HW12 Solutions
Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

1.

a. $P(X = 1, Y = 1) = p(1,1) = .20$

b. $P(X \le 1 \text{ and } Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .42$

c. At least one hose is in use at both islands. $P(X \ne 0 \text{ and } Y \ne 0) = p(1,1) + p(1,2) + p(2,1) + p(2,2) = .70$

d. By summing row probabilities, $p_x(x) = .16, .34, .50$ for $x = 0, 1, 2$, and by summing column probabilities, $p_y(y) = .24, .38, .38$ for $y = 0, 1, 2$. $P(X \le 1) = p_x(0) + p_x(1) = .50$

e. $P(0,0) = .10$, but $p_x(0) \cdot p_y(0) = (.16)(.24) = .0384 \ne .10$, so X and Y are not independent.

7.

a. $p(1,1) = .030$

b. $P(X \le 1 \text{ and } Y \le 1 = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .120$

c. $P(X = 1) = p(1,0) + p(1,1) + p(1,2) = .100$; $P(Y = 1) = p(0,1) + \ldots + p(5,1) = .300$

d. $P(\text{overflow}) = P(X + 3Y > 5) = 1 - P(X + 3Y \le 5) = 1 - P[(X,Y)=(0,0) \text{ or } \ldots \text{or } (5,0) \text{ or } (0,1) \text{ or } (1,1) \text{ or } (2,1)] = 1 - .620 = .380$

e. The marginal probabilities for X (row sums from the joint probability table) are $p_x(0) = .05$, $p_x(1) = .10$, $p_x(2) = .25$, $p_x(3) = .30$, $p_x(4) = .20$, $p_x(5) = .10$; those for Y (column sums) are $p_y(0) = .5$, $p_y(1) = .3$, $p_y(2) = .2$. It is now easily verified that for every (x,y), $p(x,y) = p_x(x) \cdot p_y(y)$, so X and Y are independent.

11.

a. $p(x,y) = \dfrac{e^{-\lambda}\lambda^x}{x!} \cdot \dfrac{e^{-\mu}\mu^y}{y!}$ for $x = 0, 1, 2, \ldots$; $y = 0, 1, 2, \ldots$

b. $p(0,0) + p(0,1) + p(1,0) = e^{-\lambda-\mu}\left[1 + \lambda + \mu\right]$

c. $P(X+Y=m) = \sum_{k=0}^{m} P(X = k, Y = m-k) = \sum_{k=0}^{m} e^{-\lambda-\mu} \dfrac{\lambda^k}{k!} \dfrac{\mu^{m-k}}{(m-k)!}$

$\dfrac{e^{-(\lambda+\mu)}}{m!} \sum_{k=0}^{m} \binom{m}{k} \lambda^k \mu^{m-k} = \dfrac{e^{-(\lambda+\mu)}(\lambda+\mu)^m}{m!}$, so the total # of errors X+Y also has a Poisson distribution with parameter $\lambda + \mu$.