Math 1431
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- There is a **Written Quiz** in lab today.
- **Online Quiz 2** is due on Monday at 11:59pm.
- An **EMCF** is due on Monday.
- **Homework 3** is due on Monday.
- **Help videos** are posted for Sections 3.1 and 3.2.
- We are in Chapter 3!

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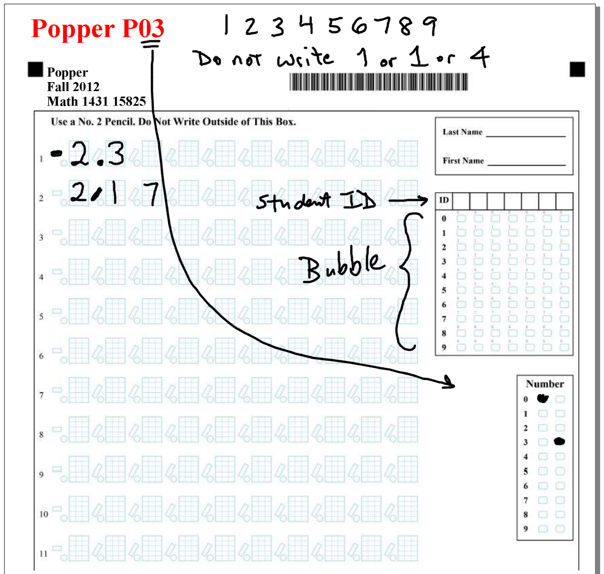
9 **Online Quizzes and Practice Tests can be attempted 20 times.** **Video help with the problems from section 2.6 in Homework 2.**	10 **Class notes, video notes, video** Deadline to enter Access Codes on **CourseWare.** Poppers Start in Lecture **EMCF05 Due at 9am – key** Homework 2 Due in Lab/Workshop **Quiz 1 Closes (2.1-2.2)**	11	12 **Class notes, video notes, video** Last day to drop without receiving a grade. **EMCF06 Due at 9am** **Homework 3 Posted** (modified 9/9 at 9:45am)	13 **Video Examples from Section 3.1** **Alpha Lambda Delta Honor Society Movie Night – Free** to everyone!! 6pm Cougar Village Rm. N111. **Snacks will be provided!**	14 **Blank Slides** **EMCF07 Due at 9am** Quiz in Lab/Workshop	15 **Video Solutions to Various Problems from Section 3.2** **Profs with Pride listing of weekly events**
16	17 **EMCF08 due at 9am** Homework 3 Due in Lab/Workshop **Quiz 2 Closes** (2.3-2.5)	18	19 **EMCF09 due at 9am** **Homework 4 Posted**	20	21 **EMCF10 due at 9am** Quiz in Lab/Workshop	22

Popper P03



The Derivative: Overview...

instantaneous rate of change

slope function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

"f prime of x"

limit definition of derivative

Other notation: $\frac{d}{dx} f(x) = f'(x) = D_x f(x)$

$= \frac{df(x)}{dx}$

Note: $f'(a)$ is the slope of the tangent line at $x = a$.

$\frac{-1}{f'(a)}$ is the slope of the normal line at $x = a$.

(provided $f'(a) \neq 0$)

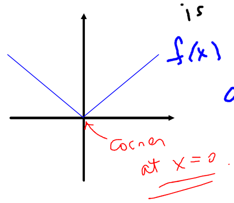
How is the derivative related to continuity?

If $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

But, the converse is not true.

ex. $f(x)=|x|$ is continuous.

$f(x)=|x|$ does not have a derivative at $x=0$.

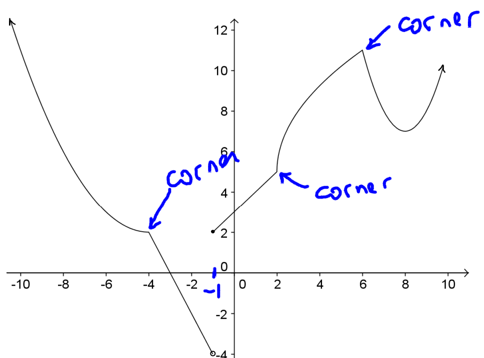


How can the graph of a function be used to determine where a function is not differentiable?

