

HW1 Solutions

Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

13.

a.

| | | | |
|----|--|---------------------------|-----------------|
| 2 | | 23 | stem units: 1.0 |
| 3 | | 2344567789 | leaf units: .10 |
| 4 | | 01356889 | |
| 5 | | 00001114455666789 | |
| 6 | | 0000122223344456667789999 | |
| 7 | | 0001223345555668 | |
| 8 | | 02233448 | |
| 9 | | 012233335666788 | |
| 10 | | 2344455688 | |
| 11 | | 2335999 | |
| 12 | | 37 | |
| 13 | | 8 | |
| 14 | | 36 | |
| 15 | | 0035 | |
| 16 | | | |
| 17 | | | |
| 18 | | 9 | |

- b. A representative value could be the median, 7.0.
- c. The data appear to be highly concentrated, except for a few values on the positive side.
- d. No, the data is skewed to the right, or positively skewed.
- e. The value 18.9 appears to be an outlier, being more than two stem units from the previous value.

33.

- a. A stem-and leaf display of this data appears below:

| | | | |
|----|--|-------|--------------|
| 32 | | 55 | stem: ones |
| 33 | | 49 | leaf: tenths |
| 34 | | | |
| 35 | | 6699 | |
| 36 | | 34469 | |
| 37 | | 03345 | |
| 38 | | 9 | |
| 39 | | 2347 | |
| 40 | | 23 | |
| 41 | | | |
| 42 | | 4 | |

The display is reasonably symmetric, so the mean and median will be close.

- b. The sample mean is $\bar{X} = 9638/26 = 370.7$. The sample median is $\tilde{X} = (369+370)/2 = 369.50$.
- c. The largest value (currently 424) could be increased by any amount. Doing so will not change the fact that the middle two observations are 369 and 170, and hence, the median will not change. However, the value $x = 424$ can not be changed to a number less than 370 (a change of $424-370 = 54$) since that *will* lower the values(s) of the two middle observations.
- d. Expressed in minutes, the mean is $(370.7 \text{ sec})/(60 \text{ sec}) = 6.18 \text{ min}$; the median is 6.16 min.

41.

a. range = 49.3 - 23.5 = 25.8

b.

| x_i | $(x_i - \bar{x})$ | $(x_i - \bar{x})^2$ | x_i^2 |
|-------|-------------------|---------------------|---------|
| 29.5 | -1.53 | 2.3409 | 870.25 |
| 49.3 | 18.27 | 333.7929 | 2430.49 |
| 30.6 | -0.43 | 0.1849 | 936.36 |
| 28.2 | -2.83 | 8.0089 | 795.24 |
| 28.0 | -3.03 | 9.1809 | 784.00 |
| 26.3 | -4.73 | 22.3729 | 691.69 |
| 33.9 | 2.87 | 8.2369 | 1149.21 |
| 29.4 | -1.63 | 2.6569 | 864.36 |
| 23.5 | -7.53 | 56.7009 | 552.25 |
| 31.6 | 0.57 | 0.3249 | 998.56 |

$\Sigma x = 310.3$ $\Sigma(x_i - \bar{x}) = 0$ $\Sigma(x_i - \bar{x})^2 = 443.801$ $\Sigma(x_i^2) = 10,072.41$

$\bar{x} = 31.03$

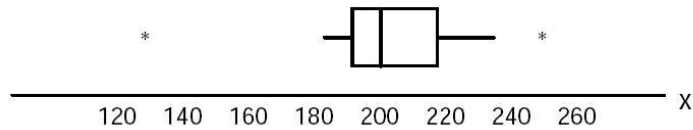
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} = \frac{443.801}{9} = 49.3112$$

c. $s = \sqrt{s^2} = 7.0222$

d. $s^2 = \frac{\Sigma x^2 - (\Sigma x)^2 / n}{n-1} = \frac{10,072.41 - (310.3)^2 / 10}{9} = 49.3112$

54.

