Math 1431 - 15825

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Course Homepage

http://www.math.uh.edu/~jmorgan/Math1431
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Reminders:

- Homework 1 is due next Wednesday in lab/workshop.
- There is a quiz today in lab/workshop.
- Test 1 and Practice Test 1 are due next Wednesday.They are available online at http://www.casa.uh.edu.
- Quiz 1 is available at http://www.casa.uh.edu.
- EMCF03 is posted, and it is due next Tuesday.
- Poppers start on Monday, September 10th.
- Purchase your Access Code from the UH Bookstore, and then log into this course at http://www.casa.uh.edu and input the code.

Take advantage of the Discussion Board on CourseWare!

Limit Theorems

Uniqueness, sums, differences, products and quotients.

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Uniqueness of the Limit

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} f(x) = M$, then $L = M$.

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The Limit of the Sum

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$,
then $\lim_{x \to c} \left(f(x) + g(x) \right) = L + M$.

is the Sum of the Limits

(provided the limits exist)

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The Limit of the Difference

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$,
then $\lim_{x \to c} \left(f(x) - g(x) \right) = L - M$.

is the Difference of the Limits

(provided the limits exist)

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The Limit of the Product

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$,
then $\lim_{x \to c} \left(f(x) \cdot g(x) \right) = L \cdot M$.

is the Product of the Limits

(provided the limits exist)

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The Limit of the Quotient

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$,
and $M \neq 0$, then $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$.

is the Quotient of the Limits

* **(provided the quotient exists)** *

(provided the limits exist)

More on Quotients

If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$,

and $L \neq 0$ and $M = 0$, then $\lim_{x\to c} \frac{f(x)}{g(x)}$ d.n.e.

Numerator $\neq 0$ denominator $= 0$

Note: If $L = 0$ and $M = 0$ then you need to do more work.

Recall: "$\frac{0}{0}$" is an indeterminant form.

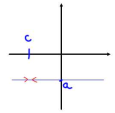
How Can We Use This Information?

Building Blocks

a is a constant

$$\lim_{x\to c} a = a$$

$$\lim_{x\to c} x = c$$



$\lim_{x\to c} x^2 = \lim_{x\to c} x \cdot x = c \cdot c = c^2$

$\lim_{x\to c} x^3 = \lim_{x\to c} x^2 \cdot x = c^2 \cdot c = c^3$

$\lim_{x\to c} x^n = c^n$ if n is a positive integer.

$\lim_{x\to 2} (2x^2 - 3x + 1)$

$= \lim_{x\to 2} (\underline{\underline{2}} \cdot x^2 - \underline{\underline{3}} \cdot x + \underline{\underline{1}})$

sums, differences + products of limits we know.

$\lim_{x\to 2} 2 = 2$ $\lim_{x\to 2} 3 = 3$

$\lim_{x\to 2} x^2 = 4$ $\lim_{x\to 2} x = 2$

$\lim_{x\to 2} 1 = 1$

$= 2 \cdot 2^2 - 3 \cdot 2 + 1$

$= 8 - 6 + 1 = 3.$

What is a Polynomial Function?

If the independent variable is x, then a polynomial in x is an expression that can be built from sums, differences and products of x with itself and scalars. ↳ #

ex. $f(x) = -3$

$g(x) = 2x - 1$

$F(x) = 3x^2 - 2x + \frac{7}{5}$

Note: $G(x) = \frac{3x}{x-1}$ is not a polynomial

$H(x) = 3x^{2/3} - 1$

Answer

Anything of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

Assuming x is the ind. variable.

$a_n, \ldots, a_0$ known real #s

n is a positive integer

For example

$f(x) = -2x^3 + x - 1$ ← 3rd degree polynomial

$g(x) = 9x^2 - 3x + \frac{1}{2}$ ← 2nd degree

$r(x) = 5x - 7$ ← 1st degree

$S(t) = -8t^5 + t^3 - 17t$

$H(x) = -3$ ← 0th degree

Degree? Highest power on x.

What is a Rational Function?

quotients of polynomials

A Rational Function is a Quotient of Polynomials

For example

$$f(x)=\frac{-2x^3+x-1}{3x^2+1} \begin{matrix}\leftarrow \text{poly.}\\ \leftarrow \text{poly}\end{matrix}$$

$$r(x)=\frac{-2}{3x+1} \begin{matrix}\leftarrow 0^{\text{th}} \text{ degree poly}\\ \leftarrow \text{poly}\end{matrix}$$

$$G(v)=\frac{v^8-3v}{2v^2+v-1} \begin{matrix}\leftarrow \text{poly}\\ \leftarrow \text{poly}\end{matrix}$$

$$H(x)=2x^2-3x+1=\frac{2x^2-3x+1}{1} \begin{matrix}\leftarrow \text{poly}\\ \leftarrow \text{poly}\end{matrix}$$

Note: Every polynomial is a rational function.

