

HW11 Solutions

Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

82.

a.  $E(X) = 3\Gamma\left(1 + \frac{1}{2}\right) = 3 \cdot \frac{1}{2} \cdot \Gamma\left(\frac{1}{2}\right) = 2.66,$

$$\text{Var}(X) = 9\left[\Gamma(1+1) - \Gamma^2\left(1 + \frac{1}{2}\right)\right] = 1.926$$

b.  $P(X \leq 6) = 1 - e^{-(6/\beta)^\alpha} = 1 - e^{-(6/3)^2} = 1 - e^{-4} = .982$

c.  $P(1.5 \leq X \leq 6) = 1 - e^{-(6/3)^2} - \left[1 - e^{-(1.5/3)^2}\right] = e^{-25} - e^{-4} = .760$

89.

a.  $E(X) = e^{3.5+(1.2)^2/2} = 68.0335; V(X) = e^{2(3.5)+(1.2)^2} \cdot (e^{(1.2)^2} - 1) = 14907.168;$   
 $\sigma_x = 122.0949$

b.  $P(50 \leq X \leq 250) = P\left(z \leq \frac{\ln(250) - 3.5}{1.2}\right) - P\left(z \leq \frac{\ln(50) - 3.5}{1.2}\right)$

$$P(Z \leq 1.68) - P(Z \leq .34) = .9535 - .6331 = .3204.$$

c.  $P(X \leq 68.0335) = P\left(z \leq \frac{\ln(68.0335) - 3.5}{1.2}\right) = P(Z \leq .60) = .7257.$  The lognormal distribution is not a symmetric distribution.

108.  $y = 1/x \rightarrow x = 1/y$  and  $x > 1 \rightarrow 1/y > 1 \rightarrow 0 < y < 1$ . Thus,  $f_Y(y) = f_X(1/y)|dx/dy| = f_X(1/y) \cdot 1/y^2 = 2y^3(1/y^2) = 2y$  for  $0 < y < 1$ . If  $X$  is the ratio of one runner's time to the winning time, then  $Y$  is the reciprocal ratio.

112.  $y = \exp(x) \rightarrow x = \ln(y)$  and  $x$  real  $\rightarrow y > 0$ . Thus,  $f_Y(y) = f_X(\ln y)|dx/dy| = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(\ln y - \mu)^2\right) \left|\frac{1}{y}\right| = \frac{1}{\sqrt{2\pi}\sigma y} \exp\left(-\frac{1}{2\sigma^2}(\ln y - \mu)^2\right)$  for  $y > 0$ .