Chapter Three: Discrete Random Variables and Probability Distributions

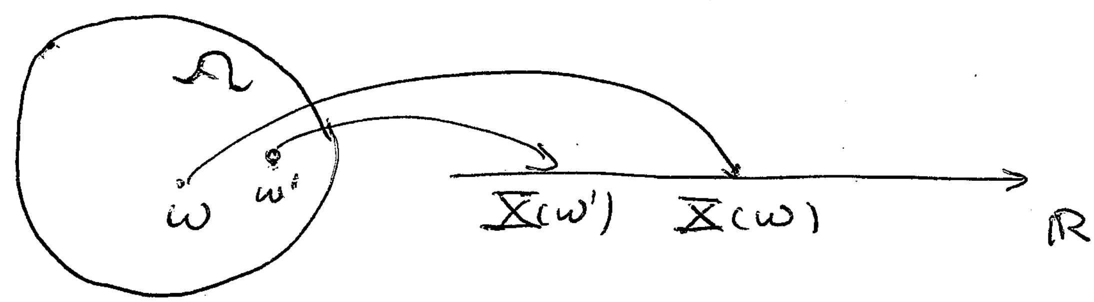
3.1 Definition of Random Variable.

* A numerically valued function X defined on a sample space is called <u>random variable (rv)</u>:

$$\omega \in \Omega: \quad \omega \longmapsto X(\omega)$$

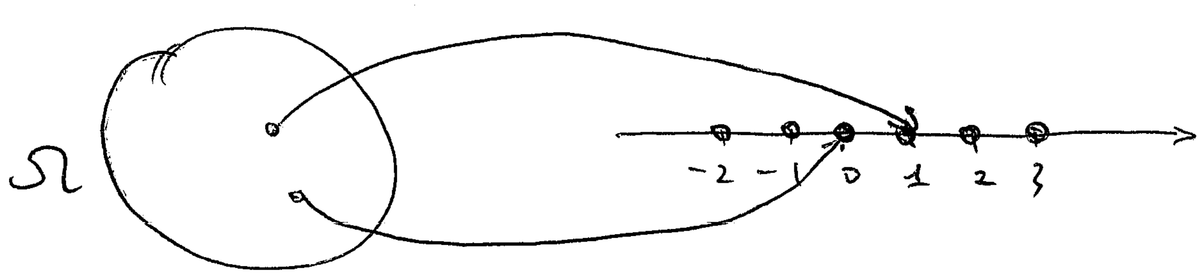
the domain of X: Ω

the exact range of X: $V_X = \bigcup_{\omega \in \Omega} \{X(\omega)\}$



* A <u>discrete</u> random variable is an rv whose possible values either constitue a finite set or a countable set.

* Let X be a (discrete) rv and let $x_1, x_2, \ldots$ be the values which it assumes; in most of what follows the x_j will be integers $\quad V_X = \{x_1, x_2, \cdots\}$



* In the case of a <u>discrete</u> sample space (i.e., Ω is a finite or countable set), we can actually tabulate any random variable $\underline{X}$ by enumerating in some order all points of the space and associating with each the corresponding value of $\underline{X}$.

* Let us introduce some convenient symbolism to denote various sets of sample points derived from random variables.

  — The singleton $\{x\}$ is defined by $\{\underline{X} = x\}$

$$\{\omega \mid \underline{X}(\omega) = x\} =: \{\underline{X} = x\}$$

* If $x \notin V_{\underline{X}}$, then $\{x\} = \phi$.

