Notes:

- This morning - EMCF09
- Friday - EMCF10, Written Quiz
- Monday - EMCF11, Homework, Quiz 3
- October 4, 5, 6 - Test 2 (in CASA). The scheduler will open on Sept. 20th at 12:01am.

We will have class on the days it is scheduled.

Recall

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \frac{d}{dx}\cos(x) = -\sin(x)$$

Consequences

$$\frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

Example: Give the derivative of $f(x) = \sin(x) - 3\tan(x)$.

$$f'(x) = \cos(x) - 3\sec^2(x)$$

Question: What does Geogebra give for this derivative? Is it correct? Yes

$$f'(x) = \cos(x) - 3\tan^2(x) - 3$$

$$= \cos(x) - 3(\tan^2(x) + 1)$$

$$= \cos(x) - 3\sec^2(x)$$

yes

Example: Give the derivative of $g(x)=\dfrac{\sin(x)}{x+\cot(x)}$.

↖ quotient

$$g'(x)=\frac{(x+\cot(x))\cos(x)-\sin(x)(1-\csc^2(x))}{(x+\cot(x))^2}$$

Using Wolfram Alpha for Homework Help

www.wolframalpha.com

Examples:

The Chain Rule

If u is differentiable at x and f is differentiable at $u(x)$, then the composition $f\circ u$ is differentiable at x and $(f\circ u)'(x)=f'(u(x))\,u'(x)$.

$$\text{i.e.}\quad \frac{d}{dx}f\big(\overbrace{u(x)}^{y}\big)=f'\big(u(x)\big)\,\frac{d}{dx}u(x)$$

$$\frac{dy}{dx}=\frac{dy}{\boxed{du}}\,\frac{\boxed{du}}{dx}$$

Why?

$$g(x)=f(u(x))\ \Rightarrow g'(x)=f'(u(x))u'(x)$$

$$g'(x)=\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(u(x+h))-f(u(x))}{h}$$

"$\frac{0}{0}$"

$$=\lim_{h\to 0}\underbrace{\boxed{\frac{f(u(x+h))-f(u(x))}{u(x+h)-u(x)}}}_{\to f'(u(x))}\cdot\underbrace{\boxed{\frac{u(x+h)-u(x)}{h}}}_{\to u'(x)}$$

Focus on

$$\lim_{h\to 0}\frac{f(\overbrace{u(x+h)}^{u(x)+k})-f(u(x))}{\underbrace{\boxed{u(x+h)-u(x)}}_{k}}$$

$$=\lim_{k\to 0}\frac{f(u(x)+k)-f(u(x))}{k}$$

$$=f'(u(x))$$

Examples:

$\frac{d}{dx}\cos(u(x)) = -\sin(u(x))\cdot u'(x)$

$\frac{d}{dx}\sin(u(x)) = \cos(u(x))\cdot u'(x)$

$\frac{d}{dx}u(x)^4 = 4u(x)^3\cdot u'(x)$

$f(x)$	$f'(x)$
$\cos(2x)$	$-\sin(2x)\cdot 2 = -2\sin(2x)$
$\sin(3x)$	$\cos(3x)\cdot 3 = 3\cos(3x)$
$(x^2+2)^4$	$4(x^2+2)^3\cdot 2x$
$\left(2x-\frac{3}{x}\right)^5$	$5\left(2x-\frac{3}{x}\right)^4\cdot\left(2+\frac{3}{x^2}\right)$
$x\sin(\pi x)$	$x\cdot\cos(\pi x)\cdot\pi + \sin(\pi x)$ $= \pi x\cos(\pi x)+\sin(\pi x)$

↑ product

Consequences

$$\frac{d}{dx}(u(x))^n = \underbrace{n(u(x))^{n-1}}\frac{du(x)}{dx},\ n\neq 0$$

$$\frac{d}{dx}\sin(u(x)) = \underbrace{\cos(u(x))}\frac{du(x)}{dx}$$

$$\frac{d}{dx}\cos(u(x)) = \underbrace{-\sin(u(x))}\frac{du(x)}{dx}$$

Example: Give the derivative of $f(x)=\sin(3x)-3\tan(x^2)$.

$$f'(x) = \cos(3x)\cdot 3 - 3\sec^2(x^2)\cdot 2x$$

$$= 3\cos(3x) - 6x\sec^2(x^2)$$

Example: Give the derivative of $g(x)=\sin^2(2x)-2\tan^3(3x)$.

Note: $g(x) = (\sin(2x))^2 - 2(\tan(3x))^3$

$$g'(x) = 2\sin(2x)\cdot\cos(2x)\cdot 2$$

$$- 2\cdot 3(\tan(3x))^2\cdot\sec^2(3x)\cdot 3$$

Example: Give the derivative of $f(x) = \left(\frac{x}{2x^2+1}\right)^4$ $= (u(x))^4$

$$f'(x) = 4(u(x))^3 \cdot u'(x)$$

$$= 4\left(\frac{x}{2x^2+1}\right)^3 \cdot \frac{(2x^2+1) - x \cdot 4x}{(2x^2+1)^2}$$

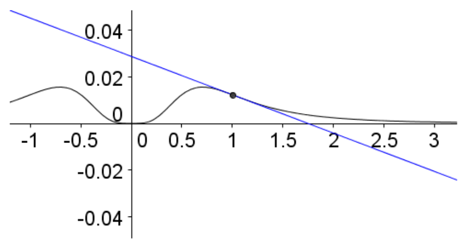
What is the equation of the tangent line at $x = 1$? $3^4 = 81$

Point: $(1, f(1)) = (1, \frac{1}{81})$

slope: $f'(1) = 4 \cdot \frac{1}{27} \cdot \left[\frac{-1}{9}\right]$

$$= \frac{-4}{243}$$

Equation: $y - \frac{1}{81} = \frac{-4}{243}(x-1)$



$f(x) = \left(\frac{x}{2x^2+1}\right)^4$ and its tangent line at $x = 1$.

Example: Suppose $G(x) = f(v(x))$, $v(1) = 2$, $f'(1) = 3$, $f'(2) = -6$, and $v'(1) = 7$. Find $G'(1)$.

1. $G'(x) = \frac{d}{dx} f(v(x))$

$$= \underbrace{f'(v(x))\, v'(x)}_{\text{chain rule}}$$

2. $G'(1) = f'(v(1))\, v'(1)$

$$= f'(2) \cdot 7 = (-6) \cdot 7$$

$$= -42.$$