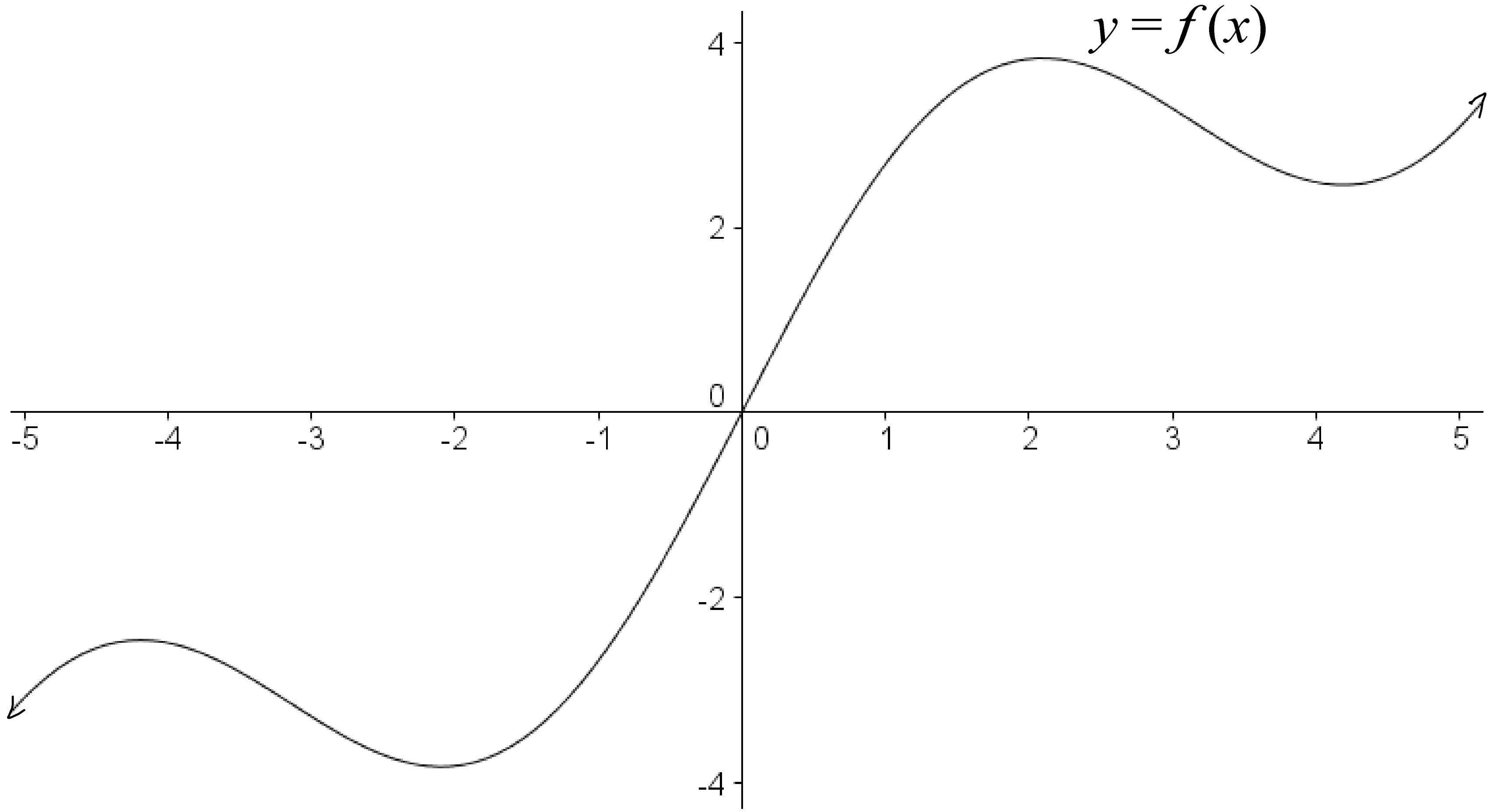
