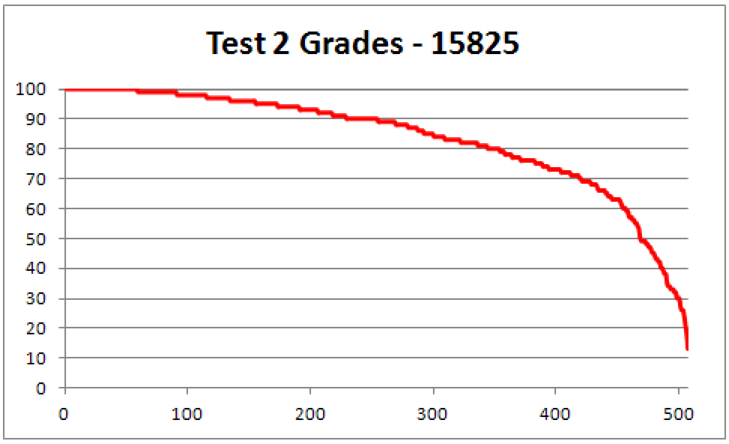
Information

- EMCFs are due MWF.
- Homework is posted for Monday, and an online quiz is due Monday night.
- The written portion of Test 2 is graded, and the grades for the multiple choice and written portions appear in a separate entries in the gradebook. You will need to add the scores to get your total score out of 100.

Test 2 Results

Section	Median	Mean	Number
15819	85	81.16	528
15825	90	83.32	507
15836	87	81.46	334
15841	77	73.4	137



A	254
B	100
C	67
D	36
F	50

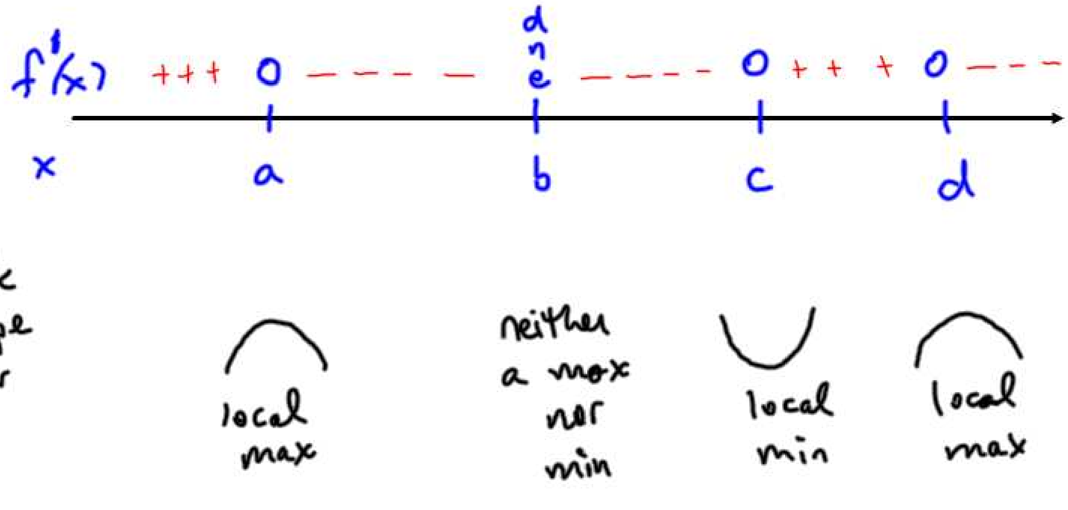
Recall: Critical Numbers

The value $x = c$ is a critical number for f if and only if c is in the domain of f and either $f\,'(c) = 0$ or $f\,'(c)$ does not exist.

How can we classify critical points as either local maximums or local minimums?

The First Derivative Test

slope chart



Example: Use the first derivative test to classify the local extrema of the function

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 4$ ← polynomial

Note: The domain is $(-\infty, \infty)$.

$f'(x) = 4x^3 - 24x^2 + 44x - 24$

↑ polynomial. $f'(x)$ exists for all x.

Set $f'(x) = 0$. $\quad 4x^3 - 24x^2 + 44x - 24 = 0$

$x^3 - 6x^2 + 11x - 6 = 0$

Note: $x = 1$ is a solution.

$$\begin{array}{r} x^2 - 5x + 6 \\ x-1 \overline{) x^3 - 6x^2 + 11x - 6} \\ -(x^3 - x^2) \\ \hline -5x^2 + 11x - 6 \\ -(-5x^2 + 5x) \\ \hline 6x - 6 \\ -(6x - 6) \\ \hline 0 \end{array}$$

$(x-1)(x^2 - 5x + 6) = 0$

$(x-1)(x-2)(x-3) = 0$

$\Rightarrow$ c.n. for f are $x = 1,\ x = 2,\ x = 3$.

