# Lecture 9 4.2 Second-Order Equations and Systems

#### Jiwen He

Department of Mathematics, University of Houston

jiwenhe@math.uh.edu math.uh.edu/~jiwenhe/math3331



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## 4.2 Second-Order Equations and Systems

Second-Order Equations

#### • Planar Systems

- yv-Phase Plane Plot
- Phase Plane Portrait



# Second-Order Equations and Planar Systems

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#### Second-order DE

$$y'' + ay' + by = 0 \quad (1)$$

$$p(\lambda) = \lambda^2 + a\lambda + b = 0$$

### yv-Phase Plane Plot

A damped unforced spring:

$$my'' + \mu y' + ky = 0$$

with m = 1,  $\mu = 0.4$ , and k = 3.

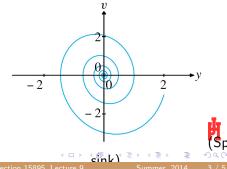
#### planar system

$$x_1 = y, \quad x_2 = v = y'$$

$$\mathbf{x}' = A\mathbf{x}, \quad A = \begin{pmatrix} 0 & 1 \\ -b & -a \end{pmatrix}$$
 (2)

 $\det(A - \lambda I) = p(\lambda)$ 

(Chapter 9)



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## Phase Plane Portrait

Ex.: 
$$y'' - y = 0$$
  $(a = 0, b = -1)$   
 $p(\lambda) = \lambda^2 - 1 \Rightarrow \lambda = \pm 1$  (saddle)  
General solution:  $y(t) = c_1 e^t + c_2 e^{-t}$   
 $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \begin{cases} \lambda_1 = 1 & \leftrightarrow \mathbf{v}_1 = [1, 1]^T \\ \lambda_2 = -1 \leftrightarrow \mathbf{v}_2 = [-1, 1]^T \end{cases}$   
 $q_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda_1 = 1 & \leftrightarrow \mathbf{v}_1 = [1, 1]^T \\ \lambda_2 = -1 \leftrightarrow \mathbf{v}_2 = [-1, 1]^T \end{cases}$   
Phase plane portrait for DE (1) = Phase plane portrait for (2)



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#### .2 Planar Systems Phase Plane

## Phase Plane Portrait

Ex.: 
$$y'' - 3y' + 2y = 0$$
  
 $p(\lambda) = \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2)$   
 $\Rightarrow$  source:  $y(t) = c_1 e^t + c_2 e^{2t}$   
 $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \rightarrow \begin{cases} \lambda_1 = 1 \leftrightarrow \mathbf{v}_1 = \begin{bmatrix} 1, 1 \end{bmatrix}^T$   
 $\lambda_2 = 2 \leftrightarrow \mathbf{v}_2 = \begin{bmatrix} 1, 2 \end{bmatrix}^T$ 

 $\begin{array}{c} 2 \\ -2 \\ -1.5 \\$ 

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