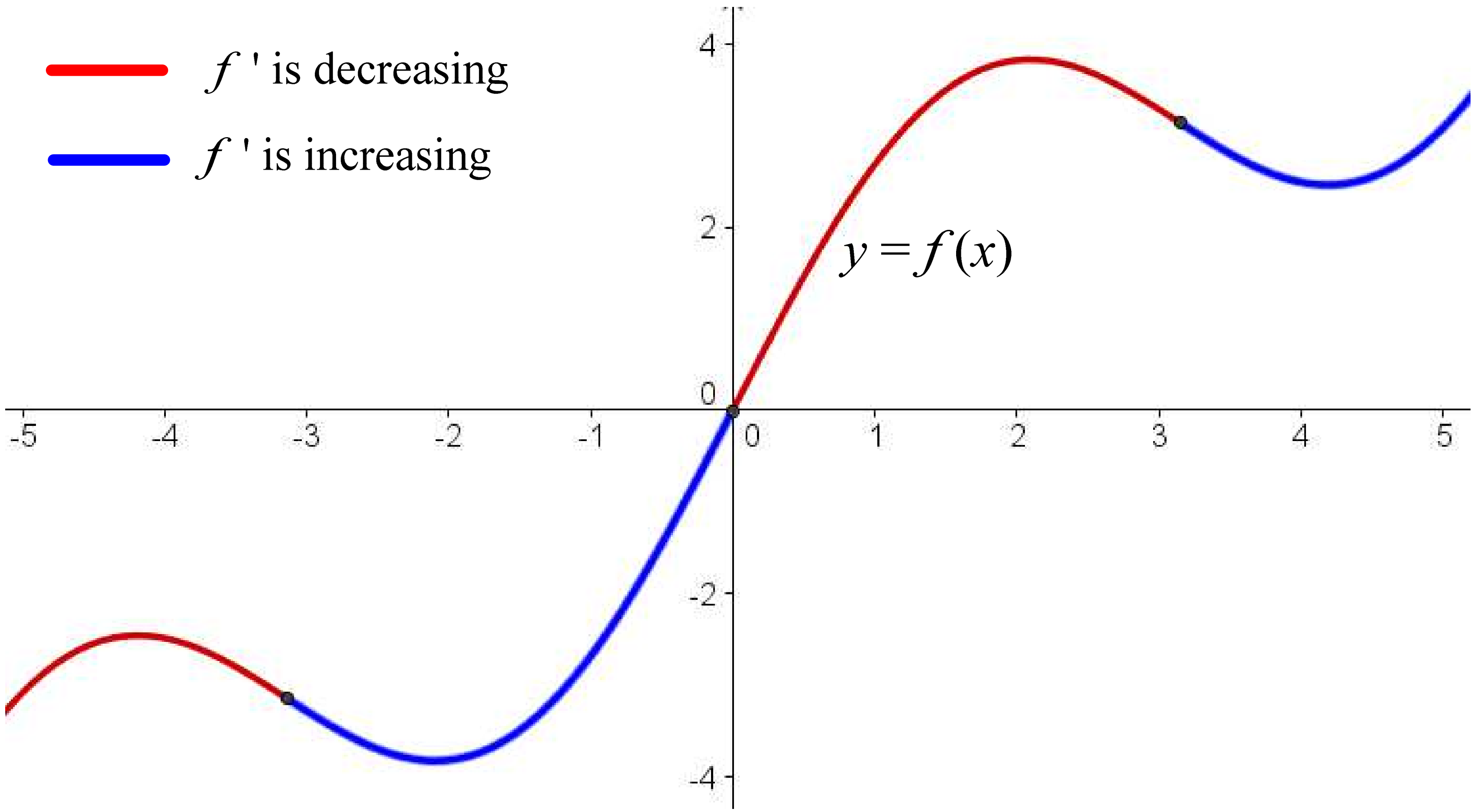
Info...