Slope chart: (First derivative test.)

$f'(x) = 4(x-1)(x-2)(x-3)$

$f'(x)$	$- - - -$	0	$+ + + +$	0	$- - -$	0	$+ + + +$
x	0	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4

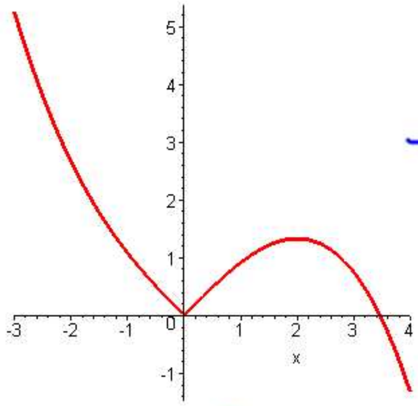
$f'(0) = -$ $\quad f'(\frac{3}{2}) = +$ $\quad f'(\frac{5}{2}) = -$ $\quad f'(4) = +$

Rough shape for $f(x)$ (locally)

local min at $x=1$ $\quad$ local max at $x=2$ $\quad$ local min at $x=3$

Example: Find and classify the critical numbers of

$$f(x) = |x| - \frac{x^3}{12}.$$



Domain of f is $(-\infty, \infty)$.

Note: $\boxed{f'(0) \text{ dne}}$.

$\therefore$ $x=0$ is a c.n.

$$f(x) = \begin{cases} -x - \frac{x^3}{12}, & x < 0 \\ x - \frac{x^3}{12}, & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -1 - \frac{x^2}{4}, & x < 0 \\ 1 - \frac{x^2}{4}, & x > 0 \end{cases}$$

Set $f'(x) = 0$. $x < 0$: $-1 - \frac{x^2}{4} = 0$

$1 + \frac{x^2}{4} = 0$ Impossible

$x > 0$: $1 - \frac{x^2}{4} = 0$

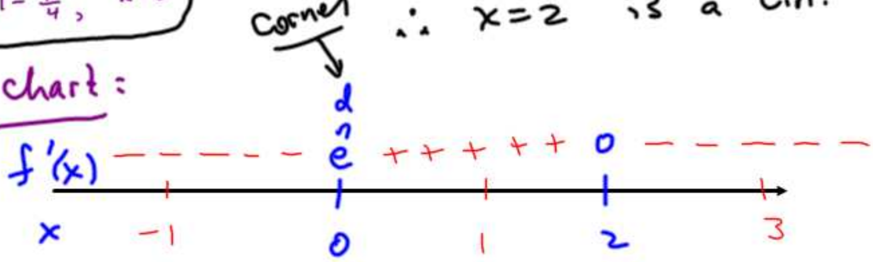
$x^2 = 4$

$\cancel{x=-2}$ or $x = 2$

$$f'(x) = \begin{cases} -1 - \frac{x^2}{4}, & x < 0 \\ 1 - \frac{x^2}{4}, & x > 0 \end{cases}$$

$\therefore$ $x = 2$ is a c.n.

slope chart:



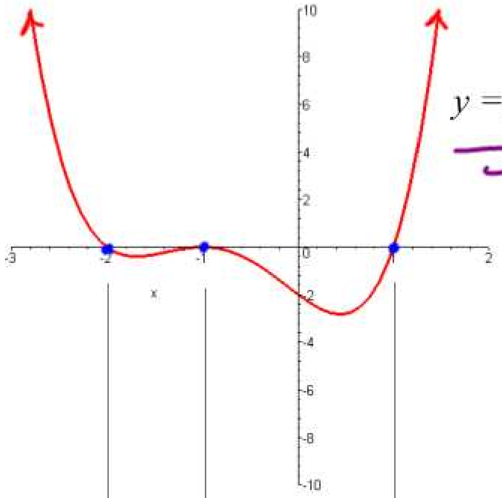
$f'(-1) = -$ $f'(1) = +$ $f'(3) = -$

Rough shape for f (local)

local min at $x=0$

local max at $x=2$

Example: The graph of $f'(x)$ is shown. Classify the critical numbers for f. In addition, list the intervals of increase and intervals of decrease for f.



Note: From the graph, it appears that the derivative exists for all values of x.

