## Lecture 26 <br> Section 11.6 Taylor Polynomials and Taylor Series in $x-a$

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## Taylor Polynomials in Powers of $x-a$

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Best Approximation

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& P_{2}(x)=f(a)+f^{\prime}(a) x+\frac{f^{\prime \prime}(a)}{2!}(x-a)^{2} .
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## Taylor's Theorem and Remainder Term

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\left|R_{n}(x)\right| \leq\left(\max _{t \in J}\left|f^{(n+1)}(t)\right|\right) \frac{|x-a|^{n+1}}{(n+1)!}, \quad J=[a, x] \text { or }[x, a] .
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f(x)=\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^{k}=\lim _{n \rightarrow \infty} \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^{k}
$$

## Taylor Polynomials and Taylor Series in $x-a$ of $f(x)=e^{x}$

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e^{x}=e^{a} \sum_{k=0}^{\infty} \frac{1}{k!}(x-a)^{k}=e^{a}+e^{a}(x-a)+\cdots+\frac{e^{a}}{n!}(x-a)^{n}+\cdots, \quad \forall x .
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## Expansion of $e^{x}$ in $x-a$

> Taylor Series in $x-a$ by Translation
> Another way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

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e^{x}=e^{a} \sum_{k=0}^{\infty} \frac{1}{k!}(x-a)^{k}, \quad \text { for all real } x .
$$

## Expansion of $e^{x}$ in $x-a$

## Taylor Series in $x-a$ by Translation

Another way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

Taylor Series in $x-a$ of the Exponential $f(x)=e^{x}$

$$
e^{x}=e^{a}+e^{a}(x-a)+\frac{e^{a}}{2!}(x-a)^{2}+\cdots+\frac{e^{a}}{n!}(x-a)^{n}+\cdots, \quad \forall x .
$$

1. Expand $e^{t+a}$ in powers of $t \Rightarrow$

$$
e^{t+a}=e^{a} e^{t}=e^{a} \sum_{k=0}^{\infty} \frac{t^{k}}{k!}=e^{a} \sum_{k=0}^{\infty} \frac{1}{k!} t^{k}
$$

2. Set $t=x-a \quad \Rightarrow$

$$
e^{x}=e^{a} \sum_{k=0}^{\infty} \frac{1}{k!}(x-a)^{k}, \quad \text { for all real } x .
$$

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

Taylor Series in $x-a$ by Translation
One way to expand $f(x)$ in in powers of $x-a$ is to expand
$f(t+a)$ in powers of $t$ and then set $t=x-a$

## Example

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
This is the approach to take when the expansion in $t$ is either
known or is readily available

## Example

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

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## Example

Expand

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
This is the approach to take when the expansion in $t$ is either known or is readily available.

## Example

## Expand $f(t+3)$ in powers of $t$ <br> $\square$ <br> 

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
This is the approach to take when the expansion in $t$ is either known or is readily available.

## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.


## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
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## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

1. Expand $f(t+3)$ in powers of $t \Rightarrow$ $f(t+3)=e^{(t+3) / 2}$

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
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## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

1. Expand $f(t+3)$ in powers of $t \quad \Rightarrow$
$f(t+3)=e^{(t+3) / 2}$

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
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## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

1. Expand $f(t+3)$ in powers of $t \Rightarrow$
$f(t+3)=e^{(t+3) / 2}=e^{3 / 2} e^{t / 2}$

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
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## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

1. Expand $f(t+3)$ in powers of $t \Rightarrow$
$f(t+3)=e^{(t+3) / 2}=e^{3 / 2} e^{t / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{(t / 2)^{k}}{k!}$

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
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## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

1. Expand $f(t+3)$ in powers of $t \Rightarrow$
$f(t+3)=e^{(t+3) / 2}=e^{3 / 2} e^{t / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{(t / 2)^{k}}{k!}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{1}{2^{k} k!} t^{k}$.

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$ ．
This is the approach to take when the expansion in $t$ is either known or is readily available．

## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$ ．
1．Expand $f(t+3)$ in powers of $t \Rightarrow$
$f(t+3)=e^{(t+3) / 2}=e^{3 / 2} e^{t / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{(t / 2)^{k}}{k!}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{1}{2^{k} k!} t^{k}$.
2．Set $t=x-3 \Rightarrow$


## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
This is the approach to take when the expansion in $t$ is either known or is readily available.

## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

1. Expand $f(t+3)$ in powers of $t \Rightarrow$
$f(t+3)=e^{(t+3) / 2}=e^{3 / 2} e^{t / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{(t / 2)^{k}}{k!}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{1}{2^{k} k!} t^{k}$.
2. Set $t=x-3 \Rightarrow$

$$
f(x)=e^{x / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{1}{2^{k} k!}(x-3)^{k}
$$

## Expansion of $f(x)$ in $x-a$ as Expansion of $f(t+a)$ in $t$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.
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## Example

Expand $f(x)=e^{x / 2}$ in powers of $x-3$.

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$f(t+3)=e^{(t+3) / 2}=e^{3 / 2} e^{t / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{(t / 2)^{k}}{k!}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{1}{2^{k} k!} t^{k}$.
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$$
f(x)=e^{x / 2}=e^{3 / 2} \sum_{k=0}^{\infty} \frac{1}{2^{k} k!}(x-3)^{k}, \quad \text { for all real } x
$$

## Expansion of $\ln x$ in $x-a, a>0$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

Taylor Series in $x$ - a of the Logarithm $f(x)=\ln x$


## Expand $\ln (t+a)$ in powers of $t$

$\square$

## Expansion of $\ln x$ in $x-a, a>0$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

Taylor Series in $x$ - a of the Logarithm $f(x)=\ln x$

$$
\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
$$

## Expand $\ln (t+a)$ in powers of $t$

$\ln (t+a)$

## Expansion of $\ln x$ in $x-a, a>0$

## Taylor Series in $x-a$ by Translation

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\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
$$

1. Expand $\ln (t+a)$ in powers of $t \quad \Rightarrow$ $\ln (t+a)$ $=\ln a(1+$ $\left.\left.\frac{t}{a}\right)\right]$

## Expansion of $\ln x$ in $x-a, a>0$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

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$$

1. Expand $\ln (t+a)$ in powers of $t \quad \Rightarrow$ $\ln (t+a)=\ln \left[a\left(1+\frac{t}{a}\right)\right]$

## Expansion of $\ln x$ in $x-a, a>0$

## Taylor Series in $x-a$ by Translation

One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

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\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
$$

1. Expand $\ln (t+a)$ in powers of $t \quad \Rightarrow$

$$
\ln (t+a)=\ln \left[a\left(1+\frac{t}{a}\right)\right]=\ln a+\ln \left(1+\frac{t}{a}\right)
$$

## Expansion of $\ln x$ in $x-a, a>0$

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One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

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$$
\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
$$

1. Expand $\ln (t+a)$ in powers of $t \quad \Rightarrow$

$$
\ln (t+a)=\ln \left[a\left(1+\frac{t}{a}\right)\right]=\ln a+\ln \left(1+\frac{t}{a}\right)=\ln a+\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(\frac{t}{a}\right)^{k}
$$

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\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
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$$

2. Set $t=x-a,-1$

## Expansion of $\ln x$ in $x-a, a>0$

Taylor Series in $x-a$ by Translation
One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

Taylor Series in $x$ - a of the Logarithm $f(x)=\ln x$

$$
\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
$$

1. Expand $\ln (t+a)$ in powers of $t \quad \Rightarrow$

$$
\ln (t+a)=\ln \left[a\left(1+\frac{t}{a}\right)\right]=\ln a+\ln \left(1+\frac{t}{a}\right)=\ln a+\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(\frac{t}{a}\right)^{k}
$$

2. Set $t=x-a$,

## Expansion of $\ln x$ in $x-a, a>0$

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One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

Taylor Series in $x$ - a of the Logarithm $f(x)=\ln x$

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\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
$$

1. Expand $\ln (t+a)$ in powers of $t \quad \Rightarrow$

$$
\ln (t+a)=\ln \left[a\left(1+\frac{t}{a}\right)\right]=\ln a+\ln \left(1+\frac{t}{a}\right)=\ln a+\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\left(\frac{t}{a}\right)^{k}
$$

2. Set $t=x-a,-1<\frac{t}{a} \leq 1$,

## Expansion of $\ln x$ in $x-a, a>0$

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One way to expand $f(x)$ in in powers of $x-a$ is to expand $f(t+a)$ in powers of $t$ and then set $t=x-a$.

