



loot Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If $\rho = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a geometric series, e.g., ∑ ρ^k, ρ > 0, then (a_k)[±]/_k is constant, i.e., ρ. If ρ < 1, then ∑ a_k converges. If ρ ≥ 1, then ∑ a_k diverges.

• If $\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho < 1$, then for large k, $a_k < \mu^k$ with $\rho < \mu < 1$. By the basic comparison test $\sum a_k$ converge



April 8, 2008 6 /

oot Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If $\rho = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a geometric series, e.g., ∑ ρ^k, ρ > 0, then (a_k)^{1/k} is constant, i.e., ρ. If ρ < 1, then ∑ a_k converges. If ρ ≥ 1, then ∑ a_k diverges.

• If $\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \rho < 1$, then for large k, $a_k < \mu^k$ with $\rho < \mu < 1$. By the basic comparison test, $\sum a_k$ converges.



April 8, 2008 6 /

oot Test

The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If $\rho = 1$, then the test is inconclusive.

Comparison with Geometric Series

• If $\sum a_k$ is a geometric series, e.g., $\sum \rho^k$, $\rho > 0$, then $(a_k)^{\frac{1}{k}}$ is constant, i.e., ρ . If $\rho < 1$, then $\sum a_k$ converges. If $\rho \ge 1$, then $\sum a_k$ diverges.

• If
$$\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \rho < 1$$
, then for large k , $a_k < \mu^k$ with $\rho < \mu < 1$. By the basic comparison test, $\sum a_k$ converges.



Examples

 $\sum \frac{k^2}{2^k}$



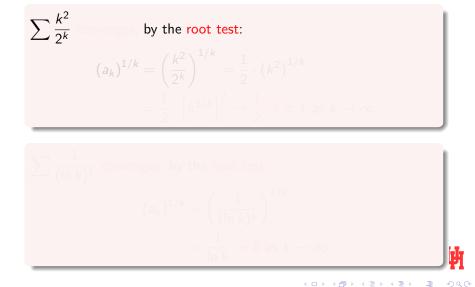
Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

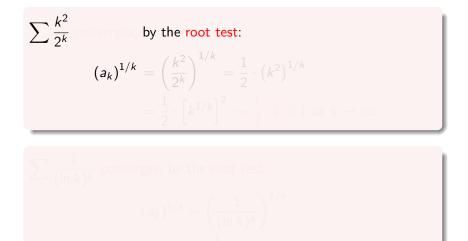
▲ 🗈 🕨 🚊 🥠 April 8, 2008 7 /

<ロ> (日) (日) (日) (日) (日)

Examples



Jiwen He, University of Houston



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 2

・ロン ・四 ・ ・ ヨン ・ ヨン

Root Test

Examples

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot \left(k^2\right)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 2

 $\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$ $(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot \left(k^2\right)^{1/k}$ $= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$



Jiwen He, University of Houston

 $\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$ $(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot (k^2)^{1/k}$ $= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \rightarrow \frac{1}{2} \cdot 1 < 1 \text{ as } k \rightarrow \infty$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 2

3

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot (k^2)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{(\ln k)^k} \text{ converges, by the root test:}$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

・ < E ト - 是 - ク April 8, 2008 - 7 /

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot (k^2)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{(\ln k)^k} \text{ converges, by the root test:}$$



Jiwen He, University of Houston

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot \left(k^2\right)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

$\sum \frac{1}{(\ln k)^k} \simeq$	inverges, by the root test:	
	$(a_k)^{1/k} = \left(\frac{1}{(\ln k)^k}\right)^{1/k}$	
	$=rac{1}{\ln k} o 0$ as $k o\infty$	#

Jiwen He, University of Houston

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot (k^2)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \rightarrow \frac{1}{2} \cdot 1 < 1 \text{ as } k \rightarrow \infty$$

 $\sum \frac{1}{(\ln k)^k} \text{ converges, by the root test:} \\ (a_k)^{1/k} = \left(\frac{1}{(\ln k)^k}\right)^{1/k} \\ = \frac{1}{\ln k} \to 0 \text{ as } k \to \infty$

Jiwen He, University of Houston

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot \left(k^2\right)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

