Lecture 24

Section 11.4 Absolute and Conditional Convergence; Alternating Series

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Basic Series that Converge or Diverge

Basic Series that Converge

Geometric series: $\sum x^k, \quad \text{if } |x| < 1$

p-series: $\sum \frac{1}{k^p}, \quad \text{if } p > 1$

Basic Series that Diverge

Any series $\sum a_k$ for which $\lim_{k\to\infty} a_k \neq 0$

p-series: $\sum \frac{1}{k^p}, \quad \text{if } p \leq 1$

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Convergence Tests (1)

Basic Test for Convergence

Keep in Mind that,

if $a_k \nrightarrow 0$, then the series $\sum a_k$ diverges;

therefore there is no reason to apply any special convergence test.

Examples

$\sum x^k$ with $|x| \geq 1$ (e.g, $\sum (-1)^k$) diverge since $x^k \nrightarrow 0$.

$$\sum \frac{k}{k+1} \quad \text{diverges since } \tfrac{k}{k+1} \to 1 \neq 0.$$

$$\sum \left(1 - \frac{1}{k}\right)^k \quad \text{diverges since } a_k = \left(1 - \tfrac{1}{k}\right)^k \to e^{-1} \neq 0.$$

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Keep in Mind that,

if $a_k \not\to 0$, then the series $\sum a_k$ diverges;

therefore there is no reason to apply any special convergence test.

Examples

$\sum x^k$ with $|x| \geq 1$ (e.g, $\sum (-1)^k$) diverge since $x^k \not\to 0$.

$$\sum \frac{k}{k+1}$$ diverges since $\frac{k}{k+1} \to 1 \neq 0$.

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Convergence Tests (2)

Comparison Tests

Rational terms are most easily handled by basic comparison or limit comparison with p-series $\sum 1/k^p$

Basic Comparison Test

$\sum \frac{1}{2k^3+1}$ converges by comparison with $\sum \frac{1}{k^3}$

$\sum \frac{k^3}{k^5+4k^4+7}$ converges by comparison with $\sum \frac{1}{k^2}$

$\sum \frac{1}{k^3-k^2}$ converges by comparison with $\sum \frac{2}{k^3}$

$\sum \frac{1}{3k+1}$ diverges by comparison with $\sum \frac{1}{3(k+1)}$

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Comparison Tests

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Limit Comparison Test

$\sum \frac{1}{k^3 - 1}$ converges by comparison with $\sum \frac{1}{k^3}$.

$\sum \frac{3k^2 + 2k + 1}{k^3 + 1}$ diverges by comparison with $\sum \frac{3}{k}$

$\sum \frac{5\sqrt{k} + 100}{2k^2\sqrt{k} - 9\sqrt{k}}$ converges by comparison with $\sum \frac{5}{2k^2}$

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Convergence Tests (3)

Root Test and Ratio Test

The root test is used only if powers are involved.

Root Test

$\sum \frac{k^2}{2^k}$ converges: $(a_k)^{1/k} = \frac{1}{2} \cdot \left[k^{1/k}\right]^2 \to \frac{1}{2} \cdot 1$

$\sum \frac{1}{(\ln k)^k}$ converges: $(a_k)^{1/k} = \frac{1}{\ln k} \to 0$

$\sum \left(1 - \frac{1}{k}\right)^{k^2}$ converges: $(a_k)^{1/k} = \left(1 + \frac{(-1)}{k}\right)^k \to e^{-1}$

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Convergence Tests (4)

Root Test and Ratio Test

The ratio test is effective with factorials and with combinations of powers and factorials.

Ratio Comparison Test

$\sum \frac{k^2}{2^k}$ converges: $\frac{a_{k+1}}{a_k} = \frac{1}{2} \cdot \frac{(k+1)^2}{k^2} \longrightarrow \frac{1}{2}$

$\sum \frac{1}{k!}$ converges: $\frac{a_{k+1}}{a_k} = \frac{1}{k+1} \longrightarrow 0$

$\sum \frac{k}{10^k}$ converges: $\frac{a_{k+1}}{a_k} = \frac{1}{10} \cdot \frac{k+1}{k} \longrightarrow \frac{1}{10}$

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$\sum \frac{2^k}{3^k - 2^k}$ converges: $\frac{a_{k+1}}{a_k} = 2 \cdot \frac{1-(2/3)^k}{3-2(2/3)^k} \longrightarrow 2 \cdot \frac{1}{3}$

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Convergence Tests (4)

Root Test and Ratio Test

The ratio test is effective with factorials and with combinations of powers and factorials.

Ratio Comparison Test

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A series $\sum a_k$ is said to converge absolutely if $\sum |a_k|$ converges.

if $\sum |a_k|$ converges, then $\sum a_k$ converges.
i.e., absolutely convergent series are convergent.

Alternating p-Series with $p > 1$

$\sum \frac{(-1)^k}{k^p}$, $p > 1$, converge absolutely because $\sum \frac{1}{k^p}$ converges.

$$\Rightarrow \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} - \cdots \text{ converge absolutely.}$$

Geometric Series with $-1 < x < 1$

$\sum (-1)^{j(k)} x^k$, $-1 < x < 1$, converge absolutely because $\sum |x|^k$ converges.

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Conditional Convergence

Conditional Convergence

A series $\sum a_k$ is said to converge conditionally if $\sum a_k$ converges while $\sum |a_k|$ diverges.

