Info...

- Complete quizzes 6, 7 and 8 asap.
- Homework is due on Monday.
- Next week's EMCF's and Homework are posted.

Recall: How do we determine the absolute extreme values of a *continuous* function on a closed bounded interval?

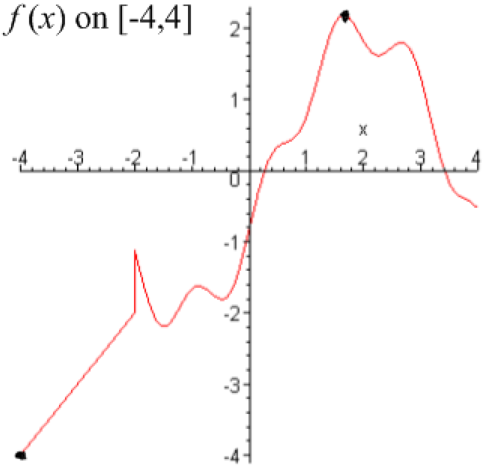
Answer:

1. Evaluate the function at the endpoints.

2. Evaluate the function at any critical numbers in the interval.

3. Compare these function values. The largest value is the absolute maximum value of the function, and the smallest value is the absolute minimum value of the function.

$f(x)$ on [-4,4]



Example: Give the absolute maximum and absolute minimum values for $f(x) = -x^3 + 6x^2 + 15x - 2$ on the interval $[-2,1]$.

Process for finding the absolute maximum and absolute minimum values of a function on a closed bounded interval.

1. Evaluate the function at the endpoints.

2. Evaluate the function at any critical numbers in the interval.

3. Compare these function values. The largest value is the absolute maximum value of the function, and the smallest value is the absolute minimum value of the function.

$f(x) = -x^3 + 6x^2 + 15x - 2$ on $[-2,1]$

polynomial $\Rightarrow f$ is continuous

1. $f(-2) = 8 + 24 - 30 - 2 = 0$

$f(1) = -1 + 6 + 15 - 2 = 18$

2. $f'(x) = -3x^2 + 12x + 15$

polynomial $\Rightarrow f'(x)$ exists everywhere

$f'(x) = 0 \Leftrightarrow -3x^2 + 12x + 15 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$\cancel{x=5}$, $x = -1$

only c.n. on $[-2,1]$

$f(-1) = 1 + 6 - 15 - 2 = -10$

3. Compare: The abs. max of f on $[-2,1]$ is 18, and it occurs at $x = 1$. The absolute min of f on $[-2,1]$ is -10, and it occurs at $x = -1$.

Question: Would classifying critical values using a slope chart lead to the same answer in the previous problem?

> Give the absolute maximum and absolute minimum values for $f(x) = -x^3 + 6x^2 + 15x - 2$ on the interval $[-2,1]$.

A: Let's see.

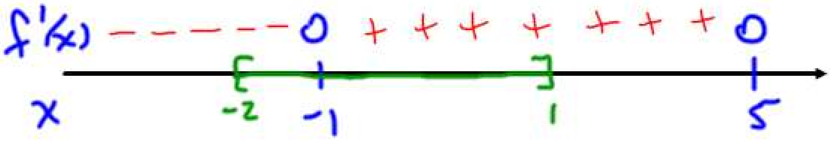
$$f'(x) = -3x^2 + 12x + 15$$

$$f'(x) = 0 \iff x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

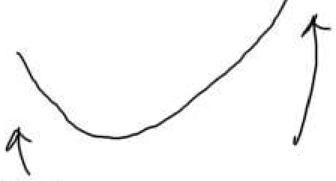
$$x = 5,\ x = -1$$

← c.n. for f.



$f'(-2) = -$ $\qquad$ $f'(0) = +$

rough shape of f



Note: We still need to evaluate the function at the endpoints, and to get the absolute minimum value, we still need to evaluate f(-1). Combining this with the work of creating the slope chart shows that this is more work than the previous process.

