Lecture 26

Section 11.6 Taylor Polynomials and Taylor Series in x - a

Jiwen He

Department of Mathematics, University of Houston

jiwenhe@math.uh.edu http://math.uh.edu/~jiwenhe/Math1432



Taylor Polynomials in Powers of x - a

The *n*th Taylor polynomial in x - a for a function *f* is

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

P_n is the polynomial that has the same value as *f* at *a* and the same first *n* derivatives:

 $P_n(a) = f(a), P'_n(a) = f'(a), P''_n(a) = f''(a), \dots, P_n^{(n)}(a) = f^{(n)}(a)$

Best Approximation

 P_n provides the best local approximation of f(x) near a by a polynomial of degree $\leq n$.

 $P_0(x) = f(a),$ $P_1(x) = f(a) + f'(a)(x - a),$ f''(a)(x - a),

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Taylor's Theorem

If f has n + 1 continuous derivatives on an open interval l that contains a, then for each $x \in I$,



 $R_n(x) = \frac{1}{n!} \int_{a} f^{(n+1)}(t)(x-t)^n dt.$

Lagrange Formula for the Remainder

where *c* is some number between *a* and

Estimate for the Remainder Term

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$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

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April 17, 2008 3 /

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April 17, 2008 3 /

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$$|R_n(x)| \le \left(\max_{t\in J} |f^{(n+1)}(t)|\right) \frac{|x-a|^{n+1}}{(n+1)!}, \quad J = [a,x] \text{ or } [x,a].$$

Taylor Polynomial and the Remainder



Taylor Series in x - a

Sigma Notation

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April 17, 2008

| / 14



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April 17, 2008

/ 14

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April 17, 2008

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Polynomials Taylor Series Powers in x -

Taylor Polynomials and Taylor Series in x - a of $f(x) = e^x$

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$$f^{(k)}(x) = e^{x}, \quad f^{(k)}(a) = e^{a}, \quad \forall k = 0, 1, 2, \cdots$$

Taylor Polynomials in x - a of the Exponential $f(x) = e^x$

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Taylor Series in x - a of the Exponential $f(x) = e^x$

$$e^{x} = e^{a} \sum_{k=0}^{\infty} \frac{1}{k!} (x-a)^{k} = e^{a} + e^{a} (x-a) + \dots + \frac{e^{a}}{n!} (x-a)^{n} + \dots, \quad \forall x.$$

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$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n;$$

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Expansion of *e^x* in *x* – *a*

Taylor Series in x - a by Translation

Another way to expand f(x) in in powers of x - a is to expand f(t + a) in powers of t and then set t = x - a.

Taylor Series in x - a of the Exponential $f(x) = e^x$

$$e^{x} = e^{a} + e^{a}(x-a) + \frac{e^{a}}{2!}(x-a)^{2} + \dots + \frac{e^{a}}{n!}(x-a)^{n} + \dots, \quad \forall x.$$

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Math 1432 - Section 26626, Lecture 26

Taylor Polynomials Taylor Series Powers in x -

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One way to expand f(x) in in powers of x - a is to expand f(t + a) in powers of t and then set t = x - a. This is the approach to take when the expansion in t is either known or is readily available.

Example

Expand $f(x) = e^{x/2}$ in powers of x - 3

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Math 1432 - Section 26626, Lecture 26

7 / 14

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$$\ln x = \ln a + \frac{1}{a}(x-a) - \frac{1}{2a^2}(x-a)^2 + \frac{1}{3a^3}(x-a)^3 - \cdots, \quad 0 < x \le 2a.$$

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Math 1432 - Section 26626, Lecture 26

/ 14

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Expansion of sin x in $x - \pi$

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$$\sin x = -(x-\pi) + \frac{1}{3!}(x-\pi)^3 - \frac{1}{5!}(x-\pi)^5 + \frac{1}{7!}(x-\pi)^7 + \cdots, \quad \forall x.$$

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April 17, 2008

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Taylor Series in $x - \pi$ of $f(x) = \cos^2 x$

$$\cos^2 x = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k 2^{2k-1}}{(2k)!} (x - \pi)^{2k}, \quad \forall x$$

1. Expand $\cos^2(t+\pi)$ in powers of $t \Rightarrow$

$$\cos^{2}(t+\pi) = \frac{1}{2} + \frac{1}{2}\cos 2(t+\pi) = \frac{1}{2} + \frac{1}{2}\cos(2t+2\pi)$$
$$= \frac{1}{2} + \frac{1}{2}\cos 2t = \frac{1}{2} + \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k)!}(2t)^{2k} = 1 + \sum_{k=1}^{\infty} \frac{(-1)^{k}2^{2k-1}}{(2k)!}t^{2k}$$
$$2. \quad \text{Set } t = x - \pi \quad \Rightarrow$$
$$\cos^{2} x = 1 + \sum_{k=1}^{\infty} \frac{(-1)^{k}2^{2k-1}}{(2k)!}(x-\pi)^{2k}, \quad \text{for all real } x.$$

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Taylor PolynomialsTaylor PolynomialsTaylor SeriesPowers in x - a	
Expansion of $(1 - x)^{-1}$ in x and Related	
Expansion of $(1 - x)^{-1}$ in x and Geometric Series	
$(1-x)^{-1} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots, x < 1.$	
Expand $f(x) = (1 - 2x)^{-1}$ in powers of $x + 2$.	j
1. Expand $f(t-2)$ in powers of $t \Rightarrow$	i.
2. Set $t = x + 2$, $-1 < \frac{2}{5}t < 1$, $-\frac{5}{2} < t < \frac{5}{2}$, $-\frac{9}{2} < t - 2 < \frac{1}{2}$, $f(x) = (1 - 2x)^{-1} = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^k (x + 2)^k$, $-\frac{9}{2} < x < \frac{1}{2}$.	H

