Math 1431 - 15825

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Office Hours: 11-noon MWF

Course Homepage

http://www.math.uh.edu/~jmorgan/Math1431

Reminders:

- Test 1, Practice Test 1, Quiz 1 are all available online.
- EMCF01 was due this morning at 9am.
- EMCF02 is due on Friday morning at 9am.
- Homework 1 is posted and due next Wednesday.
- Written Quiz 1 will be given in lab/workshop on Friday.
- **Purchase your Access Code NOW** from the UH Bookstore, and input it on *CourseWare* at http://www.casa.uh.edu. **Pick up your Popper Forms by the end of next week.**

tinyurl.com/math1431
@morgancalculus

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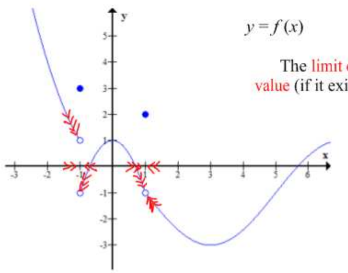
Read the Syllabus

Access all Online Quizzes, Practice Test 1, Test 1, answer sheets for EMCFs and the Discussion Board on *CourseWare* at **http://www.casa.uh.edu**. All practice tests count as online quizzes, and all online quizzes and online tests expire at 11:59pm on the stated date.

Note: The test dates listed below are subject to change.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
August 26 **Purchase your Course Access Code from the UH Bookstore ASAP** and input it on CourseWare (www.casa.uh.edu)	27 **Notes, Video** **Practice Test 1, Test 1, and Online Quizzes are Open** **Note:** Test 1 is online, and there are only 2 attempts.	28	29 **Blank Slides** **EMCF01 Due Online at 9am** **Homework 1 Posted**	30	31 **EMCF02 Due Online at 9am** Quiz in Lab/Workshop	September 1
2 **Note:** There are only 2 attempts for online Test 1.	3 Labor Day Holiday	4 EMCF03 Due Online at 9am Last day to add a class via myUH.	5 **Practice Test 1 and Test 1 Close.** Homework 1 Due in Lab/Workshop Homework 2 Posted	6	7 **EMCF04 Due Online at 9am** **Quiz in Lab/Workshop**	8

Limits - Basic Ideas and Review



The limit of $f(x)$ as x approaches a is given by the value (if it exists) we expect from $f(x)$ as x approaches a.

$\lim_{x\to 1} f(x) = -1$

$\lim_{x\to -1} f(x) = \text{DNE}$

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$\lim_{x\to 1^-} f(x) = -1$

$\lim_{x\to 1^+} f(x) = -1$

Are $=$

Not $=$

$\lim_{x\to -1^-} f(x) = 1$

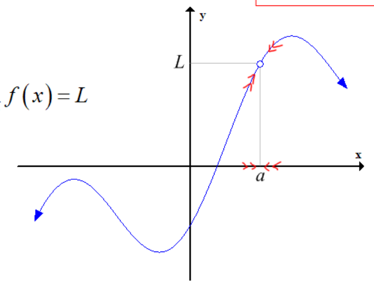
$\lim_{x\to -1^+} f(x) = -1$

The limit of $f(x)$ as x approaches a is L if and only if $f(x)$ can be made arbitrarily close to L by making $x \neq a$ sufficiently close to a.

Otherwise, we say the limit does not exist and we write *dne*.

" The limit of $f(x)$ as x approaches a is L if and only if " is equivalent to the notation $\boxed{\lim\limits_{x \to a} f(x) = L.}$

$\lim\limits_{x \to a} f(x) = L$



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One Sided Limits, *in words...*

The limit of $f(x)$ as x approaches a from the left is L if and only if $f(x)$ can be made arbitrarily close to L by making $x < a$ sufficiently close to a.

The limit of $f(x)$ as x approaches a from the left is L, is equivalent to the notation $\lim\limits_{x \to a^-} f(x) = L.$

The limit of $f(x)$ as x approaches a from the right is L if and only if $f(x)$ can be made arbitrarily close to L by making $x > a$ sufficiently close to a.