$f'(x) = 0$ at $x = -2, -1, 1$ ← c.n. for f

slope chart

$f'(x)$ +++ 0 - - 0 - - - - - 0 + + + + +

x: -2, -1, 1

Basic shape of f (local)

local max at $x=-2$

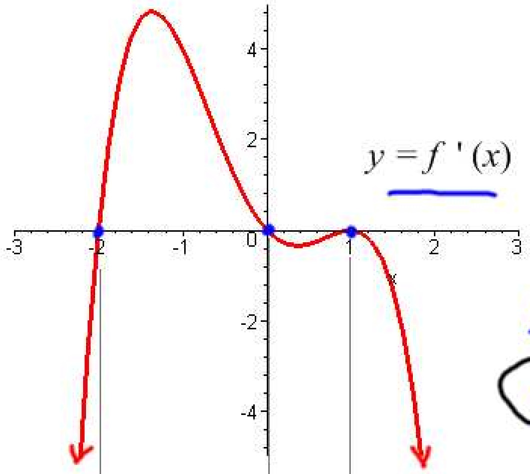
neither a local max nor a local min at $x=-1$

local min at $x=0$.

f is increasing on $(-\infty,-2]$ and $[1,\infty)$. f is decreasing on $[-2,1]$.	f is increasing on $(-\infty,-2)$ and $(1,\infty)$. f is decreasing on $(-2,1)$

either answer is acceptable.

Example: The graph of $f\,'(x)$ is shown. Classify the critical numbers for f. In addition, list the intervals of increase and intervals of decrease for f.



Note: It appears that the derivative exists for all values of x.

$f'(x) = 0$ for $x = -2, 0, 1$

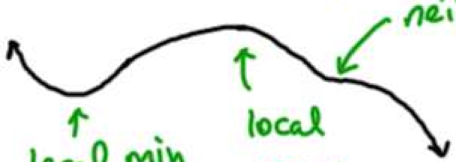
c.n. for f

Slope Chart

$f'(x)$ − − − 0 + + + + 0 − − 0 − − − − − −

x -2 0 1

Rough Shape of f



local min at $x = -2$

local max at $x = 0$

neither a local max nor a local min at $x = 1$.

f is increasing on $[-2, 0]$

f is decreasing on $(-\infty, -2]$ and $[0, \infty)$

f is increasing on $(-2, 0)$

f is decreasing on $(-\infty, -2)$ and $(0, \infty)$

Either answer is acceptable.

New: How do we determine the absolute extreme values of a function on a closed bounded interval?

Spse $f(x)$ is a continuous function on $[a,b]$.

1. Evaluate $f(x)$ at $x=a$ and $x=b$.

   i.e. Get $f(a)$ and $f(b)$

2. Find all c.n. for f in $[a,b]$, and evaluate f at these values.

3. Compare the values.

Example: Find the absolute extreme values for

$$f(x) = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x - 3 \text{ on the interval } [-4,1].$$

polynomial
$\therefore$ continuous

1. $f(-4) = \frac{64}{3} + 8 - 24 - 3 = \frac{64}{3} - 19 = \frac{7}{3}$

   $f(1) = -\frac{1}{3} + \frac{1}{2} + 6 - 3 = \frac{1}{6} + 3 = \frac{19}{6}$

2. Evaluate f at c.n. in $[-4,1]$.

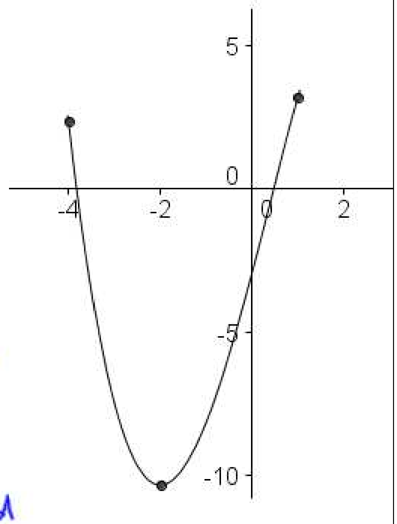
   $f'(x) = -x^2 + x + 6$ exists for all x.

   Set $f'(x) = 0$.

   $$-x^2 + x + 6 = 0$$
   $$x^2 - x - 6 = 0$$
   $$(x-3)(x+2) = 0$$
   $$\cancel{x=3},\ x=-2$$

   $\boxed{f(x) = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x - 3}$

   $f(-2) = \frac{8}{3} + 2 - 12 - 3 = \frac{8}{3} - 13 = -\frac{31}{3}$



3. Compare. The absolute maximum value is $\frac{19}{6}$, and it occurs at $x=1$. The absolute minimum value is $-\frac{31}{3}$, and it occurs at $x=-2$.

Example: A rectangle with with its base on x-axis and its left side on the y-axis has its upper right hand vertex on the line $2x + y = 3$. Give the dimensions of the rectangle with the largest possible area.

Next Time!!