Math 1432 – Section 26626, Lecture 26

April 17, 2008



Math 1432 – Section 26626, Lecture 20

April 17, 2008

$$\begin{aligned} & \text{Expansion of } (1-x)^{-1} \text{ in } x \text{ and Related} \\ \\ & \text{Expansion of } (1-x)^{-1} \text{ in } x \text{ and Geometric Series} \\ & (1-x)^{-1} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots, \quad |x| < 1. \\ & \text{Expand } f(x) = (1-2x)^{-1} \text{ in powers of } x + 2. \\ & \text{I. Expand } f(t-2) \text{ in powers of } t \Rightarrow \\ & f(t-2) = [1-2(t-2)]^{-1} = (5-2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t \right) \right]^{-1} \\ & = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}t \right)^k = 1 \\ & \text{Set } t = x + 2 \\ & \text{Set } t = x + 2 \\ & \text{$$

Math 1432 - Section 26626, Lecture 20

April 17, 2008

$$\begin{aligned} & \text{Expansion of } (1-x)^{-1} \text{ in } x \text{ and Related} \\ \\ & \text{Expansion of } (1-x)^{-1} \text{ in } x \text{ and Geometric Series} \\ & (1-x)^{-1} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots, \quad |x| < 1. \\ & \text{Expand } f(x) = (1-2x)^{-1} \text{ in powers of } x + 2. \\ & 1. & \text{Expand } f(t-2) \text{ in powers of } t \Rightarrow \\ & f(t-2) = [1-2(t-2)]^{-1} = (5-2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t\right) \right]^{-1} \\ & = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}t\right)^k = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^k t^k \end{aligned}$$

Math 1432 - Section 26626, Lecture 20

April 17, 2008

Expansion of
$$(1 - x)^{-1}$$
 in x and Related
Expansion of $(1 - x)^{-1}$ in x and Geometric Series
 $(1 - x)^{-1} = \sum_{k=0}^{\infty} x^{k} = 1 + x + x^{2} + \cdots, |x| < 1.$
Expand $f(x) = (1 - 2x)^{-1}$ in powers of $x + 2.$
1. Expand $f(t - 2)$ in powers of $t \Rightarrow$
 $f(t - 2) = [1 - 2(t - 2)]^{-1} = (5 - 2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t\right) \right]^{-1}$
 $= \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}t\right)^{k} = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^{k} t^{k}$

Math 1432 – Section 26626, Lecture 2

April 17, 2008

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Taylor Polynomials Taylor Series Powers in x - aExpansion of
$$(1 - x)^{-1}$$
 in x and RelatedExpansion of $(1 - x)^{-1}$ in x and Geometric Series $(1 - x)^{-1} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots, \quad |x| < 1.$ Expand $f(x) = (1 - 2x)^{-1}$ in powers of $x + 2$.1. Expand $f(t - 2)$ in powers of $t \Rightarrow$ $f(t - 2x)^{-1}$ in $f(t - 2)$ in powers of $t \Rightarrow$

 $f(t-2) = \left[1 - 2(t-2)\right]^{-1} = (5-2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t\right)\right]^{-1}$ $= \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}t\right)^k = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^k t^k$

2. Set t = x + 2, $-1 < \frac{2}{5}t < 1$, $-\frac{5}{2} < t < \frac{5}{2}$, $-\frac{9}{2} < t - 2$



 $f(t-2) = \left[1 - 2(t-2)\right]^{-1} = (5-2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t\right)\right]^{-1}$ $= \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}t\right)^{k} = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^{k} t^{k}$

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1. Expand f(t-2) in powers of $t \Rightarrow$ $f(t-2) = [1-2(t-2)]^{-1} = (5-2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t\right) \right]^{-1}$ $= \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}t\right)^k = \frac{1}{5} \sum_{k=0}^{\infty} \left(\frac{2}{5}\right)^k t^k$

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| Taylor Polynomials Taylor Polynomials Taylor Series Powers in $x - a$ | | | |
|--|--|--|--|
| Expansion of $(1-x)^{-1}$ in x and Related | | | |
| Expansion of $(1 - x)^{-1}$ in x and Geometric Series | | | |
| $(1-x)^{-1} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots, x < 1.$ | | | |
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| 1. Expand $f(t-2)$ in powers of $t \Rightarrow$ | | | |

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	Taylor Polynomials	Taylor Polynomials Taylor Series Powers in x — a		
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(1 - x)	$y^{-1} = \sum_{k=0}^{\infty} x^k = 1 + $	$+x+x^2+\cdots, x <1.$		
Expand $f(x) = (1 - 2x)^{-1}$ in powers of $x + 2$.				
1. Expand $f(t-2)$ in powers of $t \Rightarrow$				
f(t-2) = [1]	$[-2(t-2)]^{-1} =$	$(5-2t)^{-1} = \frac{1}{5} \left[1 - \left(\frac{2}{5}t\right) \right]^{-1}$		
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Math 1432 - Section 26626, Lecture 26



Math 1432 - Section 26626, Lecture 26



Math 1432 - Section 26626, Lecture 26



$$(1-x)^{-m} = \frac{1}{(m-1)!} \sum_{k=0}^{\infty} (k+1) \cdots (k+m-1)!$$

Expand $f(x) = (1 - 2x)^{-3}$ in powers of x + 2.



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Outline

- Taylor Polynomials in x a
 - Taylor Polynomials in x a
 - Taylor Series in x a
 - Powers in x a by Translation