Info...

- **Homework** and **EMCFs** are posted for the week.
- An **EMCF** was due this morning, and an **Online Quiz** is due tonight.
- **Practice Test 3** is posted.
- **Test 3** includes material from 3.9 to 4.8.

New Material

4.8

Using Calculus to graph a function $y = f(x)$

1. Domain

2. Asymptotes and behavior at the edges.

3. First Derivative
   - critical numbers
   - slope chart
   - intervals of increase/decrease
   - classify c.n.

4. Second Derivative
   - intervals of concavity
   - inflection

5. Graph!! Use the information above to determine the shape, and then place the graph by plotting the points associated with critical values, inflection, y-intercept and x-intercept(s) (if possible).

Example: Graph $f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

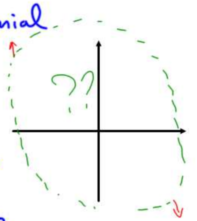
← polynomial

1. Domain: $(-\infty, \infty)$

2. No VA or HA.

Edge $\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to \infty} f(x) = -\infty$



$f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

3. $f'(x) = -3x^2 - 3x + 6$

← polynomial

$f'(x)$ exists for all x.

c.n.: set $f'(x) = 0$

$-3x^2 - 3x + 6 = 0$

$-3(x^2 + x - 2) = 0$

$-3(x+2)(x-1) = 0$

$\boxed{x = -2 \text{ or } x = 1}$ ← c.n.

slope chart:

$f'(x)$: $- - - - \; 0 \; + + + + \; 0 \; - - - -$

x: $-3 \quad -2 \quad 0 \quad 1 \quad 2$

$f'(-3) = -$ $\quad f'(0) = +$ $\quad f'(2) = -$

local min at $x = -2$

local max at $x = 1$

f is increasing on $[-2, 1]$.

f is decreasing on $(-\infty, -2]$ and $[1, \infty)$.

$f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

$f'(x) = -3x^2 - 3x + 6$

4. $f''(x) = -6x - 3$

↖ polynomial

$f''(x) = 0 \iff -6x - 3 = 0$

$x = -\frac{1}{2}$

Concavity chart

$f''(x)$: + + + + + 0 — — — —

x: -1, $-\frac{1}{2}$, 0

$f''(-1) = +$ $f''(0) = -$

f is C.U. on $(-\infty, -\frac{1}{2}]$

f is C.D. on $[-\frac{1}{2}, \infty)$

f has inflection at $x = -\frac{1}{2}$.

local min at $x = -2$ local max at $x = 1$

f is increasing on $[-2, 1]$.

f is decreasing on $(-\infty, -2]$ and $[1, \infty)$.

f is C.U. on $(-\infty, -\frac{1}{2}]$

f is C.D. on $[-\frac{1}{2}, \infty)$

f has inflection at $x = -\frac{1}{2}$.

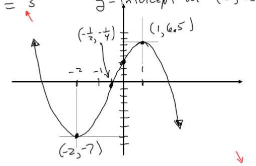
5. $f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$

$f(-2) = 8 - 6 - 12 + 3 = -7$. local min at $(-2, -7)$ ✓

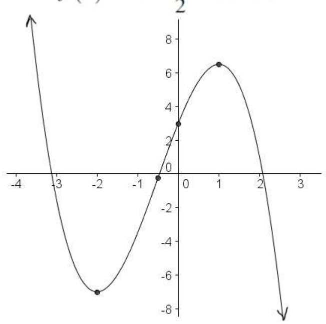
$f(1) = -1 - \frac{3}{2} + 9 = \frac{13}{2}$. local max at $(1, 13/2)$ ✓

$f(-\frac{1}{2}) = \frac{1}{8} - \frac{3}{8} - 3 + 3 = -\frac{1}{4}$ inflection occurs at $(-\frac{1}{2}, -\frac{1}{4})$

$f(0) = 3$ y-intercept at $(0, 3)$



$f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 3$



Example: Graph $f(x) = \frac{x^3}{x^2 - 3}$

↖ rational function

1. Note $x^2 - 3 = 0$ iff $x = \pm\sqrt{3}$

Domain: $(-\infty, -\sqrt{3}) \cup (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)$.

2. H.A.? $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^3(1)}{x^2(1 - 3/x^2)}$

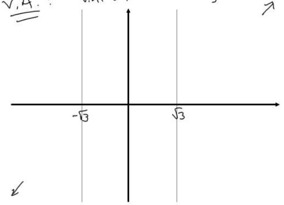
$= \lim_{x\to\infty} x \left(\frac{1}{1 - 3/x^2}\right) = \infty$

$f(x) = \frac{x^3}{x^2 - 3}$

$\lim_{x\to-\infty} f(x) = \dots = -\infty$

No H.A.

V.A.? V.A. at $x = -\sqrt{3}$, $x = \sqrt{3}$.



$f(x) = \frac{x^3}{x^2-3}$

3. $f'(x) = \frac{(x^2-3)\cdot 3x^2 - x^3\cdot 2x}{(x^2-3)^2}$

$$f'(x) = \frac{x^4-9x^2}{(x^2-3)^2} \leftarrow \text{rational function}$$

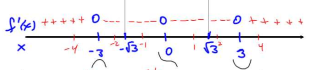
$f'(x)$ exists everywhere except $x = -\sqrt{3}$, $x = \sqrt{3}$. ← These values are NOT in the domain of f.

