Math 1431

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Office Hours: 11:00-Noon MWF

~~No Office Hours Today~~

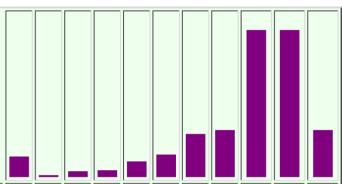
http://www.math.uh.edu/~jmorgan/Math1431
tinyurl.com/math1431
@morgancalculus

Next Monday is an Important Day!

- **Homework 2** due in lab/workshop.
- **EMCF05** is due online at 9:00am.
- **Online Quiz 1** is due at 11:59pm.
- **Poppers** start in lecture.
- **Access Codes** are due at 12:01am.

Purchase your Popper forms and Access Code from the UH Bookstore in the University Center.

Test 1 Grades



number of students	22	2	6	7	17	24	47	51	160	160	51
score range	0 to 10	10 to 20	20 to 30	30 to 40	40 to 50	50 to 60	60 to 70	70 to 80	80 to 90	90 to 100	100 to 110

did not take the test

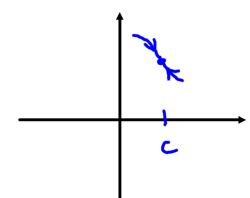
$\boxed{\text{Median} = 88}$

525 students took Test 1
283 students made 88 or higher

Review of Continuity

Graphical Description

f(x) is continuous at $x = c$

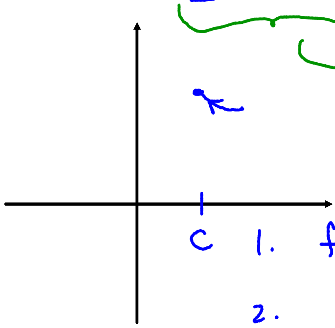


Definition in Terms of Limits

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

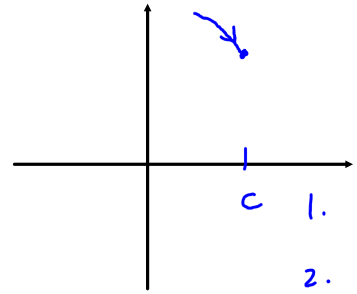
Question: What do you think it means to say that a function $f(x)$ is right continuous at $x = c$?

continuous from the right at c.



1. $f(c)$ exists
2. $\lim_{x \to c^+} f(x)$ exists
3. $\lim_{x \to c^+} f(x) = f(c)$

Question: What do you think it means to say that a function $f(x)$ is left continuous at $x = c$?



1. $f(c)$ exists
2. $\lim_{x \to c^-} f(x)$ exists
3. $\lim_{x \to c^-} f(x) = f(c)$

Example: Describe the continuity of the function below at $x = 2$.

piecewise

$$g(x) = \begin{cases} 2x - 3, & x < 2 \\ x - x^2, & x \geq 2 \end{cases}$$

line

parabola

$g(2) = -2$

$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (2x - 3) = 1$

$\neq$

$=$

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (x - x^2) = -2$

cont. and left cont. fail.

$\therefore$ g is right continuous at $x = 2$.

Q: What sort of discont do we have at $x = ?$

A: Jump.

Question: What do you think it means to say that a function $f(x)$ is continuous on an interval?

(1)

$[a, b] = \{x \mid a \leq x \leq b\}$

* f is right cont. at $x = a$
* f is left cont. at $x = b$
* f is continuous at each $x = c$ with $a < c < b$.

(2)

$(a, b] = \{x \mid a < x \leq b\}$

* f is left cont. at $x = b$
* f is continuous at each $x = c$ with $a < c < b$.

③

$[a,b) = \{x \mid a \le x < b\}$

* f is right cont. at $x=a$

* f is continuous at each $x=c$ with $a<c<b$.

④

$(a,b) = \{x \mid a < x < b\}$

* f is continuous at each $x=c$ with $a<c<b$.

An Important Fact Concerning Continuous Functions

Sums, differences, products, quotients and compositions of continuous functions are continuous on intervals on which they are defined.

(you are on your own with piecewise)

Functions that are Continuous on their Domains of Definition

polynomials, rational functions, $|x|$, $\sqrt{x}$, x^r (with $r \neq 0$), $\cos(x)$, $\sin(x)$, $\sec(x)$, $\csc(x)$, $\tan(x)$, $\cot(x)$

Example: Determine where $f(x) = \dfrac{\sqrt{x}-2}{x^2-4}$ is continuous, and describe any discontinuity.

Note: f is created from sqrt, diff, quotients and polynomials.

Domain: Not $x<0$ ← no sqrt of neg.

Not $x^2-4=0$ ← denom cannot be 0.
$x=-2, x=2$

Everything else is OK.

$[0,2) \cup (2,\infty)$

$f(x) = \dfrac{\sqrt{x}-2}{x^2-4}$ is cont. here

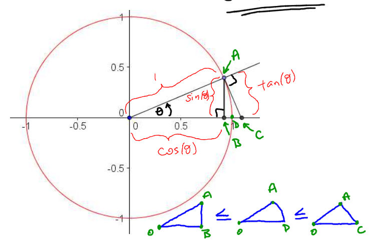
Look at the discont at $x=2$.

$$\lim_{x\to 2} \frac{\sqrt{x}-2}{x^2-4} = \text{dne}$$

"$\dfrac{\sqrt{2}-2}{0}$" $\longleftrightarrow$ vert asympt at $x=2$

$\Rightarrow$ infinite discont at $x=2$.

Geometric Exploration: θ in radians



$0 < \theta < \frac{\pi}{2}$

$$\cancel{\tfrac{1}{2}}\sin(\theta)\cos(\theta) \le \frac{\theta}{\cancel{2\pi}}\cdot\cancel{\pi} \le \cancel{\tfrac{1}{2}}\tan(\theta)$$

$$\cos(\theta) \le \frac{\theta}{\sin(\theta)} \le \frac{1}{\cos(\theta)}$$

$$\frac{1}{\cos(\theta)} \ge \frac{\sin(\theta)}{\theta} \ge \cos(\theta)$$

Let $\theta \to 0^+$

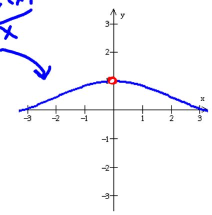
$$1 \ge \lim_{\theta\to 0^+}\frac{\sin(\theta)}{\theta} \ge 1$$

$$\Rightarrow \lim_{\theta\to 0^+}\frac{\sin(\theta)}{\theta} = 1$$

More generally: $\displaystyle\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1.$

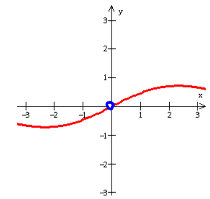
Two Important Limits

$y = \frac{\sin(x)}{x}$



$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x} = 0$$



See the video

Examples: $\displaystyle\lim_{x\to 0}\frac{2\sin(2x)}{2x} = \lim_{x\to 0} 2\cdot\boxed{\frac{\sin(2x)}{2x}} = 2$

$\displaystyle\lim_{u\to 0}\frac{\sin(u)}{u} = 1$

$$\lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)} = \lim_{x\to 0} 2\boxed{\frac{\sin(2x)}{2x}}\cdot\boxed{\frac{3x}{\sin(3x)}}\cdot\frac{1}{3} = \frac{2}{3}$$

$$\lim_{x\to 0}\frac{x^2}{1-\cos^2(x)} =$$

$$\lim_{x\to 0}\frac{\tan(3x)}{2x^2-2x} =$$

Video