Lecture 24

Section 11.4 Absolute and Conditional Convergence; Alternating Series

Jiwen He

Department of Mathematics, University of Houston

jiwenhe@math.uh.edu http://math.uh.edu/~jiwenhe/Math1432













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Basic Series that Converge

Geometric series:	$\sum x^k$,	if $ x < 1$
<i>p</i> -series:	$\sum \frac{1}{k^p}$,	if p>1

Basic Series that Diverge





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Any series
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 for which $\lim_{k o \infty} a_k
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Basic Test for Convergence

Keep in Mind that, if $a_k \not\rightarrow 0$, then the series $\sum a_k$ diverges; therefore there is no reason to apply any special convergence test.

Examples

 $\sum x^{k} \text{ with } |x| \ge 1 \text{ (e.g. } \sum (-1)^{k} \text{) diverge since } x^{k} \to 0.$ $\sum \frac{k}{k+1} \text{ diverges since } \frac{k}{k+1} \to 1 \neq 0.$ $\sum \left(1 - \frac{1}{k}\right)^{k} \text{ diverges since } a_{k} = \left(1 - \frac{1}{k}\right)^{k} \to e^{-1} \neq 0.$



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Comparison Tests

Rational terms are most easily handled by basic comparison or limit comparison with *p*-series $\sum 1/k^p$

Basic Comparison Test



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Limit Comparison Test

$$\sum \frac{1}{k^3 - 1} \text{ converges by comparison with } \sum \frac{1}{k^3}.$$

$$\sum \frac{3k^2 + 2k + 1}{k^3 + 1} \text{ diverges by comparison with } \sum \frac{3}{k}$$

$$\sum \frac{5\sqrt{k} + 100}{2k^2\sqrt{k} - 9\sqrt{k}} \text{ converges by comparison with } \sum \frac{5}{2k^2}$$



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Root Test and Ratio Test

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$$\sum \frac{k^2}{2^k} \text{ converges: } (a_k)^{1/k} = \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1$$
$$\sum \frac{1}{(\ln k)^k} \text{ converges: } (a_k)^{1/k} = \frac{1}{\ln k} \to 0$$
$$\sum \left(1 - \frac{1}{k}\right)^{k^2} \text{ converges: } (a_k)^{1/k} = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1}$$



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April 10, 2008 5 /
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April 10, 2008 5 /

Convergence Tests Absolute Convergence Alternating Series F

Convergence Tests (4)

Root Test and Ratio Test

The ratio test is effective with factorials and with combinations of powers and factorials.

Ratio Comparison Test

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5 / 16

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5 / 16

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5 / 16

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$$\sum \frac{1}{k!} \text{ converges: } \frac{a_{k+1}}{a_k} = \frac{1}{k+1} \rightarrow 0$$

$$\sum \frac{k}{10^k} \text{ converges: } \frac{a_{k+1}}{a_k} = \frac{1}{10} \cdot \frac{k+1}{k} \rightarrow \frac{1}{10}$$

$$\sum \frac{k^k}{k!} \text{ diverges: } \frac{a_{k+1}}{a_k} = (1+\frac{1}{k})^k \rightarrow e$$

$$\sum \frac{2^k}{3^k - 2^k} \text{ converges: } \frac{a_{k+1}}{a_k} = 2 \cdot \frac{1-(2/3)^k}{3-2(2/3)^k} \rightarrow 2 \cdot \frac{1}{3}$$

$$\sum \frac{1}{\sqrt{k!}} \text{ converges: } \frac{a_{k+1}}{a_k} = \sqrt{\frac{1}{k+1}} \rightarrow 0$$

Root Test and Ratio Test

The ratio test is effective with factorials and with combinations of powers and factorials.

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A series $\sum a_k$ is said to converge absolutely if $\sum |a_k|$ converges.

if $\sum_{k=1}^{k} |a_k|$ converges, then $\sum_{k=1}^{k} a_k$ converges.

., absolutely convergent series are convergent.