- **Test 3** is scheduled for November 1-5. The scheduler opens on October 18th.
- **EMCF**s and **Homework** are posted.
- **Test 3 Review** is posted.

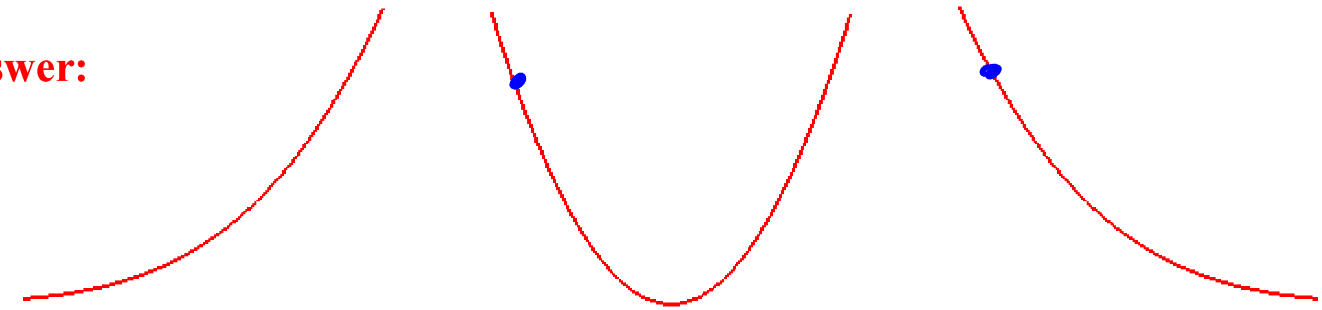
Where is $f\,'$ increasing/decreasing?

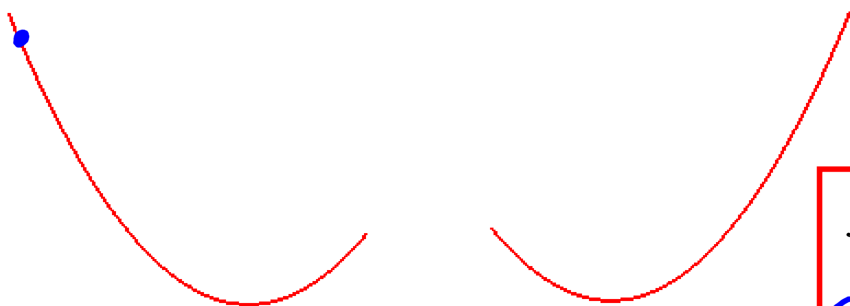



f ' is decreasing
f ' is increasing
y = f(x)
4
2
0
-2
-4
-5
-4
-3
-2
-1
0
1
2
3
4
5

Question: Suppose $f\,'$ is increasing on an interval. What are the possible shapes for the graph of f over this interval?

Answer:





