Name and ID: ______________________________

50 points 1. Use the Laplace Transform to find the solution of the following initial-value problems

a. $y'' + y = \sin 2t, \quad y(0) = 0, \quad y'(0) = 1.$

b. $y'' - y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 0.$

Hint: Table of the Laplace Transform:

$$F = \mathcal{L}(f) \quad \Leftrightarrow \quad f = \mathcal{L}^{-1}(F)$$

$F(s)$	$\mathcal{L}^{-1}\{F(s)\}(t)$
$\frac{1}{s-c}$	e^{ct}
$\frac{1}{(s-c)^k}$	$\frac{t^{k-1}}{(k-1)!}e^{ct}$
$\frac{1}{(s-\alpha)^2+\beta^2}$	$\frac{e^{\alpha t}\sin\beta t}{\beta}$
$\frac{s-\alpha}{(s-\alpha)^2+\beta^2}$	$e^{\alpha t}\cos\beta t$

50 points 2. Consider the initial value problem

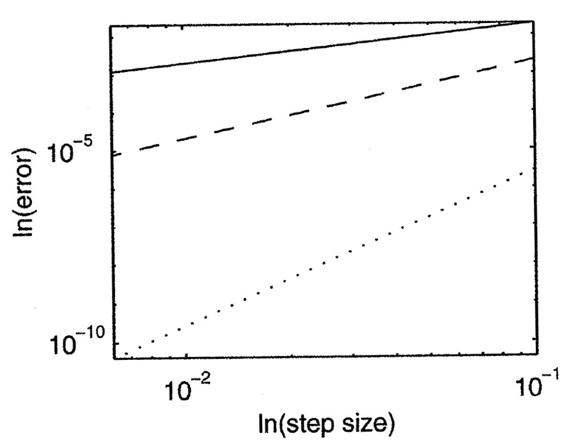
$$x' = x + t, \quad x(0) = 1. \tag{1}$$

Carry out one step calculation of the Euler, RK2 and RK4 methods with step size $h = 0.5$ to approximate the value of $x(0.5)$ and compute the error of your numerical solution.

20 points 3. (**BONUS PROBLEM**) Consider the initial value problem

$$x' = -tx^2, \quad 0 \le t \le 2, \quad x(0) = 3. \tag{2}$$

The equation is separable and the solution is $x(t) = 6/(3t^2+2)$. We used the Euler, RK2 and RK4 methods to compute the value of $x(2)$ and constructed a plot of the logarithm of the error versus the logarithm of the step size for each numerical method. The slope of the solid line is 1.0135, the slope of the dashed line is 2.0303, and the slope of the dotted line is 4.0256. Indicate each line by its corresponding numerical method and explain your answer.



Name and ID: ____________________

Problem 1.

Name and ID: ______________________________

Problem 2.

Name and ID: ____________________

Problem 3.

When you finish this exam, you should go back and reexamine your work for any errors that you may have made.