Notes:

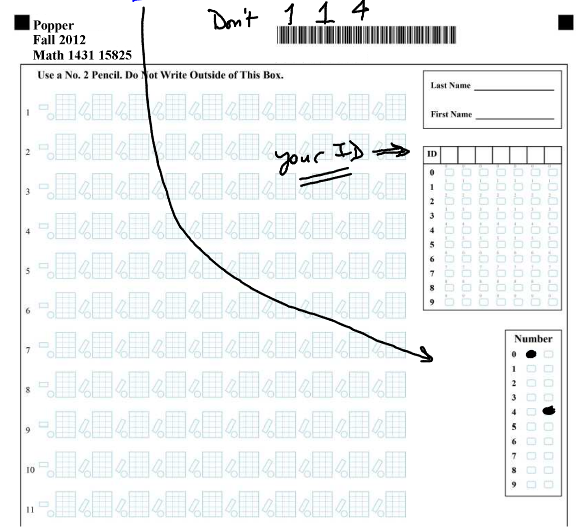
Homework is due in lab/workshop.
EMCF08 was due at 9:00am.
Online Quiz 2 is due at 11:59pm.
EMCF09 is due Wednesday at 9:00am.

Test 2 is tentatively scheduled for Oct. 4 - 6 in CASA. Use the online scheduler **SOON** to set your test time. It should open by 12:01am on September 20th.

Popper P04

1 2 3 4 5 6 7 8 9 0

Don't 1 1 4



Review – Differentiation Formulas

Sums $\frac{d}{dx}(f(x)+g(x))=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$

Differences $\frac{d}{dx}(f(x)-g(x))=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$

Scalar Mult $\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x)$

product rule

$$\frac{d}{dx}(f(x)\cdot g(x))=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

quotient

More…

Power Rule

$$\frac{d}{dx}x^n=nx^{n-1},\quad n\neq 0$$

$$\frac{d}{dx}c=0, \text{ whenever } c \text{ is a constant}$$

Review Examples...

$\frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}3x^{-1} = 3(-1)x^{-2}$

$f(x)$	$f'(x)$
x^2	$2x$
$-2x^3$	$-6x^2$
$4x^2 - 2x + 5$	$8x - 2$
7	0
$\sqrt{x} + 3x - 1$	$\frac{1}{2\sqrt{x}} + 3$
$\frac{3}{x} - 4x^3 + 7x$	$\frac{-3}{x^2} - 12x^2 + 7$

Popper P04

1. Give the slope of the tangent line to the graph of $f(x) = 3x^3 - 9x$ at the point where $x = 2$.

2. Give the y-intercept of the tangent line to the graph of $f(x) = 3x^3 - 9x$ at the point where $x = 2$.

3. Give the slope of the normal line to the graph of $f(x) = 3x^3 - 9x$ at the point where $x = 2$.

4. Give the y-intercept of the normal line to the graph of $f(x) = 3x^3 - 9x$ at the point where $x = 2$.

Example: Find the derivative of $T(x) = \underbrace{(x^3 - 2x^2 - x)(x^5 - 3x + 1)}_{\text{product}}$

$$\frac{d}{dx}\left(u(x)v(x)\right) = u(x)v'(x) + v(x)u'(x)$$

$$T'(x) = (x^3 - 2x^2 - x)(5x^4 - 3) + (x^5 - 3x + 1)(3x^2 - 4x - 1)$$

Example: Find the derivative of $f(x) = \underbrace{\frac{3x^2 - 1}{2x^3 + 1}}_{\text{quotient}}$

$$\frac{d}{dx}\frac{u(x)}{v(x)} = \frac{v(x)u'(x) - u(x)v'(x)}{v(x)^2}$$

$$f'(x) = \frac{(2x^3 + 1)6x - (3x^2 - 1)6x^2}{(2x^3 + 1)^2}$$

$$= \frac{-6x^4 + 6x + 6x^2}{(2x^3 + 1)^2}$$

3.3 Higher Order Derivatives

" The second derivative of f(x) wrt x "

"f double prime of x"

$$f'(x),\ f''(x),\ f'''(x),\ f^{(4)}(x)$$

derivative of f(x) wrt x

derivative of f'(x) wrt x

derivative of f''(x) wrt x

you know.

$$\frac{d}{dx}f(x),\ \frac{d^2}{dx^2}f(x),\ \frac{d^3}{dx^3}f(x),\ \frac{d^4}{dx^4}f(x)$$

Question: How do we indicate "evaluation" at a specific value of x?

A: e.g. $f'(2)$ ← evaluate $f'(x)$ at $x = 2$

Do NOT write $\frac{d}{dx}f(2)$

Instead, write

$$\left.\frac{d}{dx}f(x)\right|_{x=2}$$

The derivative of f(x) evaluated at x = 2

$$\boxed{\left.\frac{d^2}{dx^2}f(x)\right|_{x=2} \quad \text{or} \quad f''(2)}$$

Example: Determine $\frac{d^2}{dx^2}\left(3x^3 - 5x^2 + 2x - 1\right) = y''$

Let $y = 3x^3 - 5x^2 + 2x - 1$

$\frac{dy}{dx} \rightarrow y' = 9x^2 - 10x + 2$

$\frac{d^2y}{dx^2} \rightarrow y'' = 18x - 10$

Note: $\left. y'' \right|_{x=1} = 8$

$\left. y' \right|_{x=2} = 18$

Example: Determine $\frac{d^3}{dx^3}\left(3x^8 + 2x^4 - 3x - 2\right) = y'''$

Let $y = 3x^8 + 2x^4 - 3x - 2$

$y' = 24x^7 + 8x^3 - 3$

$y'' = 168x^6 + 24x^2$

$y''' = 1008x^5 + 48x$

$$\boxed{\begin{array}{r} 168 \\ \times 6 \\ \hline 1008 \end{array}}$$

3.6 Derivatives of Trig Functions

$\frac{d}{dx}\sin(x) = \cos(x)$ $\qquad$ $\frac{d}{dx}\cos(x) = -\sin(x)$

$$\frac{d}{dx}\sin(x) = \lim_{h\to 0}\frac{\sin(x+h) - \sin(x)}{h} \qquad \text{"}\tfrac{0}{0}\text{"}$$

$\sin(x+h) = \sin(x)\cos(h) + \sin(h)\cos(x)$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h\to 0}\left[\sin(x)\cdot\underbrace{\boxed{\frac{\cos(h)-1}{h}}}_{\to 0} + \underbrace{\boxed{\frac{\sin(h)}{h}}}_{\to 1}\cdot\cos(x)\right]$$

$= \cos(x)$

Example: Give the derivative of $f(x) = \sin(x) - 2\cos(x)$

$$f'(x) = \cos(x) - 2(-\sin(x))$$

$$= \cos(x) + 2\sin(x)$$

Example: Give the slope of the tangent line to $f(x) = \sin(x) - \underbrace{2x\cos(x)}$ at the point where $x = \pi$.

$$f'(x) = \cos(x) - \left[2x(-\sin(x)) + \cos(x)\cdot 2\right]$$

$$f'(\pi) = -1 - \left[2\pi(-0) + (-1)\cdot 2\right]$$

$$= 1$$

Other Trig Functions

$$\frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \cdots$$