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\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
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$$

2. Set $t=x-a,-1<\frac{t}{a} \leq 1,-a<t \leq a$, $\square$

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\ln x=\ln a+\frac{1}{a}(x-a)-\frac{1}{2 a^{2}}(x-a)^{2}+\frac{1}{3 a^{3}}(x-a)^{3}-\cdots, \quad 0<x \leq 2 a .
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\ln x=\ln a+\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k a^{k}}(x-a)^{k}
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$$

2. Set $t=x-a,-1<\frac{t}{a} \leq 1,-a<t \leq a, 0<t+a \leq 2 a$, $\quad \Rightarrow$

$$
\ln x=\ln a+\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k a^{k}}(x-a)^{k}, \quad \text { for } 0<x \leq 2 a
$$

## Expansion of $\sin x$ in $x-\pi$

## Taylor Series in $x-\pi$ of the Sine $f(x)=\sin x$



## Expansion of $\sin x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Sine $f(x)=\sin x$

$$
\sin x=-(x-\pi)+\frac{1}{3!}(x-\pi)^{3}-\frac{1}{5!}(x-\pi)^{5}+\frac{1}{7!}(x-\pi)^{7}+\cdots, \quad \forall x .
$$

> Expand $\sin (t+\pi)$ in powers of $t$
> $\sin (t+\pi)=\sin t \cos \pi+\cos t \sin \pi$

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$$
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Taylor Series in $x-\pi$ of the Sine $f(x)=\sin x$

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$$

1. Expand $\sin (t+\pi)$ in powers of $t \quad \Rightarrow$

$$
\sin (t+\pi)=\sin t \cos \pi+\cos t \sin \pi=-\sin t
$$

$$
=-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k+1)!} t^{2 k+1}
$$

2. Set $t=x-\pi$

## Expansion of $\sin x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Sine $f(x)=\sin x$

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\sin x=-(x-\pi)+\frac{1}{3!}(x-\pi)^{3}-\frac{1}{5!}(x-\pi)^{5}+\frac{1}{7!}(x-\pi)^{7}+\cdots, \quad \forall x .
$$

1. Expand $\sin (t+\pi)$ in powers of $t \quad \Rightarrow$

$$
\begin{aligned}
\sin (t+\pi) & =\sin t \cos \pi+\cos t \sin \pi=-\sin t \\
& =-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k+1)!} t^{2 k+1}=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k+1)!} t^{2 k+1}
\end{aligned}
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## Expansion of $\sin x$ in $x-\pi$

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\sin x=-(x-\pi)+\frac{1}{3!}(x-\pi)^{3}-\frac{1}{5!}(x-\pi)^{5}+\frac{1}{7!}(x-\pi)^{7}+\cdots, \quad \forall x .
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$$

2. Set $t=x-\pi \Rightarrow$


## Expansion of $\sin x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Sine $f(x)=\sin x$

$$
\sin x=-(x-\pi)+\frac{1}{3!}(x-\pi)^{3}-\frac{1}{5!}(x-\pi)^{5}+\frac{1}{7!}(x-\pi)^{7}+\cdots, \quad \forall x .
$$

1. Expand $\sin (t+\pi)$ in powers of $t \quad \Rightarrow$

$$
\begin{aligned}
\sin (t+\pi) & =\sin t \cos \pi+\cos t \sin \pi=-\sin t \\
& =-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k+1)!} t^{2 k+1}=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k+1)!} t^{2 k+1}
\end{aligned}
$$

2. Set $t=x-\pi \Rightarrow$

$$
\sin x=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k+1)!}(x-\pi)^{2 k+1}
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\end{aligned}
$$

2. Set $t=x-\pi \Rightarrow$

$$
\sin x=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k+1)!}(x-\pi)^{2 k+1}, \quad \text { for all real } x
$$

## Expansion of $\cos x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Cosine $f(x)=\cos x$


## Expansion of $\cos x$ in $x-\pi$

$$
\begin{aligned}
& \text { Taylor Series in } x-\pi \text { of the Cosine } f(x)=\cos x \\
& \cos x=-1+\frac{1}{2!}(x-\pi)^{2}-\frac{1}{4!}(x-\pi)^{4}+\frac{1}{6!}(x-\pi)^{6}-\cdots, \quad \forall x .
\end{aligned}
$$

