

# EMCF 38

## Question 1

Compute the lower Riemann sum for the given function  $f(x) = 3 - x^2$  over the interval  $x \in [0, 1]$  with respect to the partition  $P = \left[0, \frac{1}{4}, \frac{3}{4}, 1\right]$

- a)   $\frac{165}{64}$
- b)   $\frac{181}{64}$
- c)   $\frac{169}{64}$
- d)   $\frac{157}{64}$
- e)   $\frac{173}{64}$

## Question 2

Given that

$$\left[ \int_0^2 f(x) \, dx = 1, \int_0^4 f(x) \, dx = 6, \int_4^7 f(x) \, dx = 3 \right]$$

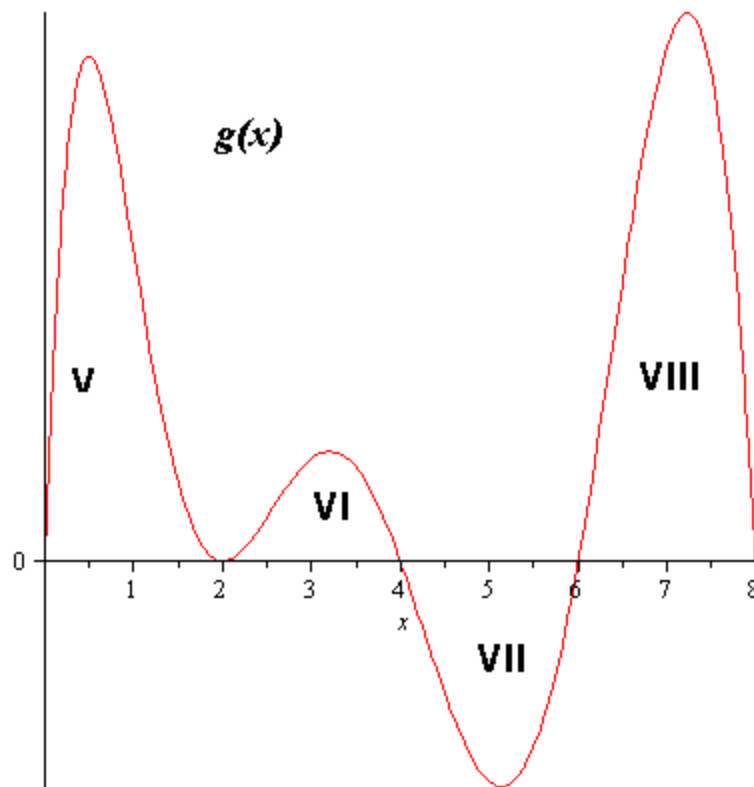
find

$$\int_0^7 f(x) \, dx$$

- a)  6
- b)  10
- c)  2
- d)  9
- e)  3

## Question 3

The graph of  $g(x)$  is shown below. Regions V, VI, VII and VIII have areas  $4$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$  and  $5$  respectively.



Compute the following integral (**pay attention to the endpoints**).

$$\int_0^6 2g(x) dx$$

- a)  16
- b)  1
- c)  18
- d)  26
- e)  6

#### Question 4

Find a formula for  $f(x)$  given that  $f$  is continuous and

$$-6x^7 + x^4 - 3x = \int_0^x \frac{f(t)}{t+1} dt$$

- a)   $f(x) = -42x^6 + 4x^3 - 3$

- b)   $f(x) = (-6x^7 + x^4 - 3x)(x + 1)$
- c)   $f(x) = (-42x^6 + 4x^3 - 3)(x + 1)$
- d)   $f(x) = -6x^7 + x^4 - 3x$
- e)   $f(x) = -\frac{3}{4}x^8 + \frac{1}{5}x^5 - \frac{3}{2}x^2 - 3$

**Question 5**

Given that  $x > -7$  and

$$F(x) = \int_4^x t\sqrt{t+7} \, dt$$

find  $F(4)$ .

- a)   $4\sqrt{11}$
- b)   $13\sqrt{11}/11$
- c)   $0$
- d)   $4$
- e)   $\sqrt{11}$

**Question 6**

Find the derivative of the function  $F$

$$F(x) = \int_0^{x \sin(x)} \sqrt{25 - t^2} \, dt$$

- a)   $(\sin(x) + x \cos(x)) \sqrt{25 - x^2}$
- b)   $(\sin(x) + x \cos(x)) \sqrt{25 - (x \sin(x))^2}$
- c)   $\sqrt{25 - x^2}$
- d)   $-\frac{(x \sin(x))}{\sqrt{25 - (x \sin(x))^2}}$
- e)   $\sqrt{25 - (x \sin(x))^2}$

**Question 7**

Evaluate the definite integral:

$$\int_1^3 (6x + x^3) dx$$

- a)  44
- b)  20
- c)   $\frac{98}{3}$
- d)   $\frac{362}{5}$
- e)  38

**Question 8**

Evaluate the definite integral:

$$\int_1^9 2\sqrt{x} dx$$

- a)  8
- b)  4
- c)   $\frac{104}{3}$
- d)   $\frac{484}{5}$
- e)   $\frac{968}{5}$