 $\sum \frac{1}{(\ln k)^k}$ by the root test: $(a_k)^{1/k} = \left(\frac{1}{(\ln k)^k}\right)^{1/k}$ $=rac{1}{\ln k}
ightarrow 0$ as $k
ightarrow\infty$ h

Jiwen He, University of Houston

∃ →

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot (k^2)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{(\ln k)^k} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(\frac{1}{(\ln k)^k}\right)^{1/k}$$
$$= \frac{1}{\ln k} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

4

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot \left(k^2\right)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{(\ln k)^k} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(\frac{1}{(\ln k)^k}\right)^{1/k}$$
$$= \frac{1}{\ln k} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

4

イロト イヨト イヨト イヨト

呐

$$\sum \frac{k^2}{2^k} \text{ converges, by the root test:}$$

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2} \cdot \left(k^2\right)^{1/k}$$

$$= \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{(\ln k)^k} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(\frac{1}{(\ln k)^k}\right)^{1/k}$$
$$= \frac{1}{\ln k} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

4

<ロ> (日) (日) (日) (日) (日)

呐

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$

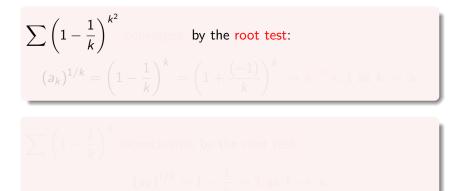
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^{k} \text{ inconclusive, by the root test:}$$

$$(a_{k})^{1/k} = 1 - \frac{1}{k} \to 1 \text{ as } k \to \infty$$
the series diverges since $a_{k} = \left(1 - \frac{1}{k}\right)^{k} \to e^{-1} \neq 0$.



Jiwen He, University of Houston



the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^k
ightarrow e^{-1}
eq 0.$



Jiwen He, University of Houston

▲ **■ ▶ ■ 少**へ April 8, 2008 8 / 1

- 4 回 ト - 4 回 ト

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

 $\sum \left(1-rac{1}{k}
ight)^k$ inconclusive, by the root test: $(a_k)^{1/k}=1-rac{1}{k} o 1$ as $k o\infty$ the series diverges since $a_k=\left(1-rac{1}{k}
ight)^k o e^{-1}
eq 0$.



Jiwen He, University of Houston

▲ **ミト ミーク**へ April 8, 2008 8 / 1

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k \text{ inconclusive, by the root test:}$$
$$(a_k)^{1/k} = 1 - \frac{1}{k} \to 1 \text{ as } k \to \infty$$
the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^k \to e^{-1} \neq 0.$



Jiwen He, University of Houston

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k$$
 inconclusive, by the root test:
 $(a_k)^{1/k} = 1 - \frac{1}{k} \to 1$ as $k \to \infty$
the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^k \to e^{-1} \neq 0$.



Jiwen He, University of Houston

April 8, 2008 8 / 3

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1-rac{1}{k}
ight)^k$$
 inconclusive, by the root test: $(a_k)^{1/k}=1-rac{1}{k} o 1$ as $k o\infty$ the series diverges since $a_k=\left(1-rac{1}{k}
ight)^k o e^{-1}
eq 0.$



Jiwen He, University of Houston

▶ **ब हे रे ब र**े २ April 8, 2008 8 / 3

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - rac{1}{k}
ight)^k$$
 inconclusive, by the root test: $(a_k)^{1/k} = 1 - rac{1}{k} o 1$ as $k o \infty$ the series diverges since $a_k = \left(1 - rac{1}{k}
ight)^k o e^{-1}
eq 0$.



Jiwen He, University of Houston

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k$$
 inconclusive, by the root test:
 $(a_k)^{1/k} = 1 - \frac{1}{k} \to 1$ as $k \to \infty$
the series diverges since $a_k = (1 - \frac{1}{k})^k \to e^{-1} \neq 0$.

仲

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k$$
 inconclusive, by the root test:
 $(a_k)^{1/k} = 1 - \frac{1}{k} \to 1$ as $k \to \infty$
the series diverges since $a_k = (1 - \frac{1}{k})^k \to e^{-1} \neq 0$.

