Math 1431

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Office Hours: 11:00-Noon MWF

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Access Codes were due at 12:01am today.

EMCF05 was due this morning at 9:00am.

Homework 2 is due Today in lab/workshop.

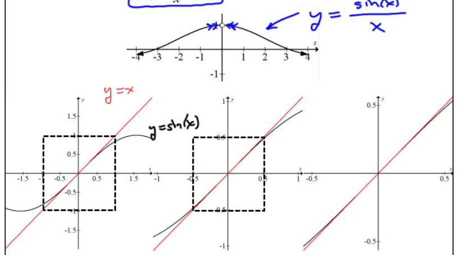
Poppers start today.

Quiz 1 expires tonight at 11:59 pm.

Video Help was posted for Sections 2.5 and 2.6.

We will finish Chapter 2 today, and start Section 3.1.
We will skip the Extreme Value Theorem in Section 2.6, and talk about it later when we need it.

Recall: $\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} = 1}$ **What does this mean?**



Note: $y = \sin(x)$ looks like $y = x$ close to $x = 0$.

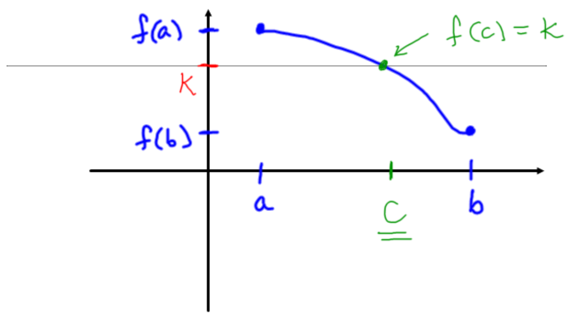
This is the essence of

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

i.e. $\sin(x) \approx x$ for $x \approx 0$.

The Intermediate Value Theorem
(common sense for continuous functions)

If $f(x)$ is a continuous function on the interval $[a,b]$ and K is a value between $f(a)$ and $f(b)$, then there is a value c between a and b so that $f(c) = K$.



Show there is a value of x between 1 and 3 so that $-3x^3 + 2x^4 = \boxed{7}$

(a good place to use the Intermediate Value Theorem)

Define $f(x) = -3x^3 + 2x^4$

Note that $f(x)$ is continuous on $[1, 3]$ b/c it is a polynomial.

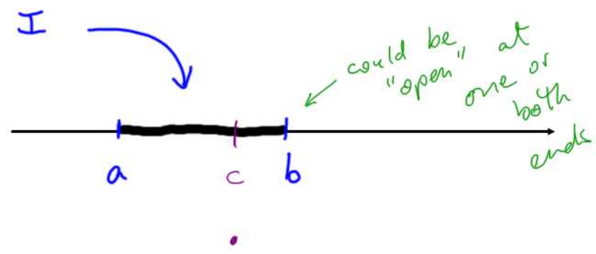
Also, $f(1) = -1$ and

$f(3) = -81 + 2 \cdot 81 = 81$

Note that 7 lies between -1 and 81. $\therefore$ From the I.V.Thm there exists at least one value of x btwn 1 and 3 so that $f(x) = 7$.

Corollary to the Intermediate Value Theorem: Suppose a function f is continuous on an interval I and $f(x)$ is not 0 at any value x in I.

- If $f(c) > 0$ at some point c in I, then $f(x) > 0$ at every x in I.
- If $f(c) < 0$ at some point c in I, then $f(x) < 0$ at every x in I.



Solve the inequality $\frac{1}{x-1} + \frac{1}{x-2} < 0$

(a good place to use the Intermediate Value Theorem)

(use the corollary)

$\frac{x-2 + x-1}{(x-1)(x-2)} < 0$

Solve: $\frac{2x-3}{(x-1)(x-2)} < 0$

Note: $f(x) = \frac{2x-3}{(x-1)(x-2)}$ is discont at $x = 1$ and $x = 2$. Also, $f(x) = 0$ when $2x - 3 = 0$. i.e. when $x = \frac{3}{2}$.