**Question 9**

Find the area bounded by the curves

$$y = 1 - x^2$$
$$y = 13 - 7x$$

- a)   $\frac{1}{6}$

b)   $-\frac{1}{12}$

c)   $\frac{1}{12}$

d)   $-\frac{1}{6}$

e)   $\frac{1}{3}$

**Question 10**

Calculate the indefinite integral:

$$\int \frac{5x^3 - 4}{x^2} dx$$

a)   $\frac{5}{2}x^2 + \frac{4}{x} + C$

b)   $\frac{5x^3 + 8}{x^3} + C$

c)   $\frac{5}{2}x^2 - 4x + C$

d)   $\frac{5}{3}x^3 - 4x + C$

e)   $5x + \frac{4}{x} + C$

**Question 11**

Calculate the indefinite integral:

$$\int \left( 2\sqrt{x} - \frac{7}{\sqrt{x}} \right) dx$$

a)   $-14\sqrt{x} + \frac{4}{3}x^{3/2} + C$

b)   $\frac{14}{3}x^{3/2} + \frac{4}{5}x^{5/2} + C$

c)   $\frac{1}{\sqrt{x}} + \frac{7}{2x^{3/2}} + C$

- d)   $-\frac{10}{3}x^{3/2} + C$
- e)   $14\sqrt{x} + \frac{4}{3}x^{3/2} + C$

**Question 12**

Calculate:

$$\int \frac{12x + 36}{\sqrt{x^2 + 6x - 2}} dx$$

- a)   $12\sqrt{x^2 + 6x - 2} + C$
- b)   $6\sqrt{x^2 + 6x - 2} + C$
- c)   $-12\sqrt{x^2 + 6x - 2} + C$
- d)   $2\sqrt{x^2 + 6x - 2} + C$
- e)   $-2\sqrt{x^2 + 6x - 2} + C$

**Question 13**

Calculate the integral:

$$\int \cos^6(2x) \sin(2x) dx$$

- a)   $\frac{1}{7} \cos^7(2x) + C$
- b)   $-\frac{2}{7} \sin^7(2x) + C$
- c)   $\frac{1}{7} \sin^7(2x) + C$
- d)   $-\frac{1}{14} \cos^7(2x) + C$
- e)   $\frac{1}{12} \cos^7(2x) + C$

**Question 14**

Evaluate:

$$\int_{-1}^0 9x^2(4+2x^3)^2 dx$$

- a)  28
- b)   $14/3$
- c)   $37/3$
- d)   $28/3$
- e)  56

### Question 15

Find  $f$  based on the following information:

$$\begin{aligned} f''(x) &= 2x^2 - x \\ f'(4) &= \frac{116}{3} \\ f(1) &= 5 \end{aligned}$$

- a)   $f(x) = \frac{1}{6}x^4 - \frac{1}{6}x^3 + 4x + 1$
- b)   $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 + \frac{29}{6}$
- c)   $f(x) = \frac{1}{6}x^4 - \frac{1}{6}x^3 + 4x + \frac{1}{6}$
- d)   $f(x) = \frac{1}{6}x^4 - \frac{1}{6}x^3 + 4x + \frac{11}{6}$
- e)   $f(x) = -\frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{29}{6}$

### Question 16

Find the average value of the function  $f(x)$  on the interval  $[0, 6]$  and determine a number  $c$  in this interval for which  $f(c)$  is equal to the average value.

$$f(x) = 12x - 6x^2$$

- a)  Average value = -36,  $c = 1 - \sqrt{7}$

- b)  Average value = -216,  $c = \frac{1}{2} - \frac{1}{2}\sqrt{7}$
- c)  Average value = 0,  $c = 0$
- d)  Average value = -216,  $c = -72$
- e)  Average value = -36,  $c = 1 + \sqrt{7}$

**Question 17**

Which of the following integrals represents the area of the region bounded by the curves,  $y = -2x^{1/2}$ ,  $y = x - 3$ , and  $y = 0$ , **in terms of  $x$** ?

- a)   $\int_0^3 (-2\sqrt{x}) dx + \int_3^4 (x-3) dx$
- b)   $\int_0^1 2\sqrt{x} dx + \int_1^3 (-x+3) dx$
- c)   $\int_0^3 2\sqrt{x} dx$
- d)   $\int_0^2 2\sqrt{x} dx + \int_2^3 (-x+3) dx$
- e)   $\int_0^1 (-2\sqrt{x}) dx + \int_1^3 (-x+3) dx$

**Question 18**

Which of the following integrals represents the area of the region bounded by the curves,  $y = -7x^{1/2}$ ,  $y = x - 8$ , and  $y = 0$ , **in terms of  $y$** ?

- a)   $\int_{-7}^0 \left( \frac{1}{49}y^2 - y - 8 \right) dy$
- b)   $\int_{-7}^0 \left( y + 8 - \frac{1}{49}y^2 \right) dy$
- c)   $\int_0^8 \frac{1}{49}y^2 dy$



d)   $\int_{-7}^0 (y + 8) \, dy$

e)   $\int_0^8 \left( y + 8 - \frac{1}{49} y^2 \right) \, dy$

### Question 19

Sketch the region bounded by the following curves and find the volume of the solid generated by revolving this region about the  $x$ -axis.

$$y = 5\sqrt{x}$$

$$y = 5x^3$$

a)   $\frac{139}{14} \pi$

b)   $\frac{181}{14} \pi$

c)   $\frac{167}{14} \pi$

d)   $\frac{125}{14} \pi$

e)   $\frac{153}{14} \pi$

### Question 20

Sketch the region bounded by the following curves and find the volume of the solid generated by revolving this region about the  $y$ -axis.

$$x = y^3$$

$$x = 1$$

$$y = 0$$

a)   $\frac{13}{7} \pi$

b)   $\frac{27}{7} \pi$

c)   $\frac{34}{7} \pi$

d)   $\frac{6}{7}\pi$

e)   $\frac{20}{7}\pi$