HW10 Solutions

Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

39.

a. $P(0 \le Z \le 2.17) = \Phi(2.17) - \Phi(0) = .4850$

b. $\Phi(1) - \Phi(0) = .3413$

c. $\Phi(0) - \Phi(-2.50) = .4938$

d. $\Phi(2.50) - \Phi(-2.50) = .9876$

e. $\Phi(1.37) = .9147$

f. $P(-1.75 < Z) + [1 - P(Z < -1.75)] = 1 - \Phi(-1.75) = .9599$

g. $\Phi(2) - \Phi(-1.50) = .9104$

h. $\Phi(2.50) - \Phi(1.37) = .0791$

i. $1 - \Phi(1.50) = .0668$

j. $P(|Z| \le 2.50) = P(-2.50 \le Z \le 2.50) = \Phi(2.50) - \Phi(-2.50) = .9876$

41.

a. $\Phi(c) = .9100 \Rightarrow c \approx 1.34$ (.9099 is the entry in the 1.3 row, .04 column)

b. 9^{th} percentile = -91^{st} percentile = -1.34

c. $\Phi(c) = .7500 \Rightarrow c \approx .675$ since .7486 and .7517 are in the .67 and .68 entries, respectively.

d. $25^{th} = -75^{th} = -.675$

e. $\Phi(c) = .06 \Rightarrow c \approx .-1.555$ (.0594 and .0606 appear as the –1.56 and –1.55 entries, respectively).

45.

a. $P(X > .25) = P(Z > -.83) = 1 - .2033 = .7967$

b. $P(X \le .10) = \Phi(-3.33) = .0004$

c. We want the value of the distribution, c, that is the 95^{th} percentile (5% of the values are higher). The 95^{th} percentile of the standard normal distribution = 1.645. So c = .30 + (1.645)(.06) = .3987. The largest 5% of all concentration values are above .3987 mg/cm^3.

60. $P(|X - \mu| \ge \sigma) = P(X \le \mu - \sigma \text{ or } X \ge \mu + \sigma)$

$= 1 - P(\mu - \sigma \le X \le \mu + \sigma) = 1 - P(-1 \le Z \le 1) = .3174$

Similarly, $P(|X - \mu| \ge 2\sigma) = 1 - P(-2 \le Z \le 2) = .0456$

And $P(|X - \mu| \ge 3\sigma) = 1 - P(-3 \le Z \le 3) = .0026$

These are considerably less than the bounds 1, .25, and .11 given by Chebyshev.

63. p = .10; n = 200; np = 20, npq = 18

a. $P(X \le 30) = \Phi\left(\dfrac{30+.5-20}{\sqrt{18}}\right) = \Phi(2.47) = .9932$

b. $P(X < 30) = P(X \le 29) = \Phi\left(\dfrac{29+.5-20}{\sqrt{18}}\right) = \Phi(2.24) = .9875$

c. $P(15 \le X \le 25) = P(X \le 25) - P(X \le 14) = \Phi\left(\dfrac{25+.5-20}{\sqrt{18}}\right) - \Phi\left(\dfrac{14+.5-20}{\sqrt{18}}\right)$

$\Phi(1.30) - \Phi(-1.30) = .9032 - .0968 = .8064$

69.

a. $\Gamma(6) = 5! = 120$

b. $\Gamma\left(\dfrac{5}{2}\right) = \dfrac{3}{2}\Gamma\left(\dfrac{1}{2}\right) = \dfrac{3}{2}\cdot\dfrac{1}{2}\cdot\Gamma\left(\dfrac{1}{2}\right) = \left(\dfrac{3}{4}\right)\sqrt{\pi} \approx 1.329$

c. F(4;5) = .371 from row 4, column 5 of Table A.4

d. F(5;4) = .735

e. $F(0;4) = P(X \le 0;\ \alpha = 4) = 0$

71.

a. $\mu = 20,\ \sigma^2 = 80 \Rightarrow \alpha\beta = 20,\ \alpha\beta^2 = 80 \Rightarrow \beta = \frac{80}{20},\ \alpha = 5$

b. $P(X \le 24) = F\left(\dfrac{24}{4};5\right) = F(6;5) = .715$

c. $P(20 \le X \le 40) = F(10;5) - F(5;5) = .411$

78. With x_p = (100p)th percentile, $p = F(x_p) = 1 - e^{-\lambda x_p} \Rightarrow e^{-\lambda x_p} = 1 - p$,

$\Rightarrow -\lambda x_p = \ln(1-p) \Rightarrow x_p = \dfrac{-[\ln(1-p)]}{\lambda}$. For p = .5, $x_{.5} = \tilde{\mu} = \dfrac{.693}{\lambda}$.