A function is not differentiable at:

1. Points of discontinuity.
2. Points where the graph has a corner.
3. Points where the graph has a cusp. $f(x)=x^{2/3}$ at $x=0$
4. Points where the graph has a vertical tangent. $f(x)=x^{1/3}$ at $x=0$

The function $f(x)$ is graphed below. Determine the values of x where f is differentiable.

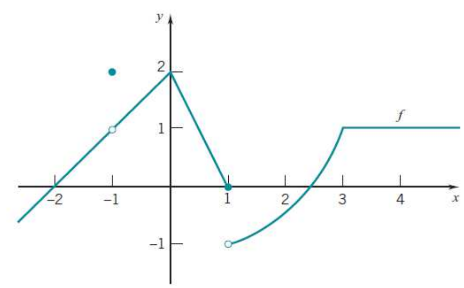


Not differentiable at $x=-4$, $x=-1$, $x=2$, $x=6$

$\therefore$ f is differentiable on $(-\infty,-4)\cup(-4,-1)\cup(-1,2)\cup(2,6)\cup(6,\infty)$

An Exercise from 3.1

21. The graph of a function f is shown in the figure.



(a) For which numbers c does f fail to be continuous? For each discontinuity, state whether it is a removable discontinuity, a jump discontinuity, or neither.

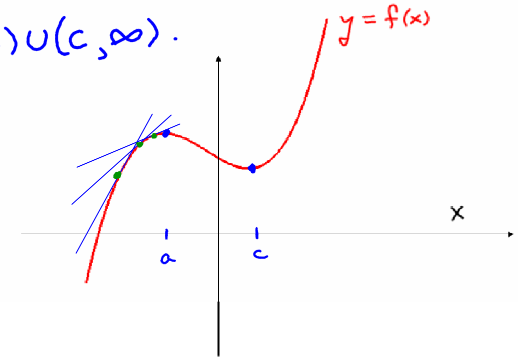
$x=-1$, removable
$x=1$, jump

(b) At which numbers c is f continuous but not differentiable?

$x=0$
$x=3$

Identify the values of x where the slope of the tangent line is **positive**.

$(-\infty, a) \cup (c, \infty)$.

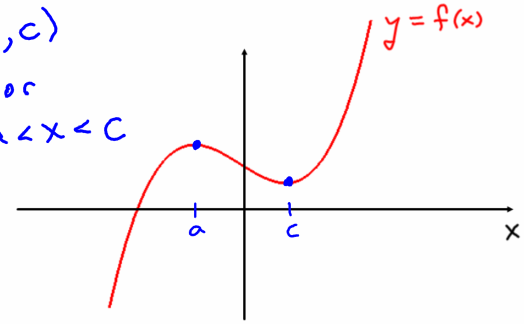


Identify the values of x where the slope of the tangent line is **negative**.

(a, c)

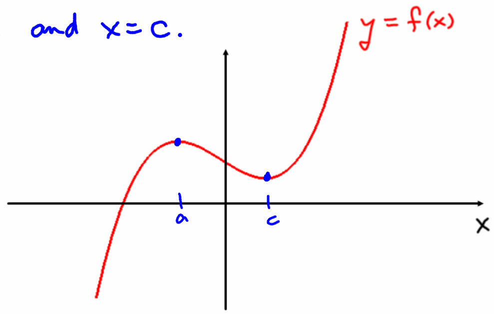
or

$a < x < c$



Identify the values of x where the slope of the tangent line is **zero**.

$x = a$ and $x = c$.



