Notes:

Homework is due in lab/workshop.
EMCF08 was due at 9:00am.
Online Quiz 2 is due at 11:59pm.
EMCF09 is due Wednesday at 9:00am.

Test 2 is tentatively scheduled for Oct. 4 - 6 in CASA. Use the online scheduler **SOON** to set your test time. It should open by 12:01am on September 20th.

Review – Differentiation Formulas

$$\frac{d}{dx}(f(x)+g(x))=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

$$\frac{d}{dx}(f(x)-g(x))=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

$$\frac{d}{dx}(cf(x))=c\frac{d}{dx}f(x)$$

product rule

$$\frac{d}{dx}(f(x)\cdot g(x))=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

quotient rule

More…

$$\frac{d}{dx}x^n = nx^{n-1}, \quad n\neq 0$$

$$\frac{d}{dx}c=0, \text{ whenever } c \text{ is a constant}$$

Review Examples...

$f(x)$	$f'(x)$
x^2	$2x$
$-2x^3$	$-6x^2$
$4x^2-2x+5$	$8x-2$
7	0
$\sqrt{x}+3x-1$	$\frac{1}{2}x^{-\frac{1}{2}}+3$ or $\frac{1}{2\sqrt{x}}+3$
$\frac{3}{x}-4x^3+7x$	$3(-1)x^{-2}-12x^2+7$ or $-\frac{3}{x^2}-12x^2+7$

Example: Find the derivative of $T(x)=(x^3-2x^2-x)(x^5-3x+1)$

product

$$\frac{d}{dx}\big(u(x)v(x)\big) = u(x)v'(x) + v(x)u'(x)$$

$\therefore$

$$T'(x) = (x^3-2x^2-x)(5x^4-3) + (x^5-3x+1)(3x^2-4x-1)$$

Example: Find the derivative of $f(x)=\dfrac{3x^2-1}{2x^3+1}$

quotient

$$\frac{d}{dx}\frac{u(x)}{v(x)} = \frac{v(x)u'(x)-u(x)v'(x)}{v(x)^2}$$

$$f'(x) = \frac{(2x^3+1)6x-(3x^2-1)6x^2}{(2x^3+1)^2}$$

3.3 Higher Order Derivatives

"f double prime of x"

$f'(x),\ f''(x),\ f'''(x),\ f^{(4)}(x)$

derivative of $f(x)$ | derivative of $f'(x)$ | derivative of $f''(x)$ | derivative of $f'''(x)$

the first derivative of f(x) wrt x

$\dfrac{d}{dx}f(x),\ \dfrac{d^2}{dx^2}f(x),\ \dfrac{d^3}{dx^3}f(x),\ \dfrac{d^4}{dx^4}f(x)$

"the second derivative of f(x) wrt x"

Question: How do we indicate "evaluation" at a specific value of x?

Answer: $f'(2)$ or $\left.\dfrac{d}{dx}f(x)\right|_{x=2}$

the derivative of $f(x)$ evaluated at $x=2$

$f''(2)$ or $\left.\dfrac{d^2}{dx^2}f(x)\right|_{x=2}$

Example: Determine $\dfrac{d^2}{dx^2}(3x^3-5x^2+2x-1)$

$$\frac{d}{dx}(3x^3-5x^2+2x-1) = 9x^2-10x+2$$

$\therefore$

$$\frac{d^2}{dx^2}(3x^3-5x^2+2x-1) = \frac{d}{dx}(9x^2-10x+2)$$

$$= 18x-10.$$

Example: Determine $\frac{d^3}{dx^3}\left(3x^8+2x^4-3x-2\right)$

Let $y = 3x^8 + 2x^4 - 3x$

Then $y' = \frac{d}{dx}y = 24x^7 + 8x^3 - 3$

$\Rightarrow y'' = \frac{d^2}{dx^2}y = 168x^6 + 24x^2$

$$\begin{array}{r} 168 \\ \times 6 \\ \hline 1008 \end{array}$$

$y''' = \frac{d^3}{dx^3}y = 1008x^5 + 48x$

3.6 Derivatives of Trig Functions

$\frac{d}{dx}\sin(x) = \cos(x)$ $\quad \frac{d}{dx}\cos(x) = -\sin(x)$

$\frac{d}{dx}\sin(x) = \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$ "$\frac{0}{0}$"

$\sin(x+h) = \sin(x)\cos(h) + \sin(h)\cos(x)$

$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$

$= \lim_{h\to 0} \left[\sin(x) \boxed{\frac{\cos(h)-1}{h}} + \boxed{\frac{\sin(h)}{h}}\cos(x) \right]$ $\quad \to 0 \quad \to 1$

$= \cos(x)$

Example: Give the derivative of $f(x) = \sin(x) - 2\cos(x)$

$f'(x) = \cos(x) - 2(-\sin(x))$

$= \cos(x) + 2\sin(x)$

Example: Give the slope of the tangent line to $f(x) = \sin(x) - 2x\cos(x)$ at the point where $x = \pi$.

$f'(x) = \cos(x) - \left[2x(-\sin(x)) + \cos(x)\cdot 2\right]$

$= \cos(x) + 2x\sin(x) - 2\cos(x)$

$\therefore f'(\pi) = -1 + 2\pi\cdot 0 - 2\cdot(-1)$

$= \boxed{1.}$

Other Trig Functions

$$\frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$= \sec^2(x)$$