HW14 Solutions

Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

1.

	$P(x_1)$	.20	.50	.30
$P(x_2)$	$x_2 \mid x_1$	25	40	65
.20	25	.04	.10	.06
.50	40	.10	.25	.15
.30	65	.06	.15	.09

a.

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	.04	.20	.25	.12	.30	.09

$$E(\bar{x}) = (25)(.04) + 32.5(.20) + \ldots + 65(.09) = 44.5 = \mu$$

b.

s^2	0	112.5	312.5	800
$P(s^2)$	.38	.20	.30	.12

$E(s^2) = 212.25 = \sigma^2$

4.

a. Possible values of M are: 0, 5, 10. M = 0 when all 3 envelopes contain 0 money, hence p(M = 0) = $(.5)^3$ = .125. M = 10 when there is a single envelope with $10, hence p(M = 10) = 1 – p(no envelopes with $10) = 1 – $(.8)^3$ = .488.
p(M = 5) = 1 – [.125 + .488] = .387.

M	0	5	10
p(M)	.125	.387	.488

An alternative solution would be to list all 27 possible combinations using a tree diagram and computing probabilities directly from the tree.

b. The statistic of interest is M, the maximum of x_1, x_2, or x_3, so that M = 0, 5, or 10. The population distribution is a s follows:

x	0	5	10
p(x)	1/2	3/10	1/5

Write a computer program to generate the digits 0 – 9 from a uniform distribution. Assign a value of 0 to the digits 0 – 4, a value of 5 to digits 5 – 7, and a value of 10 to digits 8 and 9. Generate samples of increasing sizes, keeping the number of replications constant and compute M from each sample. As n, the sample size, increases, p(M = 0) goes to zero, p(M = 10) goes to one. Furthermore, p(M = 5) goes to zero, but at a slower rate than p(M = 0).

11. $\mu = 12$ cm $\sigma = .04$ cm

a. n = 16 $E(\overline{X}) = \mu = 12cm \quad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{4} = .01cm$

b. n = 64 $E(\overline{X}) = \mu = 12cm \quad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{8} = .005cm$

c. $\overline{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\overline{X}$ with a larger sample size.

13.

a. $\mu_{\overline{X}} = \mu = 50$, $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{1}{\sqrt{100}} = .10$

$$P(49.75 \le \overline{X} \le 50.25) = P\left(\frac{49.75 - 50}{.10} \le Z \le \frac{50.25 - 50}{.10}\right)$$

= P(-2.5 ≤ Z ≤ 2.5) = .9876

b. $$P(49.75 \le \overline{X} \le 50.25) \approx P\left(\frac{49.75 - 49.8}{.10} \le Z \le \frac{50.25 - 49.8}{.10}\right)$$

= P(-.5 ≤ Z ≤ 4.5) = .6915

17. X ~ N(10,1), n = 4

$\mu_{T_0} = n\mu = (4)(10) = 40$ and $\sigma_{T_0} = \sigma\sqrt{n} = (2)(1) = 2,$

We desire the 95th percentile: 40 + (1.645)(2) = 43.29

21.

a. With Y = # of tickets, Y has approximately a normal distribution with $\mu = \lambda = 50$, $\sigma = \sqrt{\lambda} = 7.071$, so P(35 ≤ Y ≤ 70) ≈ $P\left(\frac{34.5-50}{7.071} \le Z \le \frac{70.5-50}{7.071}\right)$ = P(-2.19 ≤ Z ≤ 2.90) = .9838

b. Here $\mu = 250$, $\sigma^2 = 250, \sigma = 15.811$, so P(225 ≤ Y ≤ 275) ≈ $P\left(\frac{224.5-250}{15.811} \le Z \le \frac{275.5-250}{15.811}\right)$ = P(-1.61 ≤ Z ≤ 1.61) = .8926