The limit of $f(x)$ as x approaches a from the left is L, is equivalent to the notation $\lim\limits_{x \to a^+} f(x) = L.$

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The Fundamental Relationship Between Left Hand Limits, Right Hand Limits, and Limits

$$\boxed{\begin{gathered}\lim_{x \to a} f(x) = L \\ \text{if and only if} \\ \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L\end{gathered}}$$

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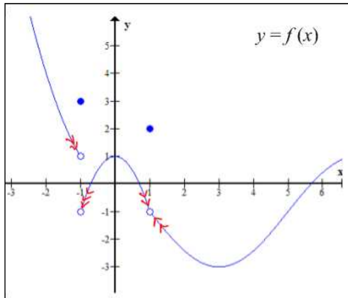

$\lim\limits_{x \to -1^-} f(x) = 1$ $\lim\limits_{x \to 1^-} f(x) = -1$

$\lim\limits_{x \to -1^+} f(x) = -1$ $\lim\limits_{x \to 1^+} f(x) = -1$

$\lim\limits_{x \to -1} f(x) = \text{DNE}$ $\lim\limits_{x \to 1} f(x) = -1$

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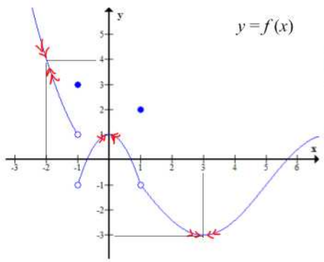

$\lim_{x\to 0} f(x) = 1$

$\lim_{x\to -1} f(x) = \text{DNE}$

$\lim_{x\to 1} f(x) = -1$

$\lim_{x\to 3} f(x) = -3$

$\lim_{x\to -2} f(x) = 4$

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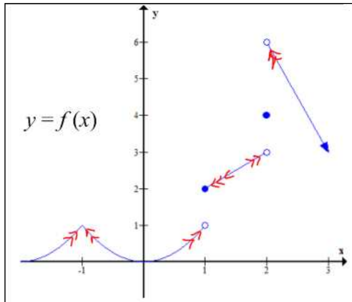

$\lim_{x\to -1^-} f(x) = 1$ $\quad$ $\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to -1^+} f(x) = 1$ $\quad$ $\lim_{x\to 1^+} f(x) = 2$

$\lim_{x\to -1} f(x) = 1$ $\quad$ $\lim_{x\to 1} f(x) = \text{DNE}$

$\lim_{x\to 2^-} f(x) = 3$ $\quad$ $\lim_{x\to 2^+} f(x) = 6$ $\quad$ $\lim_{x\to 2} f(x) = \text{DNE}$

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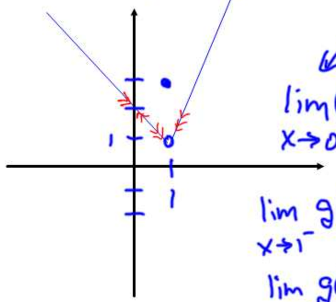
Suppose $g(x) = \begin{cases} -x+2, & x<1 \\ 3 & x=1. \\ 3x-2, & x>1 \end{cases}$ Find $\lim_{x\to 0} g(x)$ and $\lim_{x\to 1} g(x)$.



$\lim_{x\to 0}(-x+2) = 2$

$= 1$

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-}(-x+2) = 1$

$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+}(3x-2) = 1$

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Intuitive Limits From Formulas...

$\lim_{x\to -2}(2x^2-1) = 7$

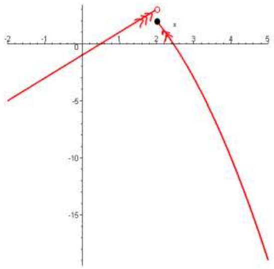
$\lim_{x\to 2} \frac{x^2-3x+2}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}}$

"$\frac{0}{0}$ 'indeterminant form"

$= \lim_{x\to 2}(x-1) = 1$

$$\lim_{x\to 2} f(x) \quad \text{where} \quad f(x)=\begin{cases}2x-1 & x<2\\ 6-x^2 & x\ge 2\end{cases}$$

" DNE



Not =

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x-1) = 3$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(6-x^2) = 2$$

$$\lim_{x\to 1}\frac{\overset{a}{\sqrt{x^2+1}}-\overset{b}{\sqrt{2}}}{x-1} = \lim_{x\to 1}\frac{\overset{(a-b)}{\left(\sqrt{x^2+1}-\sqrt2\right)}\overset{(a+b)}{\left(\sqrt{x^2+1}+\sqrt2\right)}}{(x-1)\left(\sqrt{x^2+1}+\sqrt2\right)}$$

