ODE

Midterm 3 **Math 3331 (Summer 2014)** **July 2, 2014**

Name and ID: _____

50 points 1. Find the solution of the initial-value problem

$$\begin{aligned} x' &= -3x \\ y' &= -5x + 6y - 4z \\ z' &= -5x + 2y \end{aligned}$$

with $x(0) = -1$, $y(0) = 0$ and $z(0) = 1$.

50 points 2. Find the solution of the initial-value problem

$$\begin{aligned} x' &= 6x - 4z \\ y' &= 8x - 2y \\ z' &= 8x - 2z \end{aligned}$$

with $x(0) = -2$, $y(0) = -1$ and $z(0) = 0$.

20 points 3. (**BONUS PROBLEM**) Classify the equilibrium point of the system $y' = Ay$. Sketch the phase portrait by hand.

$$(1)\ A = \begin{pmatrix} -16 & 9 \\ -18 & 11 \end{pmatrix} \qquad (2)\ A = \begin{pmatrix} 2 & 1 \\ -10 & -5 \end{pmatrix}$$

Eigenvalues and eigenvectors of matrices

- The eigen-pairs of $A = \begin{pmatrix} 2 & 1 \\ -10 & -5 \end{pmatrix}$ are

$$\lambda_1 = 0, \quad v_1 = (1, -2)^T,$$
$$\lambda_2 = -3, \quad v_2 = (1, -5)^T.$$

- The eigen-pairs of $A = \begin{pmatrix} -16 & 9 \\ -18 & 11 \end{pmatrix}$ are

$$\lambda_1 = -7, \quad v_1 = (1, 1)^T,$$
$$\lambda_2 = 2, \quad v_2 = (1, 2)^T.$$

- The eigen-pairs of $A = \begin{pmatrix} 6 & 0 & -4 \\ 8 & -2 & 0 \\ 8 & 0 & -2 \end{pmatrix}$ are

$$\lambda_1 = -2, \quad v_1 = (0, 1, 0)^T,$$
$$\lambda_2 = 2 + 4i, \quad v_2 = (1 + i, 2, 2)^T,$$
$$\lambda_3 = 2 - 4i, \quad v_3 = (1 - i, 2, 2)^T.$$

- The eigen-pairs of $A = \begin{pmatrix} -3 & 0 & 0 \\ -5 & 6 & -4 \\ -5 & 2 & 0 \end{pmatrix}$ are

$$\lambda_1 = 4, \quad v_1 = (0, 2, 1)^T,$$
$$\lambda_2 = -3, \quad v_2 = (1, 1, 1)^T,$$
$$\lambda_3 = 2, \quad v_3 = (0, 1, 1)^T.$$

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Problem 1.

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Problem 2.

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Problem 3. (BONUS PROBLEM)

When you finish this exam, you should go back and reexamine your work for any errors that you may have made.