Jiwen He, University of Houston

▶ **▲ ≣ ▶ ≣ ∽** ۹ April 8, 2008 8 / 1

イロト イヨト イヨト イヨト

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1-rac{1}{k}
ight)^k$$
 inconclusive, by the root test: $(a_k)^{1/k}=1-rac{1}{k} o 1$ as $k o\infty$

the series diverges since $a_k = (1 - \frac{1}{\nu})^k \rightarrow e^{-1} \neq 0$.

Jiwen He, University of Houston

イロト イヨト イヨト イヨト

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - rac{1}{k}
ight)^k$$
 inconclusive, by the root test: $(a_k)^{1/k} = 1 - rac{1}{k} o 1$ as $k o \infty$

the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^{\kappa} \to e^{-1} \neq 0$.

Jiwen He, University of Houston

イロト イ団ト イヨト イヨト

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k$$
 inconclusive, by the root test: $(a_k)^{1/k} = 1 - \frac{1}{k} o 1$ as $k o \infty$

the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^{\kappa}
ightarrow e^{-1}
eq 0.$

Jiwen He, University of Houston

(日) (同) (三) (三)

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1-rac{1}{k}
ight)^k$$
 inconclusive, by the root test: $(a_k)^{1/k} = 1-rac{1}{k} o 1$ as $k o \infty$

the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^k \rightarrow e^{-1} \neq 0$.

Jiwen He, University of Houston

- 4 回 2 - 4 □ 2 - 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □ 0 − 4 □

Ý

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k \text{ inconclusive, by the root test:}$$
$$(a_k)^{1/k} = 1 - \frac{1}{k} \to 1 \text{ as } k \to \infty$$
the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^k \to e^{-1} \neq 0.$

Jiwen He, University of Houston

イロト イヨト イヨト イヨト

$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges, by the root test:}$$
$$(a_k)^{1/k} = \left(1 - \frac{1}{k}\right)^k = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1} < 1 \text{ as } k \to \infty$$

$$\sum \left(1 - \frac{1}{k}\right)^k \text{ inconclusive, by the root test:}$$
$$(a_k)^{1/k} = 1 - \frac{1}{k} \to 1 \text{ as } k \to \infty$$
the series diverges since $a_k = \left(1 - \frac{1}{k}\right)^k \to e^{-1} \neq 0.$

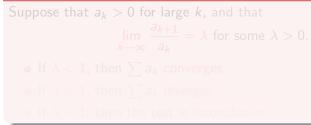
Jiwen He, University of Houston

イロト イヨト イヨト イヨト

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test



Comparison with Geometric Series



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008 9 / 1

屮

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

 $\max_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$

- If $\lambda < 1$, then $\sum a_k$ converges.
- If $\lambda > 1$, then $\sum a_k$ diverges.

• If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series



屮

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

• If $\lambda < 1$, then $\sum a_k$ converges.

• If $\lambda > 1$, then $\sum a_k$ diverges.

• If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series



H

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

• If $\lambda < 1$, then $\sum a_k$ converges.

• If $\lambda > 1$, then $\sum a_k$ diverges.

• If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a multiple of a geometric series, e.g., ∑ c λ^k, λ > 0, then a_{k+1}/a_k is constant, i.e., λ. If λ < 1, then ∑ a_k converges. If λ ≥ 1, then ∑ a_k diverges.
If lim a_{k+1}/a_k = λ < 1, then for large k, a_k < cμ^k with λ < μ < 1. By the basic comparison test, ∑ a_k converges.



Math 1432 - Section 26626, Lecture 23

屮

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

- If $\lambda < 1$, then $\sum a_k$ converges.
- If $\lambda > 1$, then $\sum a_k$ diverges.
- If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a multiple of a geometric series, e.g., ∑ c λ^k, λ > 0, then a_{k+1}/a_k is constant, i.e., λ. If λ < 1, then ∑ a_k converges. If λ ≥ 1, then ∑ a_k diverges.
If lim a_{k+1}/a_k = λ < 1, then for large k, a_k < cμ^k with λ < μ < 1. By the basic comparison test, ∑ a_k converges.

屮

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{n \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

- If $\lambda < 1$, then $\sum a_k$ converges.
- If $\lambda > 1$, then $\sum a_k$ diverges.
- If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a multiple of a geometric series, e.g., ∑ c λ^k, λ > 0, then a_{k+1}/a_k is constant, i.e., λ. If λ < 1, then ∑ a_k converges. If λ ≥ 1, then ∑ a_k diverges.
If lim a_{k+1}/a_k = λ < 1, then for large k, a_k < cμ^k with λ < μ < 1. By the basic comparison test, ∑ a_k converges.

Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008 9 / 1

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

- If $\lambda < 1$, then $\sum a_k$ converges.
- If $\lambda > 1$, then $\sum a_k$ diverges.
- If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a multiple of a geometric series, e.g., ∑ c λ^k, λ > 0, then a_{k+1}/a_k is constant, i.e., λ. If λ < 1, then ∑ a_k converges. If λ ≥ 1, then ∑ a_k diverges.

• If $\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda < 1$, then for large k, $a_k < c\mu^k$ with $\lambda < \mu < 1$. By the basic comparison test, $\sum a_k$ converges.

Ħ

Ratio Test

The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

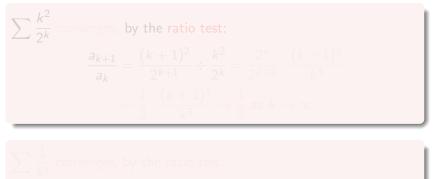
$$\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

- If $\lambda < 1$, then $\sum a_k$ converges.
- If $\lambda > 1$, then $\sum a_k$ diverges.
- If $\lambda = 1$, then the test is inconclusive.

Comparison with Geometric Series

If ∑ a_k is a multiple of a geometric series, e.g., ∑ c λ^k, λ > 0, then a_{k+1}/a_k is constant, i.e., λ. If λ < 1, then ∑ a_k converges. If λ ≥ 1, then ∑ a_k diverges.
If lim a_{k+1}/a_k = λ < 1, then for large k, a_k < cμ^k with λ < μ < 1. By the basic comparison test, ∑ a_k converges.

버

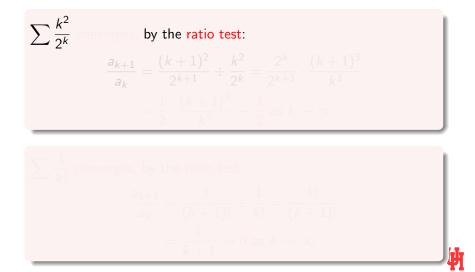


$$\frac{a_{k+1}}{a_k} = \frac{1}{(k+1)!} \div \frac{1}{k!} = \frac{k!}{(k+1)!}$$
$$= \frac{1}{k+1} \to 0 \text{ as } k \to \infty$$

搟

Jiwen He, University of Houston

3



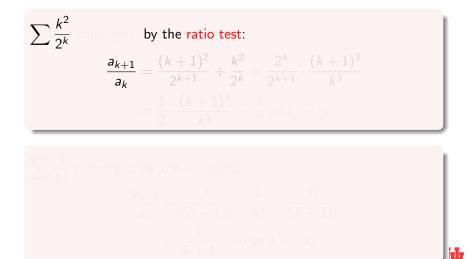
Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

pril 8, 2008 10

3

イロト イヨト イヨト イヨト



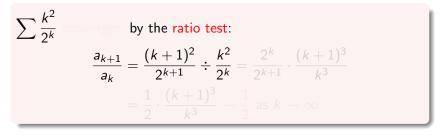
Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

pril 8, 2008 1

3

- 4 同 6 4 日 6 4 日 6





Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

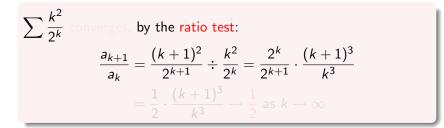
oril 8, 2008 10

3

- 4 週 ト - 4 三 ト - 4 三 ト

Ratio Test

Examples





Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

pril 8, 2008 1

Ratio Test

Examples

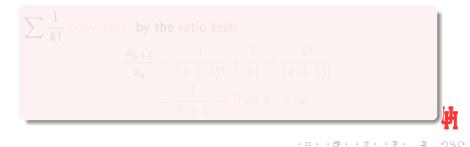
$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Ratio Test