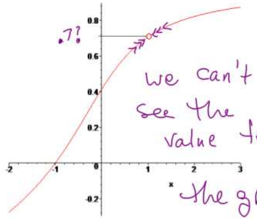
$$=\lim_{x\to 1}\frac{\overset{a^2-b^2}{x^2+1-2}}{(x-1)\left(\sqrt{x^2+1}+\sqrt2\right)}$$

$$=\lim_{x\to 1}\frac{x^2-1}{(x-1)\left(\sqrt{x^2+1}+\sqrt2\right)}$$

$$=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}\left(\sqrt{x^2+1}+\sqrt2\right)}$$

$$=\frac{2}{2\sqrt2}=\frac{1}{\sqrt2}=\frac{\sqrt2}{2}$$

our limit

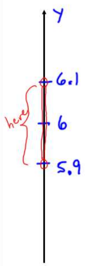


.7?

we can't see the value from the graph.

$$(a-b)(a+b)=a^2-b^2$$

Limits Done Right!



What does it mean to say that y is within .1 of 6?

strictly

$$5.9 < y < 6.1$$
$$-6 \quad -6 \quad -6$$

$$-.1 < y-6 < .1$$

$$|y-6| < .1$$

"y is within .1 of 6"

$$|y-7| < .215$$

"y is within .215 of 7"

$$|\overset{y-(-4)}{y+4}| < .01$$

"y is within .01 of −4"

What does it mean to say that x is within .01 of 12, but not equal to 12?

$$|x-12| < .01 \quad \text{and} \quad x \neq 12$$

$$0 < |x-12| < .01$$

"x is within .01 of 12, but not equal to 12"

$$0 < |x-7| < .001$$

"x is within .001 of 7, but not equal to 7"

What does it mean to say that
$f(x)$ is within ε of L?

y

$\varepsilon > 0$

$$|f(x) - L| < \varepsilon$$

"$f(x)$ is within ε of L"

What does it mean to say that
$\boxed{x \text{ is within } \delta \text{ of } c,}$
$\boxed{\text{but not equal to } c?}$

$\delta > 0$

$$0 < |x - c| < \delta$$

"x is within δ of c,
but not equal to c"

Let's use this language to discuss limits...

$$\lim_{x \to a} f(x) = L$$

$$\Leftrightarrow$$

??

The limit of $f(x)$ as x approaches a is L if and only if $\boxed{f(x) \text{ can be made arbitrarily close to } L \text{ by making } x \neq a \text{ sufficiently close to } a.}$

Let $\varepsilon > 0$. Then there is a value $\delta > 0$ so that
$$|f(x) - L| < \varepsilon$$
whenever
$$0 < |x - a| < \delta.$$

Give the largest δ that works with ε=.1 for the limit

$$\boxed{\lim_{x \to -1}(1 - 2x) = 3}$$

$f(x) = 1 - 2x$

$L = 3 \quad a = -1$

Let $\varepsilon > 0$. Then there is a value $\delta > 0$ so that
$|f(x) - L| < \varepsilon$
whenever
$\underline{0 < |x - a| < \delta}.$

$\underbrace{0 < |x - -1| < \delta}$

$0 < |x + 1| < \delta$

Hm... we need $\delta > 0$ that forces $\underline{|(1 - 2x) - 3| < .1}$

$|-2x - 2| < .1$

$|-2(x+1)| < .1$

$2|x+1| < .1$

$0 < |x+1| < \underbrace{.05}_{\delta}$

Use $\underline{\delta = .05}$.

Prove that $\lim_{x \to -1} (1 - 2x) = 3$.

Hm... Scratch work.

Let $\varepsilon > 0$. Then there is a value $\delta > 0$ so that

$$|f(x) - L| < \varepsilon$$

whenever

$$0 < |x - a| < \delta.$$

Need to force

$$\boxed{|(1-2x) - 3| < \varepsilon}$$

by making

$$0 < |x - -1| < \delta$$

$$|-2(x+1)| < \varepsilon$$

$$2|x+1| < \varepsilon$$

$$0 < |x - -1| < \frac{\varepsilon}{2} \leftarrow \delta$$

Let $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{2}$. If $0 < |x - -1| < \delta$ then

$$0 < |x - -1| < \frac{\varepsilon}{2} \Rightarrow 2|x+1| < \varepsilon$$

$$\Rightarrow |-2(x+1)| < \varepsilon \Rightarrow |(1-2x) - 3| < \varepsilon.$$

$$\therefore \lim_{x \to -1} (1-2x) = 3.$$