## 1. Expand $\cos (t+\pi)$ in powers of $t \quad \Rightarrow$

 $\cos (t+\pi)=\cos t \cos \pi-\sin t \sin \pi$
## Expansion of $\cos x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Cosine $f(x)=\cos x$

$$
\cos x=-1+\frac{1}{2!}(x-\pi)^{2}-\frac{1}{4!}(x-\pi)^{4}+\frac{1}{6!}(x-\pi)^{6}-\cdots, \quad \forall x .
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1. Expand $\cos (t+\pi)$ in powers of $t \Rightarrow$

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## Expansion of $\cos x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Cosine $f(x)=\cos x$

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$$

1. Expand $\cos (t+\pi)$ in powers of $t \quad \Rightarrow$

$$
\cos (t+\pi)=\cos t \cos \pi-\sin t \sin \pi=-\cos t
$$

$$
=-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!} t^{2 k}
$$

## Expansion of $\cos x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Cosine $f(x)=\cos x$

$$
\cos x=-1+\frac{1}{2!}(x-\pi)^{2}-\frac{1}{4!}(x-\pi)^{4}+\frac{1}{6!}(x-\pi)^{6}-\cdots, \quad \forall x .
$$

1. Expand $\cos (t+\pi)$ in powers of $t \quad \Rightarrow$

$$
\cos (t+\pi)=\cos t \cos \pi-\sin t \sin \pi=-\cos t
$$

$$
=-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!} t^{2 k}=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k)!} t^{2 k}
$$

2. Set $t=x-\pi$

## Expansion of $\cos x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Cosine $f(x)=\cos x$

$$
\cos x=-1+\frac{1}{2!}(x-\pi)^{2}-\frac{1}{4!}(x-\pi)^{4}+\frac{1}{6!}(x-\pi)^{6}-\cdots, \quad \forall x .
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\cos (t+\pi)=\cos t \cos \pi-\sin t \sin \pi=-\cos t
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$$
=-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!} t^{2 k}=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k)!} t^{2 k}
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2. Set $t=x-\pi \Rightarrow$

for all real

## Expansion of $\cos x$ in $x-\pi$

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1. Expand $\cos (t+\pi)$ in powers of $t \Rightarrow$

$$
\begin{aligned}
\cos (t+\pi) & =\cos t \cos \pi-\sin t \sin \pi=-\cos t \\
& =-\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!} t^{2 k}=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k)!} t^{2 k}
\end{aligned}
$$

2. Set $t=x-\pi \Rightarrow$

$$
\cos x=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k)!}(x-\pi)^{2 k}
$$

## Expansion of $\cos x$ in $x-\pi$

Taylor Series in $x-\pi$ of the Cosine $f(x)=\cos x$

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\cos x=-1+\frac{1}{2!}(x-\pi)^{2}-\frac{1}{4!}(x-\pi)^{4}+\frac{1}{6!}(x-\pi)^{6}-\cdots, \quad \forall x .
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\end{aligned}
$$

2. Set $t=x-\pi \Rightarrow$

$$
\cos x=\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2 k)!}(x-\pi)^{2 k}, \quad \text { for all real } x
$$

## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$


## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

$$
\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \forall x
$$

1. Expand $\cos ^{2}(t+\pi)$ in powers of $t \Rightarrow$


## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

$$
\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \forall x
$$

1. Expand $\cos ^{2}(t+\pi)$ in powers of $t \quad \Rightarrow$ $\cos ^{2}(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos 2(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos (2 t+2 \pi)$

## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

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\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \forall x
$$

1. Expand $\cos ^{2}(t+\pi)$ in powers of $t \Rightarrow$

$$
\cos ^{2}(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos 2(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos (2 t+2 \pi)
$$

## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

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\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \forall x
$$

1. Expand $\cos ^{2}(t+\pi)$ in powers of $t \quad \Rightarrow$
$\cos ^{2}(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos 2(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos (2 t+2 \pi)$
$=\frac{1}{2}+\frac{1}{2} \cos 2 t$

## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

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\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \forall x
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$\cos ^{2}(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos 2(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos (2 t+2 \pi)$
$=\frac{1}{2}+\frac{1}{2} \cos 2 t=\frac{1}{2}+\frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!}(2 t)^{2 k}$
2. Set $t=x-\pi$

## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

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$\cos ^{2}(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos 2(t+\pi)=\frac{1}{2}+\frac{1}{2} \cos (2 t+2 \pi)$
$=\frac{1}{2}+\frac{1}{2} \cos 2 t=\frac{1}{2}+\frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!}(2 t)^{2 k}=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!} t^{2 k}$
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## Expansion of $\cos ^{2} x$ in $x-\pi$

