Info...

- **Homework** and **EMCFs** are posted for the week.
- An **EMCF** was due this morning, and an **Online Quiz** is due tonight.
- **Practice Test 3** is posted.
- **Test 3** includes material from 3.9 to 4.8.

New Material 4.8

Using Calculus to graph a function $y = f(x)$

1. Domain

2. Asymptotes and behavior at the edges.

3. First Derivative
   - critical numbers
   - slope chart
   - intervals of increase/decrease
   - classify c.n.

4. Second Derivative
   - intervals of concavity
   - inflection

5. Graph!! Use the information above to determine the shape, and then place the graph by plotting the points associated with critical values, inflection, y-intercept and x-intercept(s) (if possible).

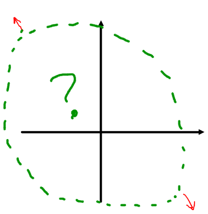
Example: Graph $f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

1. Domain: $(-\infty, \infty)$

2. No H.A. . No V.A..

Edge: $\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to \infty} f(x) = -\infty$



3. $f'(x) = -3x^2 - 3x + 6$

Exists everywhere.

Set $f'(x) = 0$

$-3x^2 - 3x + 6 = 0$

$-3(x^2 + x - 2) = 0$

$-3(x+2)(x-1) = 0$

$\boxed{x = -2,\ x = 1}$ c.n.

Slope chart

$f'(x)$	$-$ $-$ $-$	0	$+$ $+$ $+$ $+$	0	$-$ $-$ $-$
x	-3	-2 ∪	0	1 ∩	2

$f'(-3) = -$ $\quad$ $f'(0) = +$ $\quad$ $f'(2) = -$

f has a local min at $x = -2$

f has a local max at $x = 1$

f is increasing on $[-2, 1]$

f is decreasing on $(-\infty, -2]$ and $[1, \infty)$

$f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

$f'(x) = -3x^2 - 3x + 6$

4. $f''(x) = -6x - 3$

$f''(x) = 0 \iff -6x - 3 = 0$

$x = -\frac{1}{2}$

Concavity chart

$f''(x)$	$+$ $+$ $+$ $+$	0	$-$ $-$ $-$ $-$
x	-1	$-\frac{1}{2}$	0

$f''(-1) = +$ $\quad$ $f''(0) = -$

f is concave up on $(-\infty, -\frac{1}{2}]$

f is concave down on $[-\frac{1}{2}, \infty)$.

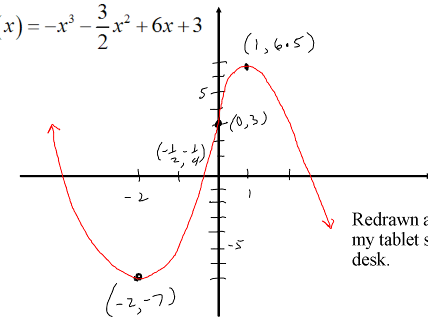
f has inflection at $x = -\frac{1}{2}$

$f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

f has a local min at $x = -2$
f has a local max at $x = 1$
f is increasing on $[-2, 1]$
f is decreasing on $(-\infty, -2]$ and $[1, \infty)$
f is concave up on $(-\infty, -\frac{1}{2}]$
f is concave down on $[-\frac{1}{2}, \infty)$.
f has inflection at $x = -\frac{1}{2}$

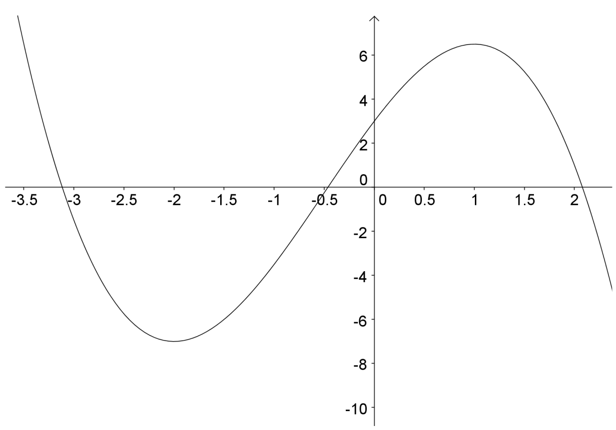
Points: c.n. $f(-2) = -7$ — $(-2, -7)$
c.n. $f(1) = 6.5 = \frac{13}{2}$ — $(1, \frac{13}{2})$
infl $f(-\frac{1}{2}) = -\frac{1}{4}$ — $(-\frac{1}{2}, -\frac{1}{4})$
y-int $f(0) = 3$ — $(0, 3)$

$f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$



$(-2, -7)$
$(1, \frac{13}{2})$
$(-\frac{1}{2}, -\frac{1}{4})$
$(0, 3)$

Redrawn after class with my tablet sitting on my desk.



Popper P19

f, $f\,'$ and $f\,''$ are graphed and labeled with numbers.

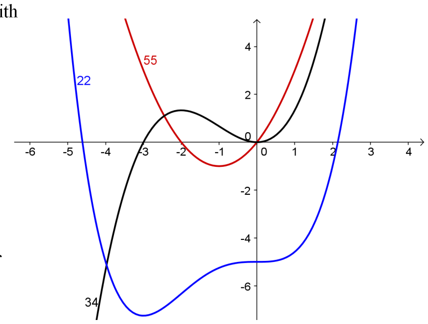
1. Which curve is the graph of f ?

2. Which curve is the graph of $f\,'$?

3. Which curve is the graph of $f\,''$?

4. Give the number of critical values of f.

5. Give the largest inflection number of f.



Example: Graph $f(x)=\dfrac{x^3}{x^2-3}$ $\leftarrow$ rational function

$x^2-3=0$ iff $x=\pm\sqrt{3}$

$f'(x)=\dfrac{x^4-9x^2}{(x^2-3)^2}$ $\quad f''(x)=x\left[\dfrac{6x^2+54}{(x^2-3)^3}\right]$

1. Domain: $(-\infty,-\sqrt{3})\cup(-\sqrt{3},\sqrt{3})\cup(\sqrt{3},\infty)$

2. V.A.: $x=\pm\sqrt{3}$

H.A.: $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{x^3}{x^2-3}$

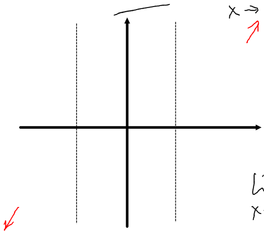
$=\lim\limits_{x\to-\infty}\dfrac{x^3}{x^2(1-3/x^2)}$

$=\lim\limits_{x\to-\infty} x\cdot\dfrac{1}{1-\underbrace{3/x^2}_{0}}$ $\to 1$

$=-\infty$

$\lim\limits_{x\to\infty} f(x)=\cdots=\infty$ $\leftarrow$ important

No H.A.



$f''(x)=x\left[\dfrac{6x^2+54}{(x^2-3)^3}\right]$

4. $f''(x)$ exists everywhere except $x=\pm\sqrt{3}$

$f''(x)=0 \iff \boxed{x=0}$ b/c $6x^2+54>0$

Concavity chart:

$f''(x)$: $- - -$ | $+ + +$ 0 $- - -$ | $+ + + +$

x: -2, $-\sqrt{3}$, -1, 0, 1, $\sqrt{3}$, 2

$f''(-2)=-$ $\quad f''(-1)=+$ $\quad f''(1)=- - -$ $\quad f''(2)=+$

See the Video