Alternating p-Series with p > 1



 $\sum_{k=1}^{k} (-1)^{k} x^{k}, -1 < x < 1, \text{ converge absolutely because } \sum_{k=1}^{k} |x|^{k}$ converges. = $1 - \frac{1}{2} - \frac{1}{2^{2}} + \frac{1}{2^{3}} - \frac{1}{2^{4}} + \frac{1}{2^{5}} + \frac{1}{2^{5}} - \cdots$ converge absolutely.

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Math 1432 - Section 26626, Lecture 24

7 / 16



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Absolute Convergence

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Geometric Series with -1 < x < 1



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Alternating p-Series with p > 1

$$\sum \frac{(-1)^k}{k^p}, \ p > 1, \ \text{converge absolutely because} \sum \frac{1}{k^p} \ \text{converges.}$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \cdots \ \text{converge absolutely.}$$

Geometric Series with -1 < x < 1

 $\sum_{\text{converges.}} (-1)^{j(k)} x^k, \ -1 < x < 1, \text{ converge absolutely because } \sum |x|^k$ $\Rightarrow \ 1 - \frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^3} - \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} - \cdots \text{ converge absolutely.}$

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Conditional Convergence

A series $\sum a_k$ is said to converge conditionally if $\sum a_k$ converges while $\sum |a_k|$ diverges.

Alternating *p*-Series with 0

$$\sum_{k \neq 0} \frac{(-1)^k}{k^p}, \ 0 diverges.
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Alternating Series

Let $\{a_k\}$ be a sequence of positive numbers.

$$(-1)^k a_k = a_0 - a_1 + a_2 - a_3 + a_4 - \cdots$$

is called an alternating series.

Alternating Series Test

Let $\{a_k\}$ be a decreasing sequence of positive numbers. If $a_k \to 0$, then $\sum (-1)^k a_k$ converges.

Alternating *p*-Series with p > 0

 $\sum \frac{(-1)^{k}}{k^{p}}, p > 0, \text{ converge since } f(x) = \frac{1}{x^{p}} \text{ is decreasing, i.e.,}$ $f'(x) = -\frac{p}{x^{p+1}} > 0 \text{ for } \forall x > 0, \text{ and } \lim_{k \to \infty} f(x) = 0.$ $\Rightarrow \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \cdots \text{ converge conditionally.}$

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Examples

$$\sum \frac{(-1)^k}{2k+1}$$
, converge since $f(x) = \frac{1}{2x+1}$ is decreasing, i.e.,
$$f'(x) = -\frac{2}{(2x+1)^2} > 0$$
 for $\forall x > 0$, and $\lim_{x \to \infty} f(x) = 0$.

$$\sum \frac{(-1)^k k}{k^2 + 10}, \text{ converge since } f(x) = \frac{x}{x^2 + 10} \text{ is decreasing, i.e.,}$$
$$f'(x) = -\frac{x^2 - 10}{(x^2 + 10)^2} > 0, \text{ for } \forall x > \sqrt{10}, \text{ and } \lim_{x \to \infty} f(x) = 0.$$



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Math 1432 – Section 26626, Lecture 2

An Estimate for Alternating Series



An Estimate for Alternating Series

Let $\{a_k\}$ be a decreasing sequence of positive numbers that tends to 0 and let $L = \sum_{k=0}^{\infty} (-1)^k a_k$. Then the sum L lies between consecutive partial sums s_n , s_{n+1} , $s_n < L < s_{n+1}$, if n is odd; $s_{n+1} < L < s_n$, if n is even. and thus s_n approximates L to within a_{n+1}

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Math 1432 – Section 26626, Lecture 24

An Estimate for Alternating Series



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Find
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 to approximate $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} \cdots$ within 10^{-2} .
Set $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$. For $|L - s_n| < 10^{-2}$, we want
 $a_{n+1} = \frac{1}{(n+1)+1} < 10^{-2} \Rightarrow n+2 > 10^2 \Rightarrow n > 98$.
Then $n = 99$ and the 99th partial sum s_{100} is
 $s_{99} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} \approx 0.6882$.
From the estimate
 $|L - s_{99}| < a_{100} = \frac{1}{101} \approx 0.00991$.
we conclude that
 $s_{99} \approx 0.6882 < \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln 2 < 0.6981 \approx s_{100}$

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Math 1432 - Section 26626, Lecture 24

April 10, 2008

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April 10, 2008

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 to approximate $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} \cdots$ within 10^{-2} .