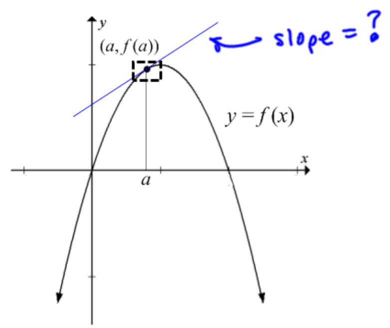
x	$(-\infty, 1)$	1	$(1, \frac{3}{2})$	$\frac{3}{2}$	$(\frac{3}{2}, 2)$	2	$(2, \infty)$
$f(x)$	$- - - - -$		$+ + + + +$	0	$- - - -$		$+ + + + + +$
	Pick $x=0$		Pick $x=\frac{5}{4}$		Pick $x=\frac{7}{4}$		Pick $x=3$
	$f(0) = \frac{-}{(-)(-)} = -$		$f(\frac{5}{4}) = \frac{-}{(+)(-)} = +$		$f(\frac{7}{4}) = \frac{+}{(+)(-)} = -$		$f(3) = \frac{+}{(+)(+)} = +$

Solution: $f(x) < 0$ if and only if $x < 1$ or $\frac{3}{2} < x < 2$

In interval notation, the solution set is $(-\infty, 1) \cup (\frac{3}{2}, 2)$.

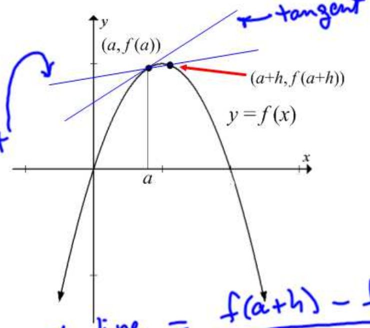
An Introduction to Derivatives: How can we approximate the slope of the tangent line to the graph at $x = a$?

I.



We can approximate the tangent line to the graph at $x = a$ by using a secant line.

II.

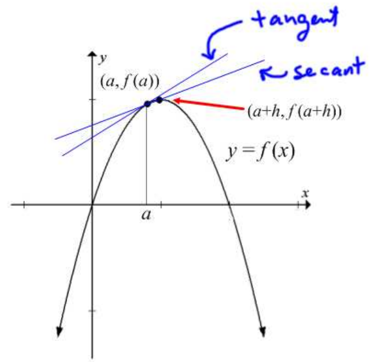


$$\text{Slope of secant line} = \frac{f(a+h) - f(a)}{a + h - a}$$

i.e. $$\text{slope of secant line} = \frac{f(a+h) - f(a)}{h}$$

We can improve this approximation by making h smaller.

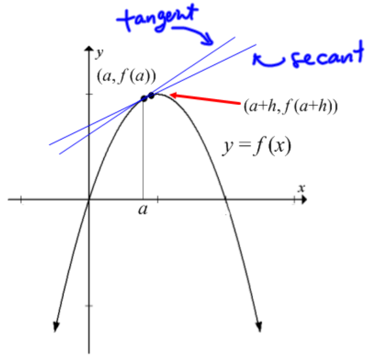
III.



$$\text{Slope of secant line} = \frac{f(a+h) - f(a)}{h}$$

The approximation will continue to improve as we make h even smaller.

IV.

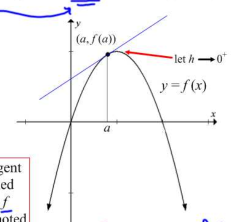


$$\text{Slope of secant line} = \frac{f(a+h) - f(a)}{h}$$

If the limit exists, then we can find the slope of the graph at $x = a$ by taking a limit.

tangent

V.



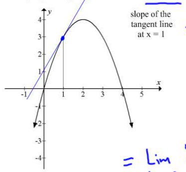
The slope of the tangent line at $x = a$ is called **the derivative** of f at $x = a$, and it is denoted by $f'(a)$.

$$\text{Slope of the graph at } x=a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) \equiv \text{Slope of the tangent line at } x=a = \text{the derivative of } f \text{ at } x = a.$$

"f prime of a"

Example: Give the slope of the tangent line to the graph of $f(x) = 4x - x^2$ at $x = 1$.



slope of the tangent line at x = 1 $= f'(1) = 2$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{4(1+h) - (1+h)^2 - 3}{h}$$

" $\frac{0}{0}$ " ind form

$$= \lim_{h \to 0} \frac{4 + 4h - (1 + 2h + h^2) - 3}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h} - h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h \to 0} (2 - h) = 2$$

To get an equation for the T.L. we need

slope $= 2$

Point $= (1, f(1)) = (1, 3)$

<u>Equation:</u> $y - 3 = 2(x - 1)$.