HW5 Solutions
Math 3338-10853: Probability (Fall 2006), Dr. Jiwen He

12.

a. In order for the flight to accommodate all the ticketed passengers who show up, no more than 50 can show up. We need $Y \leq 50$.
$P(Y \leq 50) = .05 + .10 + .12 + .14 + .25 + .17 = .83$

b. Using the information in a. above, $P(Y > 50) = 1 - P(Y \leq 50) = 1 - .83 = .17$

c. For you to get on the flight, at most 49 of the ticketed passengers must show up. $P(Y \leq 49) = .05 + .10 + .12 + .14 + .25 = .66$. For the 3rd person on the standby list, at most 47 of the ticketed passengers must show up. $P(Y \leq 47) = .05 + .10 + .12 = .27$

14.

a. $\sum_{y=1}^{5} p(y) = K[1 + 2 + 3 + 4 + 5] = 15K = 1 \Rightarrow K = \frac{1}{15}$

b. $P(Y \leq 3) = p(1) + p(2) + p(3) = \frac{6}{15} = .4$

c. $P(2 \leq Y \leq 4) = p(2) + p(3) + p(4) = \frac{9}{15} = .6$

d. $\sum_{y=1}^{5}\left(\frac{y^2}{50}\right) = \frac{1}{50}[1 + 4 + 9 + 16 + 25] = \frac{55}{50} \neq 1$; No

17.

a. $P(2) = P(Y = 2) = P(1^{st}$ 2 batteries are acceptable)
$= P(AA) = (.9)(.9) = .81$

b. $p(3) = P(Y = 3) = P(UAA \text{ or } AUA) = (.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$

c. The fifth battery must be an A, and one of the first four must also be an A. Thus, $p(5) = P(AUUUA \text{ or } UAUUA \text{ or } UUAUA \text{ or } UUUAA) = 4[(.1)^3(.9)^2] = .00324$

d. $P(Y = y) = p(y) = P(\text{the } y^{th} \text{ is an A and so is exactly one of the first } y - 1)$
$= (y - 1)(.1)^{y-2}(.9)^2$, $y = 2,3,4,5,\ldots$

18.

a. $p(1) = P(M = 1) = P[(1,1)] = \frac{1}{36}$

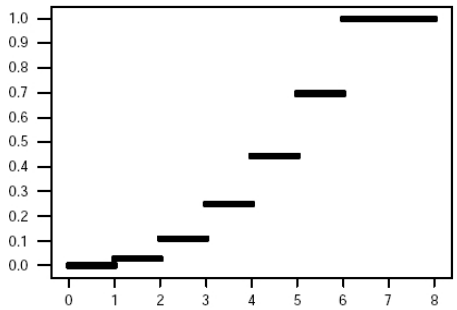
$p(2) = P(M = 2) = P[(1,2) \text{ or } (2,1) \text{ or } (2,2)] = \frac{3}{36}$

$p(3) = P(M = 3) = P[(1,3) \text{ or } (2,3) \text{ or } (3,1) \text{ or } (3,2) \text{ or } (3,3)] = \frac{5}{36}$

Similarly, $p(4) = \frac{7}{36}$, $p(5) = \frac{9}{36}$, and $p(6) = \frac{11}{36}$

b. $F(m) =$ 0 for $m < 1$, $\frac{1}{36}$ for $1 \le m < 2$,

$$F(m) = \begin{cases} 0 & m < 1 \\ \frac{1}{36} & 1 \le m < 2 \\ \frac{4}{36} & 2 \le m < 3 \\ \frac{9}{36} & 3 \le m < 4 \\ \frac{16}{36} & 4 \le m < 5 \\ \frac{25}{36} & 5 \le m < 6 \\ 1 & m \ge 6 \end{cases}$$



27. If $x_1 < x_2$, $F(x_2) = P(X \le x_2) = P(\{X \le x_1\} \cup \{x_1 < X \le x_2\})$

$= P(X \le x_1) + P(x_1 < X \le x_2) \ge P(X \le x_1) = F(x_1)$.

$F(x_1) = F(x_2)$ when $P(x_1 < X \le x_2) = 0$.