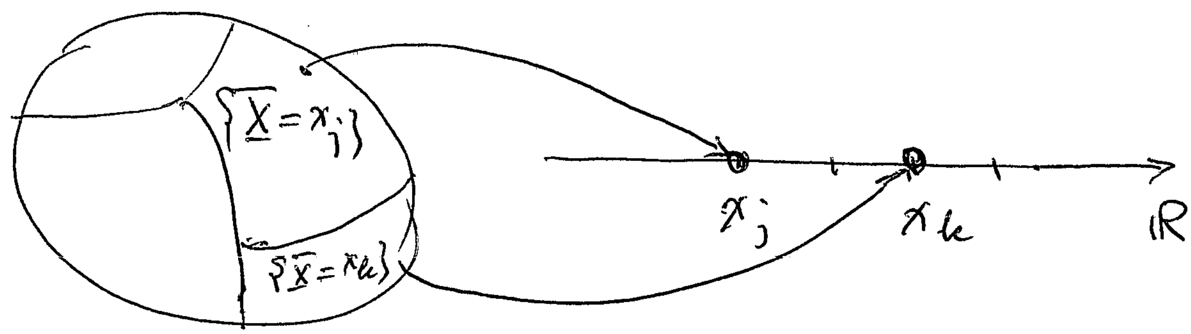
* Let $V_{\underline{X}} = \{x_j\}_j$. We have

$$\bigcup_j \{\underline{X} = x_j\} = \Omega$$

Since the x_j's are distinct, the sets $\{\underline{X} = x_j\}$ must be <u>disjoint</u>.

<u>Partition of Ω</u>:

$$\sum_j \{\underline{X} = x_j\} = \Omega$$



— $\{\underline{X} \le x\}$ is defined by

$$\{\omega \mid \underline{X}(\omega) \le x\} =: \{\underline{X} \le x\}$$

$$= \bigcup_{y \le x} \{\underline{X} = y\}$$

* Let $V_X = \{x_j\}$.

where $x_1 \le x_2 \le \cdots$

Then $\{\underline{X} \le x_j\} = \sum_{i \le j} \{\underline{X} = x_i\}$

— $\{y \le \underline{X} \le x\}$ is defined by

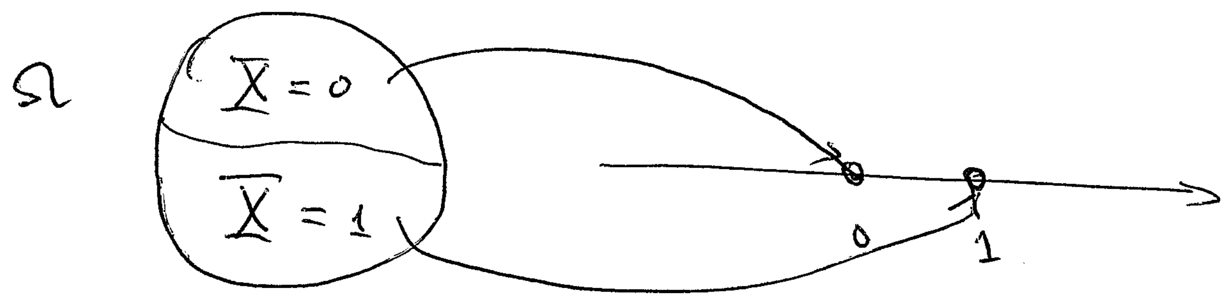
$$\{\omega \mid y \le \underline{X}(\omega) \le x\} =: \{y \le \underline{X} \le x\}$$

$$= \bigcup_{y \le z \le x} \{\underline{X} = z\}$$

— $\{\underline{X} \in A\}$ is defined by

$$\{\omega \mid \underline{X}(\omega) \in A\} =: \{\underline{X} \in A\}.$$

* Any random variable whose only possible values are 0 and 1 is called a <u>Bernoulli r v</u>.



$V_X = \{0, 1\}$, $\Omega = \{X=0\} + \{X=1\}$.

* Starting with some random variables, we can make new ones by operating on them in various ways.

— If X and Y are random variables, then so are

$$X+Y, \quad X-Y, \quad XY, \quad X/Y \; (Y \neq 0)$$

and $aX + bY$ where a and b are two numbers.

— The sum of n random variables,

$$S_n = \sum_{i=1}^{n} X_i$$

is a random variable.