Algebraic Properties of the Derivative

1. Derivatives of sums, differences, scalar multiples, products and quotients.

2. Power rule.

Sums, Differences and Scalar Multiples

If f and g are differentiable and c is a scalar, then $f+g$, f-g and cf are differentiable. Furthermore,

1. $\frac{d}{dx}(f(x)+g(x))=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$
2. $\frac{d}{dx}(f(x)-g(x))=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$
3. $\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x)$

Products and Quotients

If f and g are differentiable then $f \cdot g$ and f/g are differentiable. Furthermore,

4. $\frac{d}{dx}(f(x)\cdot g(x))=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$ product rule
5. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$ quotient rule

Examples:

Give the derivative of $f(x)=x^2$.

$$f'(x)=\frac{d}{dx}x^2=\frac{d}{dx}(x\cdot x)$$
$$=x\cdot\frac{dx}{dx}+x\cdot\frac{dx}{dx}$$
$$=x\cdot 1+x\cdot 1$$
$$=2x$$

Give the derivative of $R(x)=x^3+x$.

$$R'(x)=\frac{d}{dx}(x^3+x)=\frac{d}{dx}x^3+\frac{d}{dx}x$$
$$=\frac{d}{dx}(x^2\cdot x)+1$$
$$=x^2\cdot\frac{d}{dx}x+x\cdot\frac{d}{dx}x^2+1$$
$$=x^2\cdot 1+x\cdot 2x+1$$
$$=3x^2+1$$

Popper P03

1. We found the derivative of $f(x)=x^2$ on the previous page. Give the slope of the **tangent** line to the graph of f at the point where $x=-2$.

2. We found the derivative of $f(x)=x^2$ on the previous page. Give the slope of the **normal** line to the graph of f at the point where $x=-2$.

3. We found the derivative of $R(x)=x^3+x$ on the previous page. Give the value of $R'(1)$.

4.
sm

$1+1=$

Power Rule

$$\frac{d}{dx}x^2 = 2x$$

$$\frac{d}{dx}x^3 = 3x^2$$

$$\frac{d}{dx}x^n = nx^{n-1}, \quad n \neq 0$$

$$\frac{d}{dx}x^4 = 4x^3$$

$$\frac{d}{dx}x^5 = 5x^4$$

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = (-1)x^{-2} = -\frac{1}{x^2}$$

Examples:

Give the derivative of $f(x) = x^4 + 2x^3$.

$$f'(x) = \frac{d}{dx}\left(x^4 + 2x^3\right) = \frac{d}{dx}x^4 + \frac{d}{dx}2x^3$$

$$= 4x^3 + 2\frac{d}{dx}x^3$$

$$= 4x^3 + 2 \cdot 3x^2 = 4x^3 + 6x^2$$

Give the derivative of $g(x) = \underbrace{\left(\sqrt{x}+1\right)}_{u(x)}\underbrace{\left(3x^3 - 2x + 2\right)}_{v(x)}$.

product

$$\frac{d}{dx}\left(u(x)v(x)\right) = u(x) \cdot v'(x) + v(x)u'(x)$$

$$\therefore g'(x) = (\sqrt{x}+1) \cdot \frac{d}{dx}(3x^3 - 2x + 2) + (3x^3 - 2x + 2)\frac{d}{dx}(\sqrt{x}+1)$$

$$= (\sqrt{x}+1)\left[3 \cdot 3x^2 - 2\right] + (3x^3 - 2x + 2)\left(\frac{1}{2\sqrt{x}} + 0\right)$$

simplify.

Example:

Give the derivative of $H(x) = 3x^3 - 5x^2 - 6x + 3$.

$$H'(x) = 9x^2 - 10x - 6$$

Example:

Give the derivative of $g(x) = \frac{2}{\sqrt{x}} - \frac{3}{x} + x^2 - 1$.

$$g(x) = 2x^{-\frac{1}{2}} - 3x^{-1} + x^2 - 1$$

$$g'(x) = 2 \cdot \left(-\tfrac{1}{2}\right)x^{-\frac{3}{2}} - 3 \cdot (-1)x^{-2} + 2x - 0$$

Popper P03

5. $2 + 0 =$

6. $8 - 4 =$

Example: Give the derivative of $f(x) = \dfrac{2x-3}{x^2+1}$.

See the video