Note:

Polynomial
and
Rational Functions
Are
Sums, Products and Quotients
of Scalars and the independent variable.

Theorem

Let $p(x)$ be a polynomial and let a be a real number. Then

$$\lim_{x\to a} p(x)=p(a)$$

i.e. We can take limit of a polynomial function by simply evaluating the function.

Example: $\lim_{x \to -2} \underbrace{\left(-5x^4 + 3x^2 + x\right)}_{\text{polynomial}} = -70$

Evaluate the polynomial at $x = -2$.

$-5(-2)^4 + 3(-2)^2 + (-2)$

$= -80 + 12 - 2$

$= -82 + 12$

$= -70$

How do we evaluate

$\lim_{x \to a} \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials?

Theorem

Let $p(x)$ and $q(x)$ be polynomials, and let a be a real number. Then

$$\lim_{x \to a} \frac{p(x)}{q(x)} = \begin{cases} \dfrac{p(a)}{q(a)}, & \text{if } q(a) \neq 0 \\ d.n.e. & \text{if } p(a) \neq 0 \text{ and } q(a) = 0 \end{cases}$$

numer. denom.

If $p(a) = q(a) = 0$ then more work is required to determine the limit.

Gives a "$\frac{0}{0}$" indeterminant form.

Example: $\lim_{x \to 2} \dfrac{-2x^3 + x - 1}{3x^2 + 1}$ ← polynomial ← polynomial

rational function

Hmm... what happens at $x = 2$?

$= \dfrac{-15}{13}$ ok

$= \dfrac{-15}{13}$

Example: $\lim_{x\to 1}\frac{x^3-1}{x^2+x-2}$ ← polynomial ← polynomial

rational function

Evaluate at $x=1$.

"$\frac{0}{0}$" indeterminant form.

Get to work!

$$\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+2)}$$

$$= \lim_{x\to 1}\frac{x^2+x+1}{x+2} \quad \begin{matrix}\leftarrow \text{poly}\\ \leftarrow \text{poly}\end{matrix}$$

$$= \frac{3}{3} = 1.$$

Example: $\lim_{x\to -1}\frac{x^3-1}{x^2+x}$ ← poly ← poly

rational function

Evaluate at $x=-1$

"$\frac{-2}{0}$"

$= \text{d.n.e.}$

b/c numer $\to \neq 0$

denom $\to 0$

Example: $f(x)=2x^2-x+1$. $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=$

"$\frac{0}{0}$" ind. form

$$= \lim_{h\to 0}\frac{2(2+h)^2-(2+h)+1-7}{h}$$

$$= \lim_{h\to 0}\frac{2(4+4h+h^2)-2-h-6}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{8}+8h+2h^2-\cancel{8}-h}{h}$$

$$= \lim_{h\to 0}\frac{7\cancel{h}+2h^{\cancel{2}}}{\cancel{h}} = \lim_{h\to 0}\frac{7+2h}{1}$$

$$= 7.$$

Example: $f(x)=\frac{x}{x+1}$. $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=$

"$\frac{0}{0}$" indeterminant form

$$\lim_{x\to 1}\frac{\frac{x}{x+1}-\frac{1}{2}}{x-1} = \lim_{x\to 1}\frac{\frac{2x-(x+1)}{2(x+1)}}{x-1}$$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b}\cdot\frac{1}{c} = \frac{a}{bc}$$

$$= \lim_{x\to 1}\frac{2x-(x+1)}{2(x+1)(x-1)}$$

$$= \lim_{x\to 1}\frac{\cancel{(x-1)}}{2(x+1)\cancel{(x-1)}}$$

$$= \lim_{x\to 1}\frac{1}{2(x+1)}$$

$$= \frac{1}{4}$$

Note: We have learned that whenever $f(x)$ is a polynomial or rational function and c is in the domain of $f(x)$, then

$$\lim_{x \to c} f(x) = f(c)$$

i.e. we can evaluate the limit by evaluating the function?

Question: Are there other functions that have this property?

A: yes. Anything built from $|\cdot|$, $\sqrt{\cdot}$, trig, … provided the expression is defined in an interval containing c.