Examples

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

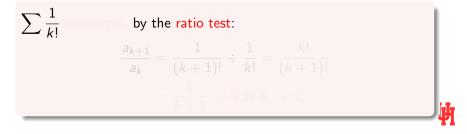
oril 8, 2008 10

3

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$

$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

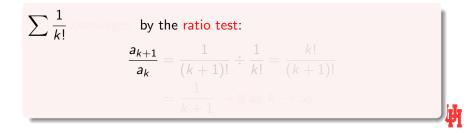
3

(本間) (本語) (本語)

Ratio Test

Examples

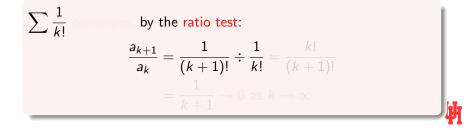
$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

oril 8, 2008 10

э

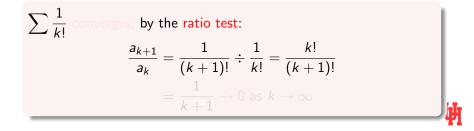
3

▲ 同 ▶ → 三 ▶

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$

$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$



Jiwen He, University of Houston

oril 8, 2008 10

< A > < 3

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$

$$\sum \frac{1}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{1}{(k+1)!} \div \frac{1}{k!} = \frac{k!}{(k+1)!}$$

$$= \frac{1}{k+1} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

3

イロト イヨト イヨト イヨト

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$

$$\sum \frac{1}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{1}{(k+1)!} \div \frac{1}{k!} = \frac{k!}{(k+1)!}$$
$$= \frac{1}{k+1} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

pril 8, 2008 10

4

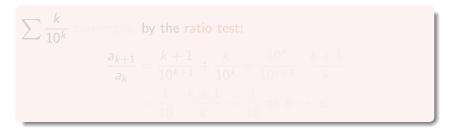
イロト イヨト イヨト イヨト

$$\sum \frac{k^2}{2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$
$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$

$$\sum \frac{1}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{1}{(k+1)!} \div \frac{1}{k!} = \frac{k!}{(k+1)!}$$
$$= \frac{1}{k+1} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

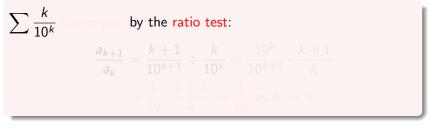
4





Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23



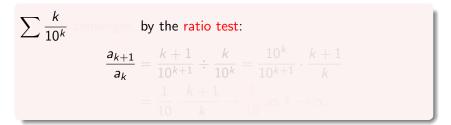


Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

oril 8, 2008 11

3

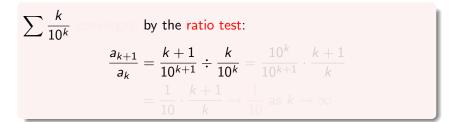




Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

oril 8, 2008 11





Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

oril 8, 2008 11

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$

$$= \frac{1}{10} \cdot \frac{k+1}{k} \rightarrow \frac{1}{10} \text{ as } k \rightarrow \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

$$\sum \frac{k}{10^{k}} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_{k}} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^{k}} = \frac{10^{k}}{10^{k+1}} \cdot \frac{k+1}{k}$$

$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

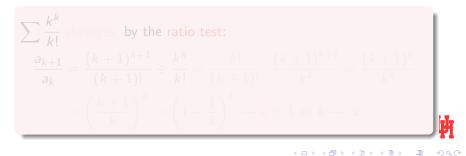
$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



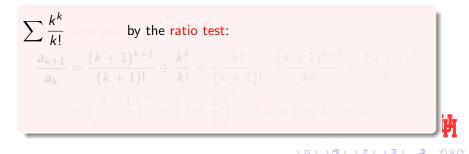
Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

$$\sum \frac{k}{10^{k}} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_{k}} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^{k}} = \frac{10^{k}}{10^{k+1}} \cdot \frac{k+1}{k}$$

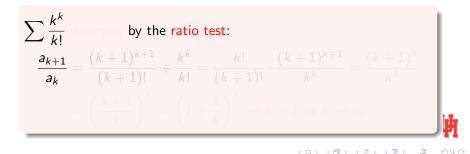
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

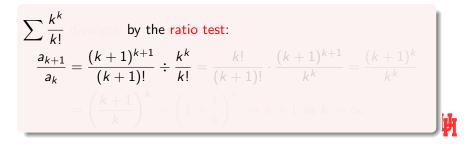
$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