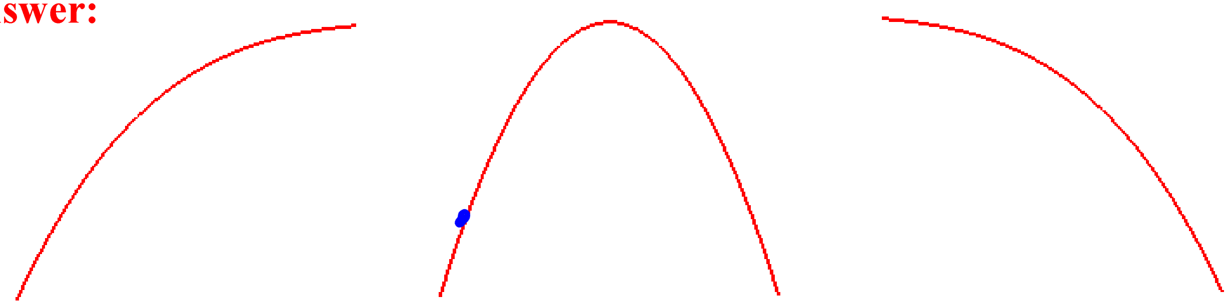
on an interval

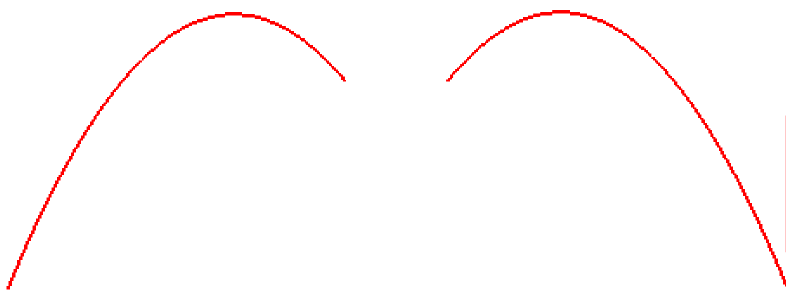
f is **concave up** when $f\,'$ is increasing

f is in an upward trend.

Question: Suppose $f\,'$ is <u>decreasing</u> on an interval. What are the possible shapes for the graph of f over this interval?

Answer:



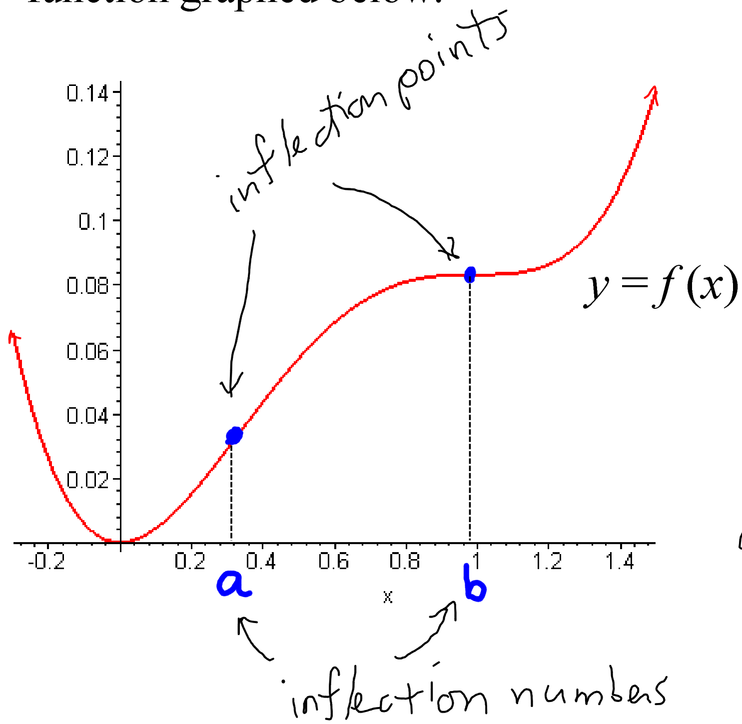


f is **concave down** when $f\,'$ is decreasing

f is in a downward trend.

Inflection occurs at a value in the domain of f where Concavity Changes!!

Example: Identify the inflection points and the intervals of cancavity of the function graphed below.



f is concave up on $(-\infty, a]$ and $[b, \infty)$.

f is concave down on $[a, b]$.

Inflection occurs at $x = a$ and $x = b$.