Set $f'(x) = 0 \Leftrightarrow x^4 - 9x^2 = 0$

$x^2(x^2-9) = 0$

$x = 0$, $x = -3$, $x = 3$ ← c.n.

slope chart



$f'(-4) = +$ $f'(-2) = -$ $f'(-1) = -$ $f'(1) = -$ $f'(2) = -$ $f'(4) = +$

$f'(x) = \frac{x^4-9x^2}{(x^2-3)^2}$

f has a local max at $x = -3$
f has neither a local max nor a local min at $x = 0$
f has a local min at $x = 3$
f is increasing on $(-\infty, -3]$ and $[3, \infty)$
f is decreasing on $[-3, -\sqrt{3})$, $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, 3]$.

$f'(x) = \frac{x^4-9x^2}{(x^2-3)^2}$

4. $f''(x) = \frac{(x^2-3)^{\cancel{2}\,1}(4x^3-18x) - (x^4-9x^2)\cdot 2\cancel{(x^2-3)}2x}{(x^2-3)^{\cancel{4}\,3}}$

$= \frac{(x^2-3)(4x^3-18x) - 4x(x^4-9x^2)}{(x^2-3)^3}$

$= x\,\frac{(x^2-3)(4x^2-18) - 4(x^4-9x^2)}{(x^2-3)^3}$

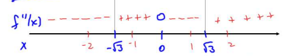
$= x\left[\frac{\cancel{4x^4} - 18x^2 - 12x^2 + 54 - \cancel{4x^4} + 36x^2}{(x^2-3)^3}\right]$

$f''(x) = x\left[\frac{6x^2+54}{(x^2-3)^3}\right]$

exists everywhere except $x = \pm\sqrt{3}$.

$f''(x) = 0 \Leftrightarrow x = 0$ (b/c $6x^2+54 > 0$)

Concavity chart:



$f''(-2) = -$ $f''(-1) = +$ $f''(1) = -$ $f''(2) = +$

f is C.U. on $(-\sqrt{3}, 0]$ and $(\sqrt{3}, \infty)$
f is C.D. on $(-\infty, -\sqrt{3})$ and $[0, \sqrt{3})$
f has inflection at $x = 0$

f has a local max at $x = -3$
f has neither a local max nor a local min at $x = 0$
f has a local min at $x = 3$
f is increasing on $(-\infty, -3]$ and $[3, \infty)$
f is decreasing on $[-3, -\sqrt{3})$, $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, 3]$.

f is C.U. on $(-\sqrt{3}, 0]$ and $(\sqrt{3}, \infty)$
f is C.D. on $(-\infty, -\sqrt{3})$ and $[0, \sqrt{3})$
f has inflection at $x = 0$

$f(x) = \frac{x^3}{x^2-3}$

$f(-3) = -\frac{27}{6} = \frac{-9}{2} \Leftrightarrow (-3, \frac{-9}{2})$ local max

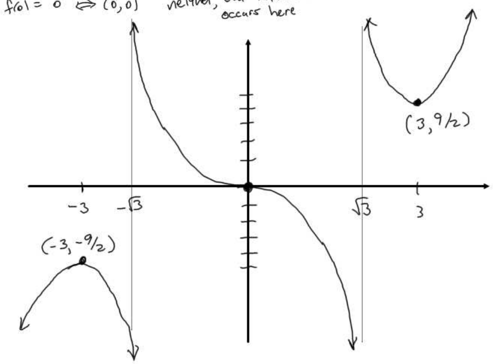
$f(3) = \frac{27}{6} = \frac{9}{2} \Leftrightarrow (3, 9/2)$ local min

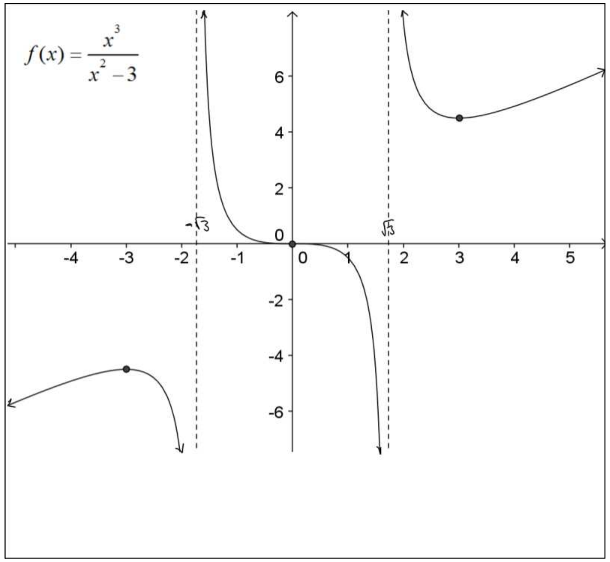
$f(0) = 0 \Leftrightarrow (0, 0)$ neither, but inflection occurs here

$f(-3) = -\frac{27}{6} = \frac{-9}{2} \Leftrightarrow (-3, \frac{-9}{2})$ local max

$f(3) = \frac{27}{6} = \frac{9}{2} \Leftrightarrow (3, 9/2)$ local min

$f(0) = 0 \Leftrightarrow (0, 0)$ neither, but inflection occurs here





$f(x)=\dfrac{x^3}{x^2-3}$
$-\sqrt{3}$
$\sqrt{3}$
6
4
2
0
-2
-4
-6
-4
-3
-2
-1
0
1
2
3
4
5