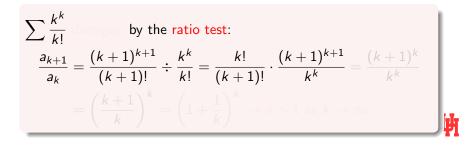
▲ 클 ▶ · 클 · ∽ ril 8, 2008 11

(日) (同) (三) (三)

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$

$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

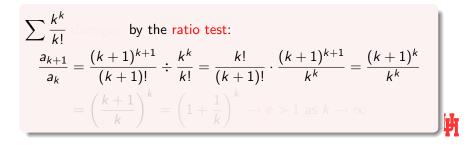
▲ 클 ▶ · 클 · ∽ ril 8, 2008 11

▲ □ ► ▲ □ ► ▲

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$

$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$



Jiwen He, University of Houston

oril 8, 2008 11

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$

$$\sum \frac{k^k}{k!} \text{ diverges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^{k+1}}{(k+1)!} \div \frac{k^k}{k!} = \frac{k!}{(k+1)!} \cdot \frac{(k+1)^{k+1}}{k^k} = \frac{(k+1)^k}{k^k}$$

$$= \left(\frac{k+1}{k}\right)^k = \left(1 + \frac{1}{k}\right)^k \rightarrow e > 1 \text{ as } k \rightarrow \infty$$

Jiwen He, University of Houston

oril 8, 2008 11

4

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$

$$\sum \frac{k^k}{k!} \text{ diverges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^{k+1}}{(k+1)!} \div \frac{k^k}{k!} = \frac{k!}{(k+1)!} \cdot \frac{(k+1)^{k+1}}{k^k} = \frac{(k+1)^k}{k^k}$$

$$= \left(\frac{k+1}{k}\right)^k = \left(1 + \frac{1}{k}\right)^k \rightarrow e > 1 \text{ as } k \rightarrow \infty$$

Jiwen He, University of Houston

4

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$

$$\sum \frac{k^k}{k!} \text{ diverges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^{k+1}}{(k+1)!} \div \frac{k^k}{k!} = \frac{k!}{(k+1)!} \cdot \frac{(k+1)^{k+1}}{k^k} = \frac{(k+1)^k}{k^k}$$

$$= \left(\frac{k+1}{k}\right)^k = \left(1 + \frac{1}{k}\right)^k \to e > 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

4

$$\sum \frac{k}{10^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{k+1}{10^{k+1}} \div \frac{k}{10^k} = \frac{10^k}{10^{k+1}} \cdot \frac{k+1}{k}$$
$$= \frac{1}{10} \cdot \frac{k+1}{k} \to \frac{1}{10} \text{ as } k \to \infty$$

$$\sum \frac{k^k}{k!} \text{ diverges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^{k+1}}{(k+1)!} \div \frac{k^k}{k!} = \frac{k!}{(k+1)!} \cdot \frac{(k+1)^{k+1}}{k^k} = \frac{(k+1)^k}{k^k}$$

$$= \left(\frac{k+1}{k}\right)^k = \left(1 + \frac{1}{k}\right)^k \to e > 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

pril 8, 2008 11

-2

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$

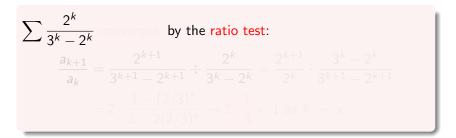
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

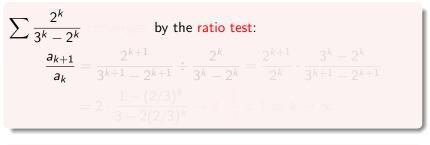




Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008





Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008

Ratio Test

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

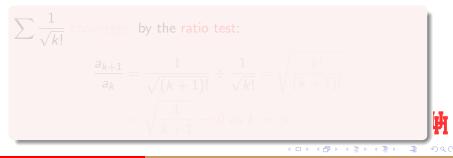
			l lelle
			- 19

Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$



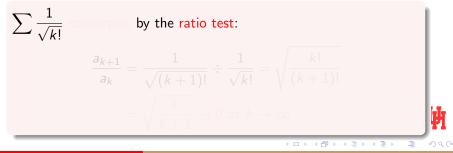
Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