Taylor Series in $x-\pi$ of $f(x)=\cos ^{2} x$

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$=\frac{1}{2}+\frac{1}{2} \cos 2 t=\frac{1}{2}+\frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!}(2 t)^{2 k}=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!} t^{2 k}$
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\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \forall x
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$$
\begin{aligned}
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& =\frac{1}{2}+\frac{1}{2} \cos 2 t=\frac{1}{2}+\frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!}(2 t)^{2 k}=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!} t^{2 k}
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& =\frac{1}{2}+\frac{1}{2} \cos 2 t=\frac{1}{2}+\frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k)!}(2 t)^{2 k}=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!} t^{2 k}
\end{aligned}
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$$
\cos ^{2} x=1+\sum_{k=1}^{\infty} \frac{(-1)^{k} 2^{2 k-1}}{(2 k)!}(x-\pi)^{2 k}, \quad \text { for all real } x
$$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

$\square$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t$


## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

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(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
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Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

$$
f(t-2)=[1-2(t-2)]^{-1}
$$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

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Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \Rightarrow$

$$
f(t-2)=[1-2(t-2)]^{-1}=(5-2 t)^{-1}
$$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

$$
f(t-2)=[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1}
$$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
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Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

$$
\begin{aligned}
f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
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Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

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& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

2. Set $t=x+2,-1$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

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\begin{aligned}
f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

2. Set $t=x+2$,

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

$$
\begin{aligned}
f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

2. Set $t=x+2,-1<\frac{2}{5} t<1$,

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

$$
\begin{aligned}
f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

2. Set $t=x+2,-1<\frac{2}{5} t<1,-\frac{5}{2}<t<\frac{5}{2}$,

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \quad \Rightarrow$

$$
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f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

2. Set $t=x+2,-1<\frac{2}{5} t<1,-\frac{5}{2}<t<\frac{5}{2},-\frac{9}{2}<t-2<\frac{1}{2}$,

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1 .
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \Rightarrow$

$$
\begin{aligned}
f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
& =\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5} t\right)^{k}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k} t^{k}
\end{aligned}
$$

2. Set $t=x+2,-1<\frac{2}{5} t<1,-\frac{5}{2}<t<\frac{5}{2},-\frac{9}{2}<t-2<\frac{1}{2}$,

$$
f(x)=(1-2 x)^{-1}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k^{2}}(x+2)^{k},
$$

## Expansion of $(1-x)^{-1}$ in $x$ and Related

Expansion of $(1-x)^{-1}$ in $x$ and Geometric Series

$$
(1-x)^{-1}=\sum_{k=0}^{\infty} x^{k}=1+x+x^{2}+\cdots, \quad|x|<1 .
$$

Expand $f(x)=(1-2 x)^{-1}$ in powers of $x+2$.

1. Expand $f(t-2)$ in powers of $t \Rightarrow$

$$
\begin{aligned}
f(t-2) & =[1-2(t-2)]^{-1}=(5-2 t)^{-1}=\frac{1}{5}\left[1-\left(\frac{2}{5} t\right)\right]^{-1} \\
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\end{aligned}
$$

2. Set $t=x+2,-1<\frac{2}{5} t<1,-\frac{5}{2}<t<\frac{5}{2},-\frac{9}{2}<t-2<\frac{1}{2}$,

$$
f(x)=(1-2 x)^{-1}=\frac{1}{5} \sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^{k^{2}}(x+2)^{k}, \quad-\frac{9}{2}<x<\frac{1}{2} .
$$

## Expansion of $(1-x)^{-m}$ in $x$ and Related

Expansion of $(1-x)^{-m}$ in $x$ for $m>1$

$$
(1-x)^{-m}=\frac{1}{(m-1)!} \sum_{k=0}^{\infty}(k+1) \cdots(k+m-1) x^{k} .
$$

## Expand $f(x)=(1-2 x)^{-3}$ in powers of $x+2$.

## Expansion of $(1-x)^{-m}$ in $x$ and Related

Expansion of $(1-x)^{-m}$ in $x$ for $m>1$

$$
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$$

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## Outline

- Taylor Polynomials in $x-a$
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- Taylor Series in $x-a$
- Powers in $x-a$ by Translation