- a. $1.5(IQR) = 1.5(216.8-196.0) = 31.2$ and $3(IQR) = 3(216.8-196.0) = 62.4$.
 Mild outliers: observations below $196-31.2 = 164.6$ or above $216.8+31.2 = 248$.
 Extreme outliers: observations below $196-62.4 = 133.6$ or above $216.8+62.4 = 279.2$. Of the observations given, 125.8 is an extreme outlier and 250.2 is a mild outlier.
- b. A boxplot of this data appears below. There is a bit of positive skew to the data but, except for the two outliers identified in part (a), the variation in the data is relatively small.



4.

- a. Event A = { RRR, LLL, SSS }
- b. Event B = { RLS, RSL, LRS, LSR, SRL, SLR }
- c. Event C = { RRL, RRS, RLR, RSR, LRR, SRR }
- d. Event D = { RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS }
- e. Event D' contains outcomes where all cars go the same direction, or they all go different directions:
 $D' = \{ RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR \}$

Because Event D totally encloses Event C, the compound event $C \cup D = D$:

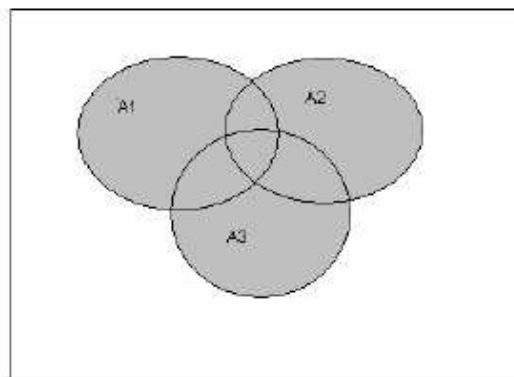
$C \cup D = \{ RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS \}$

Using similar reasoning, we see that the compound event $C \cap D = C$:

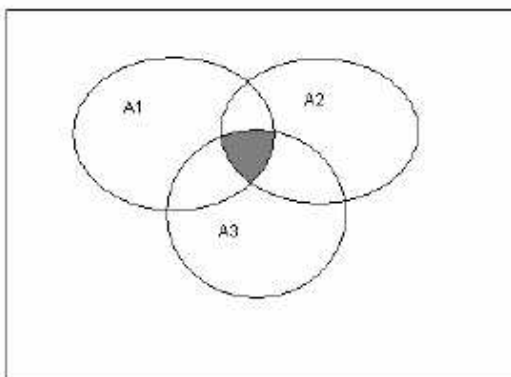
$C \cap D = \{ RRL, RRS, RLR, RSR, LRR, SRR \}$

10.

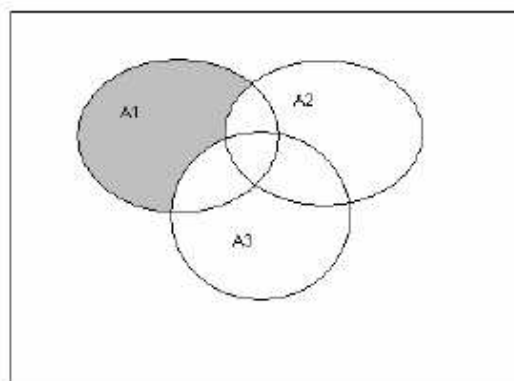
a. $A_1 \cup A_2 \cup A_3$



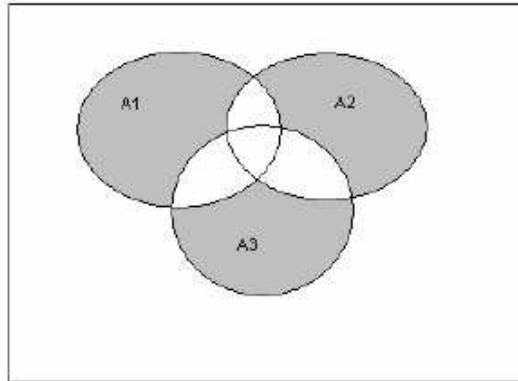
b. $A_1 \cap A_2 \cap A_3$



c. $A_1 \cap A_2' \cap A_3'$



d. $(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$



e. $A_1 \cup (A_2 \cap A_3)$