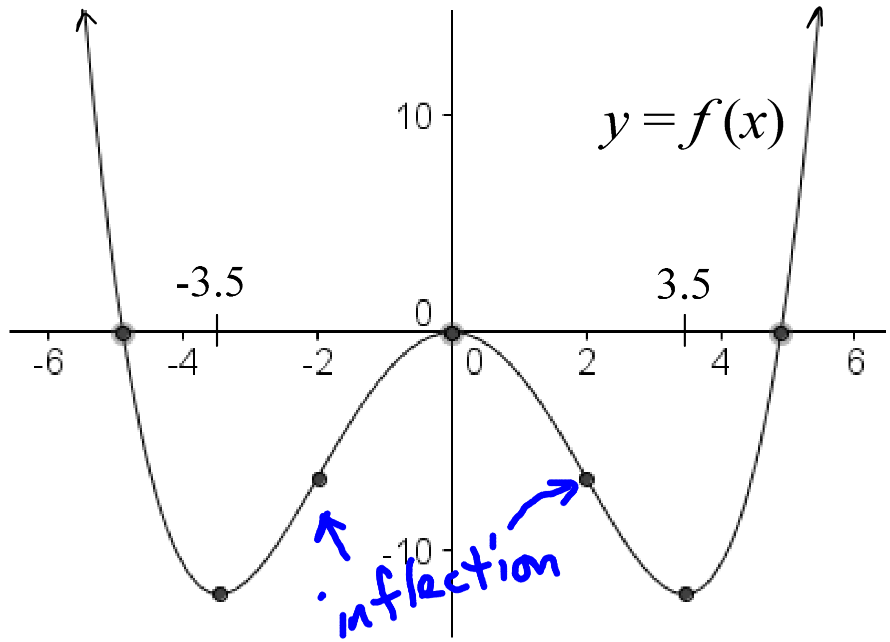
Popper P17

The graph of $y = f(x)$ is shown.

1. Give the smallest critical number of f.

2. Give the smallest inflection number of f.

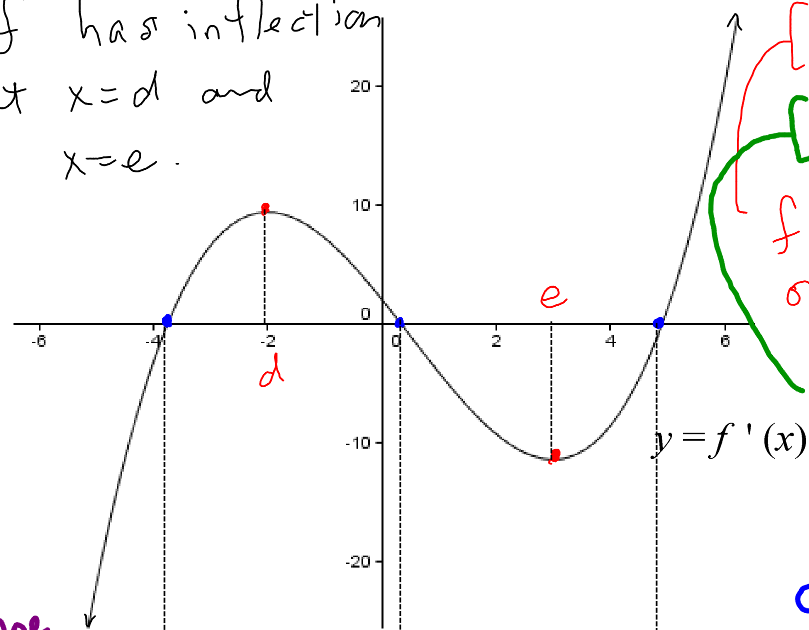
3. Give the left endpoint of the interval on which f is concave down.



c.n. $x = -3.5, 0, 3.5$

Example: The graph of $f\,'$ is shown below. List the intervals of increase, decrease, concave up and concave down for f, and classify the critical values for f and list any inflection for f.

f has inflection at $x=d$ and $x=e$.



[C.U. $\equiv f'$ increasing

[C.D. $\equiv f'$ decreasing

f is concave up on $(-\infty, d]$ and $[e, \infty)$.

f is concave down on $[d, e]$.

c.n. are $x=a, b, c$

slope chart

$f'(x)$	$- - -$	0	$+ + + +$	0	$- - - -$	0	$+ + +$
x		a		b		c	
Shape of f		$\cup$		$\cap$		$\cup$	

f has local mins at $x=a$ and $x=c$.

f has a local max at $x=b$.

f is increasing on $[a,b]$ and $[c,\infty)$

f is decreasing on $(-\infty,a]$ and $[b,c]$.

