Lecture 23

Section 11.3 The Root Test; The Ratio Test

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Basic Series that Converge or Diverge

$$\sum_{k=1}^{\infty} a_k \text{ converges} \quad \text{iff} \quad \sum_{k=j}^{\infty} a_k \text{ converges, } \forall j \geq 1.$$

In determining whether a series converges, it does not matter where the summation begins. Thus, we will omit it and write $\sum a_k$.

Basic Series that Converge

Geometric series: $\sum x^k$, if $|x| < 1$

p-series: $\sum \frac{1}{k^p}$, if $p > 1$

Basic Series that Diverge

Any series $\sum a_k$ for which $\lim_{k\to\infty} a_k \neq 0$

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1. $\sum \frac{1}{n}$ (a) converges, (b) diverges.
2. $\sum \frac{1}{\sqrt{n}}$ (a) converges, (b) diverges.

1. $\sum \frac{1}{n}$ Harmonic series diverges.
2. $\sum \frac{1}{\sqrt{n}}$ p-series with $p = \frac{1}{2}$ diverges.

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Basic Comparison Test

Suppose that $0 \le a_k \le b_k$ for sufficiently large k.

$$\text{If } \sum b_k \text{ converges, then so does } \sum a_k.$$
$$\text{If } \sum a_k \text{ diverges, then so does } \sum b_k.$$

Limit Comparison Test

Suppose that $a_k > 0$ and $b_k > 0$ for sufficiently large k, and that $\lim_{k\to\infty} \frac{a_k}{b_k} = L$ for some $L > 0$.

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3. $\sum \frac{1}{n^2+1}$ (a) converges, (b) diverges.

4. $\sum \frac{2n}{\sqrt{3n^3+5}}$ (a) converges, (b) diverges.

3. $\sum \frac{1}{n^2+1}$ converges by comparison with $\sum \frac{1}{n^2}$.

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The Root Test: Comparison with Geometric Series

Root Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \rho \text{ for some } \rho > 0.$$

- If $\rho < 1$, then $\sum a_k$ converges.
- If $\rho > 1$, then $\sum a_k$ diverges.
- If $\rho = 1$, then the test is inconclusive.

Comparison with Geometric Series

- If $\sum a_k$ is a geometric series, e.g., $\sum \rho^k$, $\rho > 0$, then $(a_k)^{\frac{1}{k}}$ is constant, i.e., ρ. If $\rho < 1$, then $\sum a_k$ converges. If $\rho \geq 1$, then $\sum a_k$ diverges.
- If $\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \rho < 1$, then for large k, $a_k < \mu^k$ with $\rho < \mu < 1$. By the basic comparison test, $\sum a_k$ converges.

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Examples

$\sum \frac{k^2}{2^k}$ converges, by the root test:

$$(a_k)^{1/k} = \left(\frac{k^2}{2^k}\right)^{1/k} = \frac{1}{2}\cdot\left(k^2\right)^{1/k}$$
$$= \frac{1}{2}\cdot\left[k^{1/k}\right]^2 \to \frac{1}{2}\cdot 1 < 1 \text{ as } k \to \infty$$

$\sum \frac{1}{(\ln k)^k}$ converges, by the root test:

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The Ratio Test: Comparison with Geometric Series

Ratio Test

Suppose that $a_k > 0$ for large k, and that

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lambda \text{ for some } \lambda > 0.$$

- If $\lambda < 1$, then $\sum a_k$ converges.
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Comparison with Geometric Series

- If $\sum a_k$ is a multiple of a geometric series, e.g., $\sum c\,\lambda^k$, $\lambda > 0$, then $\dfrac{a_{k+1}}{a_k}$ is constant, i.e., λ. If $\lambda < 1$, then $\sum a_k$ converges. If $\lambda \geq 1$, then $\sum a_k$ diverges.
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$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^2}{2^{k+1}} \div \frac{k^2}{2^k} = \frac{2^k}{2^{k+1}} \cdot \frac{(k+1)^3}{k^3}$$

$$= \frac{1}{2} \cdot \frac{(k+1)^3}{k^3} \to \frac{1}{2} \text{ as } k \to \infty$$

$\sum \frac{1}{k!}$ converges, by the ratio test:

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$\sum \frac{1}{2k+1}$ inconclusive, by the ratio test:

the series diverges by the limit comparison test:

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Examples

$\sum \frac{k^2 2^k}{k!}$ converges, by the ratio test:

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Outline

- Comparison Tests
- The Root Test
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