Set
$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$$
. For $|L - s_n| < 10^{-2}$, we want
 $a_{n+1} = \frac{1}{(n+1)+1} < 10^{-2} \Rightarrow n+2 > 10^2 \Rightarrow n > 98$.
Then $n = 99$ and the 99th partial sum s_{100} is

$$s_{99} = 1 - rac{1}{2} + rac{1}{3} - rac{1}{4} + \dots + rac{1}{99} - rac{1}{100} pprox 0.6882.$$

From the estimate

$$|L - s_{99}| < a_{100} = \frac{1}{101} \approx 0.00991.$$

we conclude that

$$s_{99} pprox 0.6882 < \sum_{k=1}^{\infty} rac{(-1)^{k+1}}{k} = \ln 2 < 0.6981 pprox s_{100}$$

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Find
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For $|L - s_n| < 10^{-2}$, we want
 $a_{n+1} = \frac{1}{(2(n+1)+1)!} < 10^{-2} \Rightarrow n \ge 1$.
Then $n = 1$ and the 2nd partial sum s_2 is
 $s_1 = 1 - \frac{1}{3!} \approx 0.8333$
From the estimate
 $|L - s_1| < a_2 = \frac{1}{5!} \approx 0.0083$.
we conclude that
 $s_1 \approx 0.8333 < \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2k+1)!} = \sin 1 < 0.8416 \approx s_2$

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13 / 16

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13 / 16

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13 / 16

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13 / 16

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3 / 16

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Rearrangement of Absolute Convergence Series



Theorem

All rearrangements of an absolutely convergent series converge absolutely to the same sum.



Rearrangement of Absolute Convergence Series



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Rearrangement of Absolute Convergence Series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} = 1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \dots = \frac{2}{3} \text{ absolutely}$$

Rearrangement $1 + \frac{1}{2^2} - \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^6} - \frac{1}{2^3} + \frac{1}{2^8} + \frac{1}{2^{10}} - \frac{1}{2^5} \dots ?$

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Rearrangement of Absolute Convergence Series

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Rearrangement of Absolute Convergence Series

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Theorem

All rearrangements of an absolutely convergent series converge absolutely to the same sum.



Rearrangement of Conditional Convergence Series



Multiply the original series by $\frac{1}{2}$



Adding the two series, we get the rearrangement

 $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \cdots$

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Rearrangement of Conditional Convergence Series





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.5 / 16

Rearrangement of Conditional Convergence Series

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Rearrangement $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} \dots ? = \ln 2$

Multiply the original series by $\frac{1}{2}$

$$\frac{1}{2}\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \frac{1}{10} + \dots = \frac{1}{2}\ln 2$$

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5 / 16

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Rearrangement of Conditional Convergence Series

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April 10, 2008

l5 / 16

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Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

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5 / 16

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Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

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Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

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Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

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Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

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Convergence Tests Absolute Convergence Alternating Series

Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

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5 / 16

Rearrangement of Conditional Convergence Series

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5 / 16

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$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} + \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \dots = \frac{3}{2} \ln 2$$

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Rearrangement of Conditional Convergence Series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \ln 2 \text{ conditionally}$$

Rearrangement $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} \dots \neq \ln 2$

Multiply the original series by $\frac{1}{2}$

$$\frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \frac{1}{10} + \dots = \frac{1}{2}\ln 2$$

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Remark

- A series that is only conditionally convergent can be rearranged to converge to any number we please.
- It can also be arranged to diverge to +∞ or -∞, or even to oscillate between any two bounds we choose.



Rearrangement of Conditional Convergence Series

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Outline

- Convergence Tests
- Absolute ConvergenceAbsolute Convergence
- Alternating Series
- Rearrangements



Jiwen He, University of Houston