Concavity and the Second Derivative

$f\,'$ is increasing on an inverval
(f is concave up on an interval)

$\Longleftarrow$

$f'' > 0$ except at finitely many places

$f\,'$ is decreasing on an inverval
(f is concave down on an interval)

$\Longleftarrow$

$f'' < 0$ except at finitely man places

Example: Determine the intervals of concavity and the inflection numbers for $f(x) = x^3 - 3x^2 + 2x - 1$

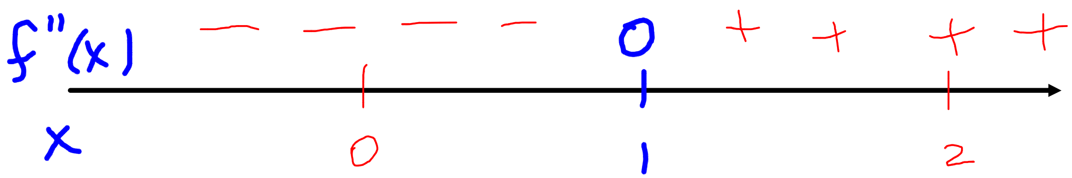
$$f'(x) = 3x^2 - 6x + 2$$

$$f''(x) = 6x - 6$$

Note: $f''(x) = 0$ iff $6x - 6 = 0$

$x = 1$

Concavity Chart



$f''(0) = -6$ $\qquad$ $f''(2) = 6$

f is concave down on $(-\infty, 1]$.

f is concave up on $[1, \infty)$.

Inflection occurs at $x = 1$.

The Second Derivative Test
for Classifying Critical Numbers

$\boxed{\text{Suppose } f\,'(c) = 0.}$

Spse f'' is continuous near c.

Question: What is the expected shape of the graph of f for a local minimum to occur at $x = c$?

C.U. shape

Question: What is the expected shape of the graph of f for a local maximum to occur at $x = c$?

C.D. shape

Question: How can the second derivative help us determine the associated shape? Does it ever fail?

$f'(c) = 0$ and $f''(c) > 0 \Rightarrow$
f has a local min at $x = c$

$f'(c) = 0$ and $f''(c) < 0 \Rightarrow$
f has a local max at $x = c$

Second derivative test.

Note: The test fails if $f''(c) = 0$.