Alternating p-Series with $0 < p \leq 1$

$\sum \frac{(-1)^k}{k^p}$, $0 < p \leq 1$, converge conditionally because $\sum \frac{1}{k^p}$ diverges.

$$\Rightarrow \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \cdots \text{ converge conditionally.}$$

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Alternating Series

Alternating Series

Let $\{a_k\}$ be a sequence of positive numbers.

$$\sum(-1)^k a_k = a_0 - a_1 + a_2 - a_3 + a_4 - \cdots$$

is called an alternating series.

Alternating Series Test

Let $\{a_k\}$ be a decreasing sequence of positive numbers.

$$\text{If } a_k \to 0, \quad \text{then } \sum(-1)^k a_k \text{ converges.}$$

Alternating p-Series with $p > 0$

$\sum \frac{(-1)^k}{k^p}$, $p > 0$, converge since $f(x) = \frac{1}{x^p}$ is decreasing, i.e.,

$f'(x) = -\frac{p}{x^{p+1}} > 0$ for $\forall x > 0$, and $\lim_{x\to\infty} f(x) = 0$.

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$\sum \frac{(-1)^k}{k^p}$, $p > 0$, converge since $f(x) = \frac{1}{x^p}$ is decreasing, i.e., $f'(x) = -\frac{p}{x^{p+1}} > 0$ for $\forall x > 0$, and $\lim_{x\to\infty} f(x) = 0$.

$$\Rightarrow \quad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \cdots \text{ converge conditionally.}$$

Examples

$\sum \frac{(-1)^k}{2k+1}$, converge since $f(x) = \frac{1}{2x+1}$ is decreasing, i.e., $f'(x) = -\frac{2}{(2x+1)^2} > 0$ for $\forall x > 0$, and $\lim_{x \to \infty} f(x) = 0$.

$\sum \frac{(-1)^k k}{k^2+10}$, converge since $f(x) = \frac{x}{x^2+10}$ is decreasing, i.e., $f'(x) = -\frac{x^2-10}{(x^2+10)^2} > 0$, for $\forall x > \sqrt{10}$, and $\lim_{x \to \infty} f(x) = 0$.

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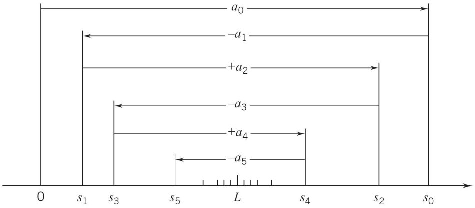

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Let $\{a_k\}$ be a decreasing sequence of positive numbers that tends to 0 and let $L = \sum_{k=0}^{\infty} (-1)^k a_k$. Then the sum L lies between consecutive partial sums s_n, s_{n+1},

$$s_n < L < s_{n+1}, \text{ if } n \text{ is odd}; \quad s_{n+1} < L < s_n, \text{ if } n \text{ is even.}$$

and thus s_n approximates L to within a_{n+1}

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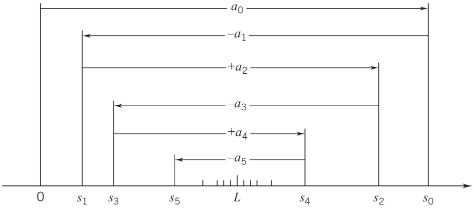

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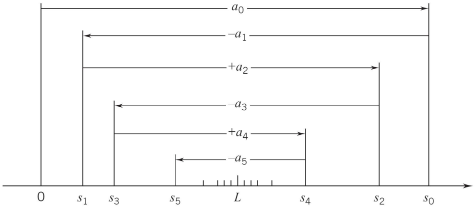

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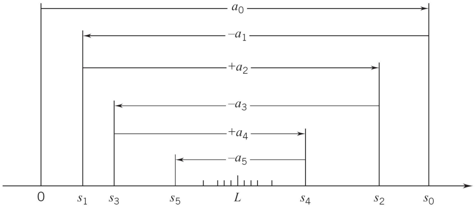

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Example

Find s_n to approximate $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} \cdots$ within 10^{-2}.

Set $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{k+1}$. For $|L - s_n| < 10^{-2}$, we want

$$a_{n+1} = \frac{1}{(n+1)+1} < 10^{-2} \quad \Rightarrow \quad n+2 > 10^2 \quad \Rightarrow \quad n > 98.$$

Then $n = 99$ and the 99th partial sum s_{100} is

$$s_{99} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{99} - \frac{1}{100} \approx 0.6882.$$

From the estimate

$$|L - s_{99}| < a_{100} = \frac{1}{101} \approx 0.00991.$$

we conclude that

$$s_{99} \approx 0.6882 < \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln 2 < 0.6981 \approx s_{100}$$

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Rearrangement of Absolute Convergence Series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} = 1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \cdots = \frac{2}{3} \text{ absolutely}$$

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Theorem

All rearrangements of an absolutely convergent series converge absolutely to the same sum.

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Why Absolute Convergence Matters: Rearrangements (2)

Rearrangement of Conditional Convergence Series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \ln 2 \text{ conditionally}$$

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Why Absolute Convergence Matters: Rearrangements (2)

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Remark

- A series that is only conditionally convergent can be rearranged to converge to any number we please.
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Outline

- Convergence Tests
- Absolute Convergence
  - Absolute Convergence
- Alternating Series
- Rearrangements