April 8, 2008

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

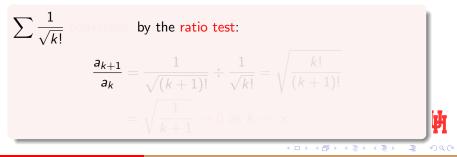


Jiwen He, University of Houston

Math 1432 - Section 26626, Lecture 23

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

 $\sum \frac{1}{\sqrt{k!}} \text{ converges, by the ratio test:}$ $\frac{a_{k+1}}{a_k} = \frac{1}{\sqrt{(k+1)!}} \div \frac{1}{\sqrt{k!}} = \sqrt{\frac{k!}{(k+1)!}}$ $= \sqrt{\frac{1}{k+1}} \to 0 \text{ as } k \to \infty$

Jiwen He, University of Houston

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

 $\sum \frac{1}{\sqrt{k!}} \text{ converges, by the ratio test:}$ $\frac{a_{k+1}}{a_k} = \frac{1}{\sqrt{(k+1)!}} \div \frac{1}{\sqrt{k!}} = \sqrt{\frac{k!}{(k+1)!}}$ $= \sqrt{\frac{1}{k+1}} \to 0 \text{ as } k \to \infty$

Jiwen He, University of Houston

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

 $\sum \frac{1}{\sqrt{k!}} \text{ converges, by the ratio test:}$ $\frac{a_{k+1}}{a_k} = \frac{1}{\sqrt{(k+1)!}} \div \frac{1}{\sqrt{k!}} = \sqrt{\frac{k!}{(k+1)!}}$ $= \sqrt{\frac{1}{k+1}} \to 0 \text{ as } k \to \infty$

Jiwen He, University of Houston

Examples

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

 $\sum \frac{1}{\sqrt{k!}} \text{ converges, by the ratio test:}$ $\frac{a_{k+1}}{a_k} = \frac{1}{\sqrt{(k+1)!}} \div \frac{1}{\sqrt{k!}} = \sqrt{\frac{k!}{(k+1)!}}$ $= \sqrt{\frac{1}{k+1}} \to 0 \text{ as } k \to \infty$

Jiwen He, University of Houston

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{2^{k+1}}{3^{k+1} - 2^{k+1}} \div \frac{2^k}{3^k - 2^k} = \frac{2^{k+1}}{2^k} \cdot \frac{3^k - 2^k}{3^{k+1} - 2^{k+1}}$$
$$= 2 \cdot \frac{1 - (2/3)^k}{3 - 2(2/3)^k} \to 2 \cdot \frac{1}{3} < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{\sqrt{k!}} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{1}{\sqrt{(k+1)!}} \div \frac{1}{\sqrt{k!}} = \sqrt{\frac{k!}{(k+1)!}}$$

$$= \sqrt{\frac{1}{k+1}} \to 0 \text{ as } k \to \infty$$

Jiwen He, University of Houston

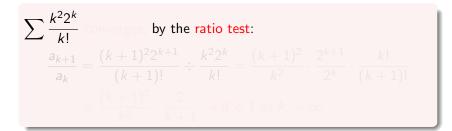




Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

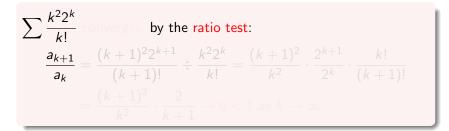




Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008





Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

Examples

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

Examples

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$
$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

Examples

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

April 8, 2008

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

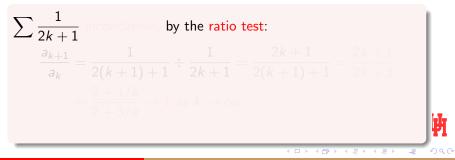


Jiwen He, University of Houston

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



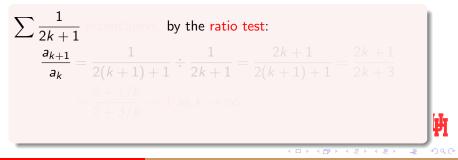
Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



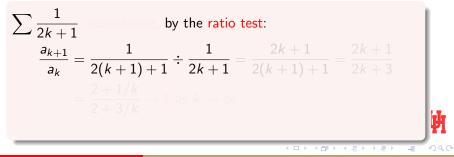
Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$



Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1} \text{ inconclusive, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{1}{2(k+1)+1} \div \frac{1}{2k+1} = \frac{2k+1}{2(k+1)+1} = \frac{2k+1}{2k+3}$$

$$= \frac{2+1/k}{2+3/k} \to 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1} \text{ inconclusive, by the ratio test:} \\ \frac{a_{k+1}}{a_k} = \frac{1}{2(k+1)+1} \div \frac{1}{2k+1} = \frac{2k+1}{2(k+1)+1} = \frac{2k+1}{2k+3} \\ = \frac{2+1/k}{2+3/k} \to 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

Math 1432 – Section 26626, Lecture 23

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1} \text{ inconclusive, by the ratio test:}
\frac{a_{k+1}}{a_k} = \frac{1}{2(k+1)+1} \div \frac{1}{2k+1} = \frac{2k+1}{2(k+1)+1} = \frac{2k+1}{2k+3}
= \frac{2+1/k}{2+3/k} \to 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

< □ >

April 8, 2008

-2

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1} \text{ inconclusive, by the ratio test:}
\frac{a_{k+1}}{a_k} = \frac{1}{2(k+1)+1} \div \frac{1}{2k+1} = \frac{2k+1}{2(k+1)+1} = \frac{2k+1}{2k+3}
= \frac{2+1/k}{2+3/k} \to 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

< □ >

April 8, 2008

-2

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$

$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1} \text{ inconclusive, by the ratio test:}
\frac{a_{k+1}}{a_k} = \frac{1}{2(k+1)+1} \div \frac{1}{2k+1} = \frac{2k+1}{2(k+1)+1} = \frac{2k+1}{2k+3}
= \frac{2+1/k}{2+3/k} \to 1 \text{ as } k \to \infty$$

Jiwen He, University of Houston

< □ >

-2

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$
$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

 $\sum \frac{1}{2k+1}$ inconclusive, by the ratio test: the series diverges by the limit comparison test: $\frac{1}{2k+1} \div \frac{1}{2k} = \frac{2k}{2k+1} \rightarrow 1 \text{ as } k \rightarrow \infty \text{ and } \sum \frac{1}{2k} \text{ diverges.}$

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$
$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1} \text{ inconclusive, by the ratio test:}$$

the series diverges by the limit comparison test:
$$\frac{1}{2k+1} \div \frac{1}{2k} = \frac{2k}{2k+1} \rightarrow 1 \text{ as } k \rightarrow \infty \text{ and } \sum \frac{1}{2k} \text{ diverges.}$$

Jiwen He, University of Houston

イロト イヨト イヨト イヨト

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$
$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1}$$
 inconclusive, by the ratio test:
the series diverges by the limit comparison test:
$$\frac{1}{2k+1} \div \frac{1}{2k} = \frac{2k}{2k+1} \rightarrow 1 \text{ as } k \rightarrow \infty \text{ and } \sum \frac{1}{2k} \text{ diverges.}$$

Jiwen He, University of Houston

-∢≣⇒

A = A + A = A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

$$\sum \frac{k^2 2^k}{k!} \text{ converges, by the ratio test:}$$
$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2 2^{k+1}}{(k+1)!} \div \frac{k^2 2^k}{k!} = \frac{(k+1)^2}{k^2} \cdot \frac{2^{k+1}}{2^k} \cdot \frac{k!}{(k+1)!}$$
$$= \frac{(k+1)^2}{k^2} \cdot \frac{2}{k+1} \to 0 < 1 \text{ as } k \to \infty$$

$$\sum \frac{1}{2k+1}$$
 inconclusive, by the ratio test:
the series diverges by the limit comparison test:
$$\frac{1}{2k+1} \div \frac{1}{2k} = \frac{2k}{2k+1} \rightarrow 1 \text{ as } k \rightarrow \infty \text{ and } \sum \frac{1}{2k} \text{ diverges.}$$

Jiwen He, University of Houston

pril 8, 2008 13 /

4

イロト イヨト イヨト イヨト

Outline

- Comparison Tests
- The Root Test
 The Root Test
- The Ratio Test
 The Ratio Test



Jiwen He, University of Houston