$f(x) = x^3$

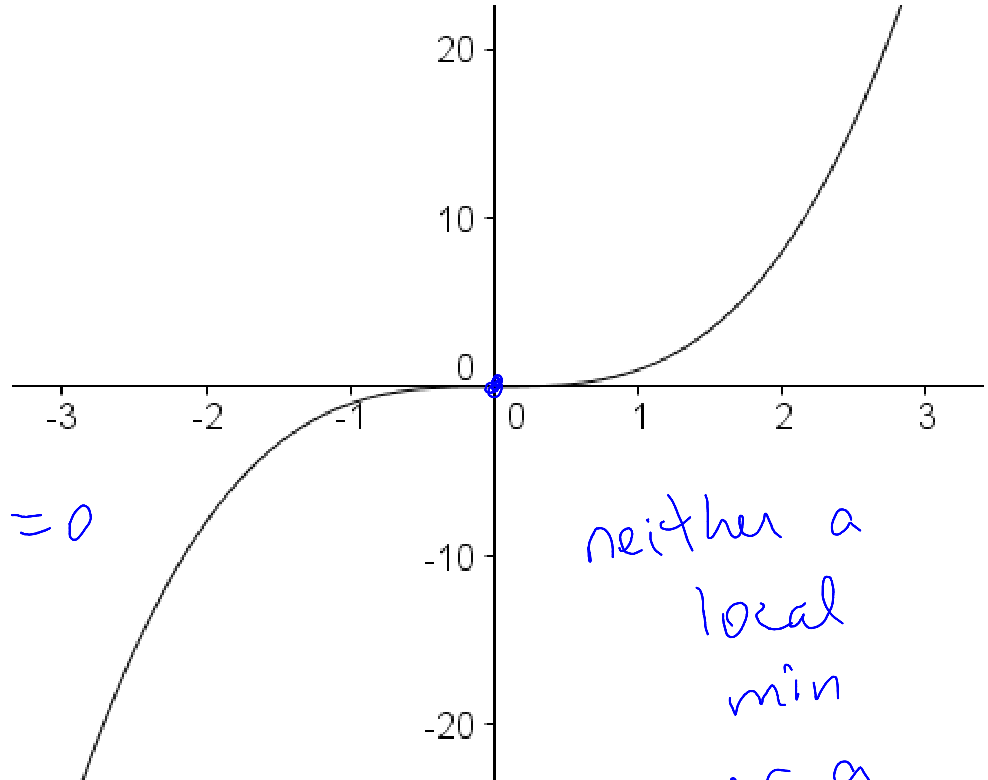
$f'(x) = 3x^2$

c.n. at $x = 0$

b/c $f'(0) = 0$

$f''(x) = 6x$

$\Rightarrow f''(0) = 0$



neither a local min nor a local max

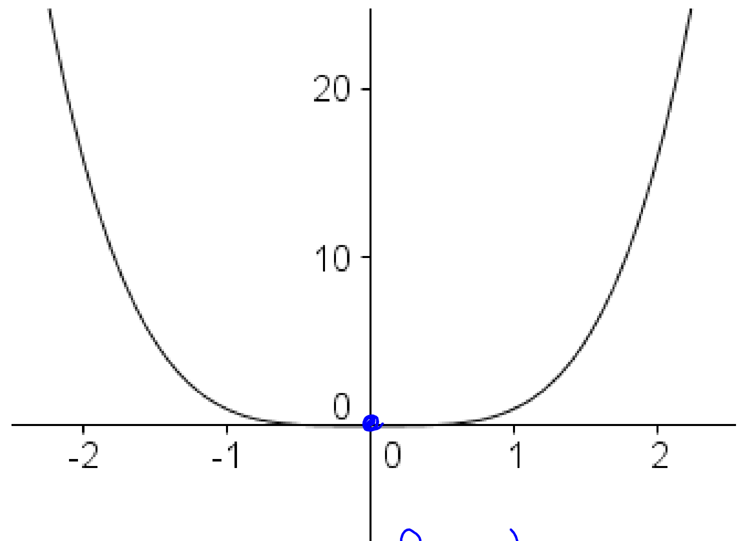
$f(x) = x^4$

$f'(x) = 4x^3$

Note: $f'(0) = 0$

$f''(x) = 12x^2$

$f''(0) = 0$



f has a local min at $x = 0$

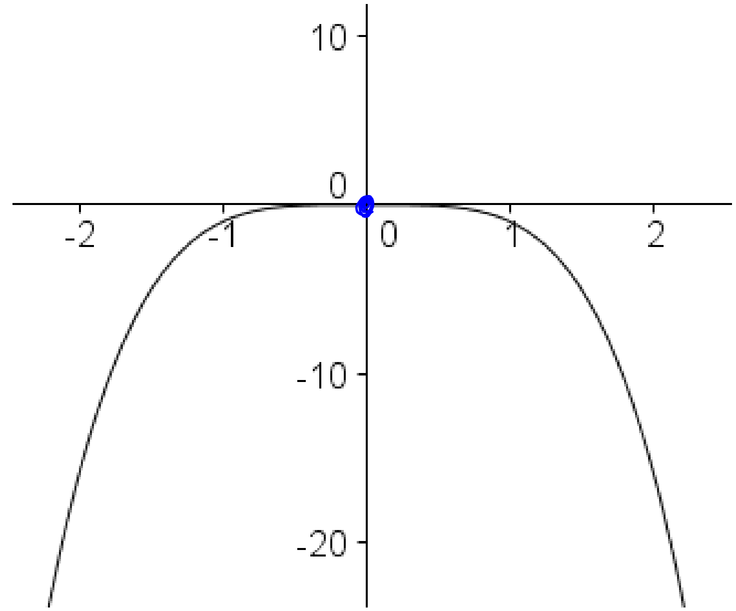
$$f(x) = -x^4$$

$$f'(x) = -4x^3$$

$$f'(0) = 0$$

$$f''(x) = -12x^2$$

$$f''(0) = 0$$



f has a local max at $x=0$.

Example: Use the second derivative test to classify the critical numbers of $f(x) = -2x^3 + 3x^2 + 6x + 2$.

See the video.