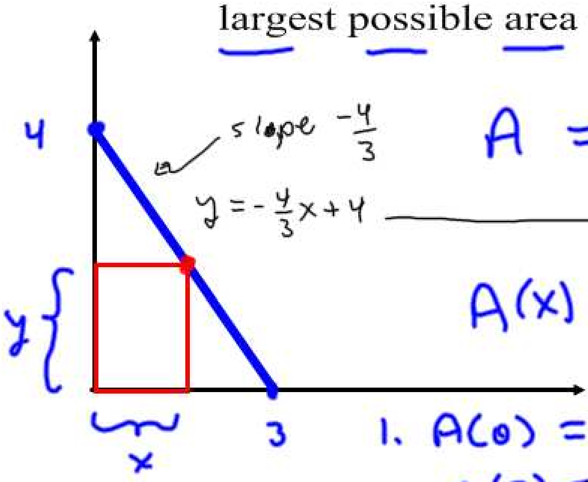
Evaluate $f(-2)$ and $f(1)$.

which is really bigger

Some Max/Min Word Problems... (section 4.5)

This is what we have already been doing, except now we have a word problem that must be translated into mathematical terms before we can find the answer.

Example: A rectangle sits in the first quadrant with its base on the x-axis and its left side on the y-axis. Its upper right hand corner is on the line passing through the points (0,4) and (3,0). What is the largest possible area of this rectangle?



slope $-\frac{4}{3}$

$y = -\frac{4}{3}x + 4$

$$A = xy = x\left(-\frac{4}{3}x + 4\right)$$

$$A(x) = -\frac{4}{3}x^2 + 4x, \quad 0 \le x \le 3$$

1. $A(0) = 0$

   $A(3) = 0$

2. $A'(x) = -\frac{8}{3}x + 4$

   $A'(x) = 0 \iff -\frac{8}{3}x + 4 = 0$

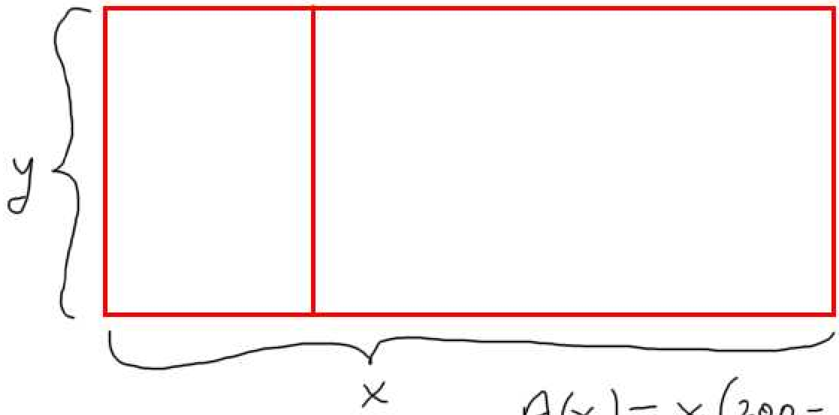
   $x = \frac{12}{8} = \frac{3}{2}$

   c.n. in $[0,3]$

$$A\left(\tfrac{3}{2}\right) = -\frac{4}{3}\cdot\frac{9}{4} + 4\cdot\frac{3}{2} = -3 + 6 = 3$$

3. Compare: The largest area is 3.

Example: A rectangular playground is to be fenced off and divided into two parts by a fence parallel to one side of the playground. Six hundred feet of fencing is used. Find the dimensions of the playground that will enclose the greatest total area.



$$A = xy$$

$$2x + 3y = 600$$

$$3y = 600 - 2x$$

$$y = 200 - \frac{2}{3}x$$

$$A(x) = x\left(200 - \frac{2}{3}x\right), \quad 0 \le x \le 300$$

$$A(x) = 200x - \frac{2}{3}x^2$$

1. $A(0) = 0$

   $A(300) = 0$

2. $A'(x) = 200 - \frac{4}{3}x$.

   $A'(x) = 0 \Leftrightarrow 200 - \frac{4}{3}x = 0$ $\quad x = 150$

   only c.n. in $[0, 300]$

$x \cdot y$

$$A(150) = 150\left(200 - \frac{2}{3}150\right)$$

$$= 15{,}000$$

3. The largest possible area is $15{,}000 \text{ ft}^2$ with $x = 150$ and $y = 100$