Information

- Test 3 is 11/01 - 11/05!!
- Practice Test 3 is posted.
- Test 3 covers sections 3.9 - 4.8.
- We will finish the review that we started on Wednesday.

Example: Find the largest possible value of xy given that x and y are both positive and $2x + y = 40$.

Maximize $f = xy$

given that $x, y > 0$ and $2x + y = 40$.

$y = 40 - 2x$

$\Rightarrow f(x) = x(40 - 2x), \quad 0 < x < 20$

$f(x) = 40x - 2x^2$

$f'(x) = 40 - 4x$ ← exists for all x.

$f'(x) = 0 \iff 40 - 4x = 0$

$x = 10$ c.n.

slope chart

$f'(x)$: + + + + + + 0 − − − − − −

x: (0   1   10   11   20)

$f'(1) = +$ $\qquad f'(11) = -$

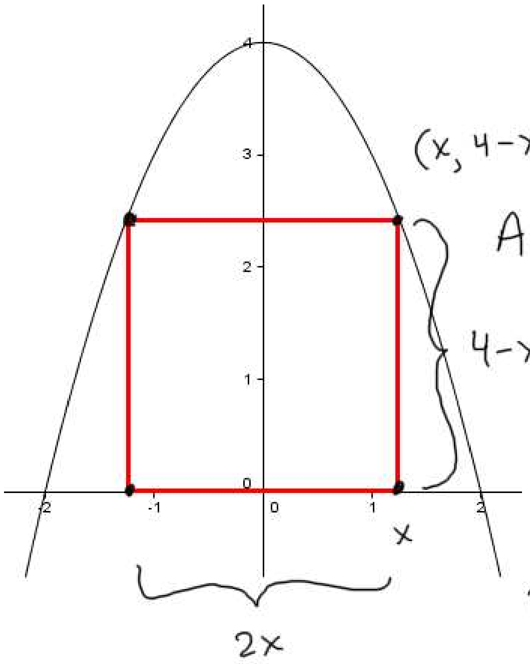
f shape

$\therefore$ f has an abs. max at $x = 10$.

The largest value of xy is

$10(40 - 2 \cdot 10) = \underline{\underline{200}}$

Example: Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 4 - x^2$.



$A(x) = 8x - 2x^3$

$A(x) = 2x(4-x^2)$, $0 \le x \le 2$

maximize

1. $A(0) = 0$

   $A(2) = 0$

2. $A'(x) = 8 - 6x^2$
   exists for all x

$A'(x) = 0 \Leftrightarrow 8 - 6x^2 = 0$

$x^2 = \frac{4}{3}$

$x = \frac{2}{\sqrt{3}}$

$A\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}\left(4 - \frac{4}{3}\right)$

$= \frac{32}{3\sqrt{3}}$

$\boxed{A(x) = 2x(4-x^2) \quad 0 \le x \le 2}$

3. Compare $\Rightarrow$ the largest possible area is $\frac{32}{3\sqrt{3}} \approx 6.1584$.

Concept	Questions/Comments

9. Concavity

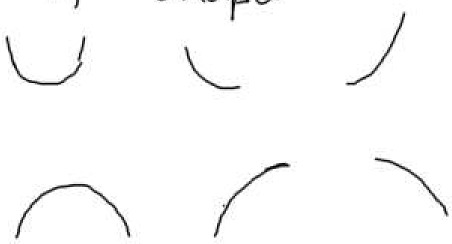
A function f if concave up on an interval I if and only if $f'(x)$ is increasing on I.

A function f if concave down on an interval I if and only if $f'(x)$ is decreasing on I.

Graphically: f shapes

C.U.

C.D



Quick Check:

f'' exists on I

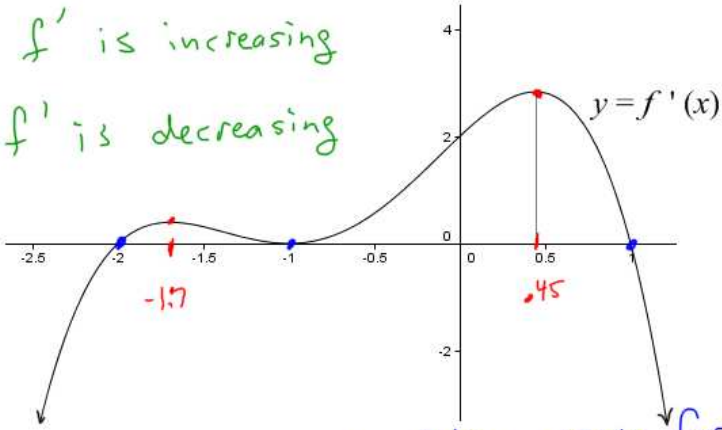
- If $f''(x) > 0$ except at finitely many values on I then f is C.U. on I.
- If $f''(x) < 0$ except at finitely many values on I then f is C.D. on I.

Concept	Questions/Comments
10. Inflection A function f has inflection at a value c provided c is in the domain of f and the concavity is different on the left of c than it is on the right of c.	Graphically: Concavity changes Quick Check: ...change in concavity...

Example: The graph of f' is shown below. Use this graph to find classify critical numbers, intervals of increase and decrease, intervals of concavity, and inflection for f. Then give a plausible graph for f.

C.U. f' is increasing

C.D. f' is decreasing

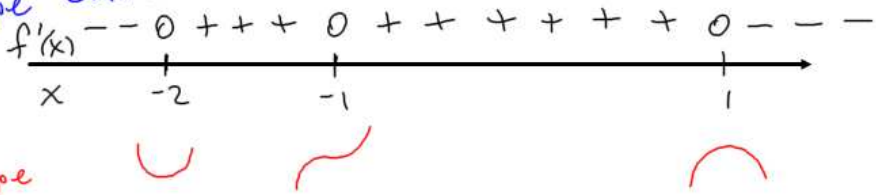


It appears as though $f'(x)$ exists for all x.

$\therefore$ C.N. occur when $f'(x) = 0$

$\Rightarrow$ f has c.n. at $\boxed{x = -2, -1, 1.}$

Slope chart



f shape

f has a local min at $x = -2$

f has a local max at $x = 1$

f has neither a local min nor local max at $x = -1$

f is increasing on $[-2, 1]$.

f is decreasing on $(-\infty, -2]$ and $[1, \infty)$.

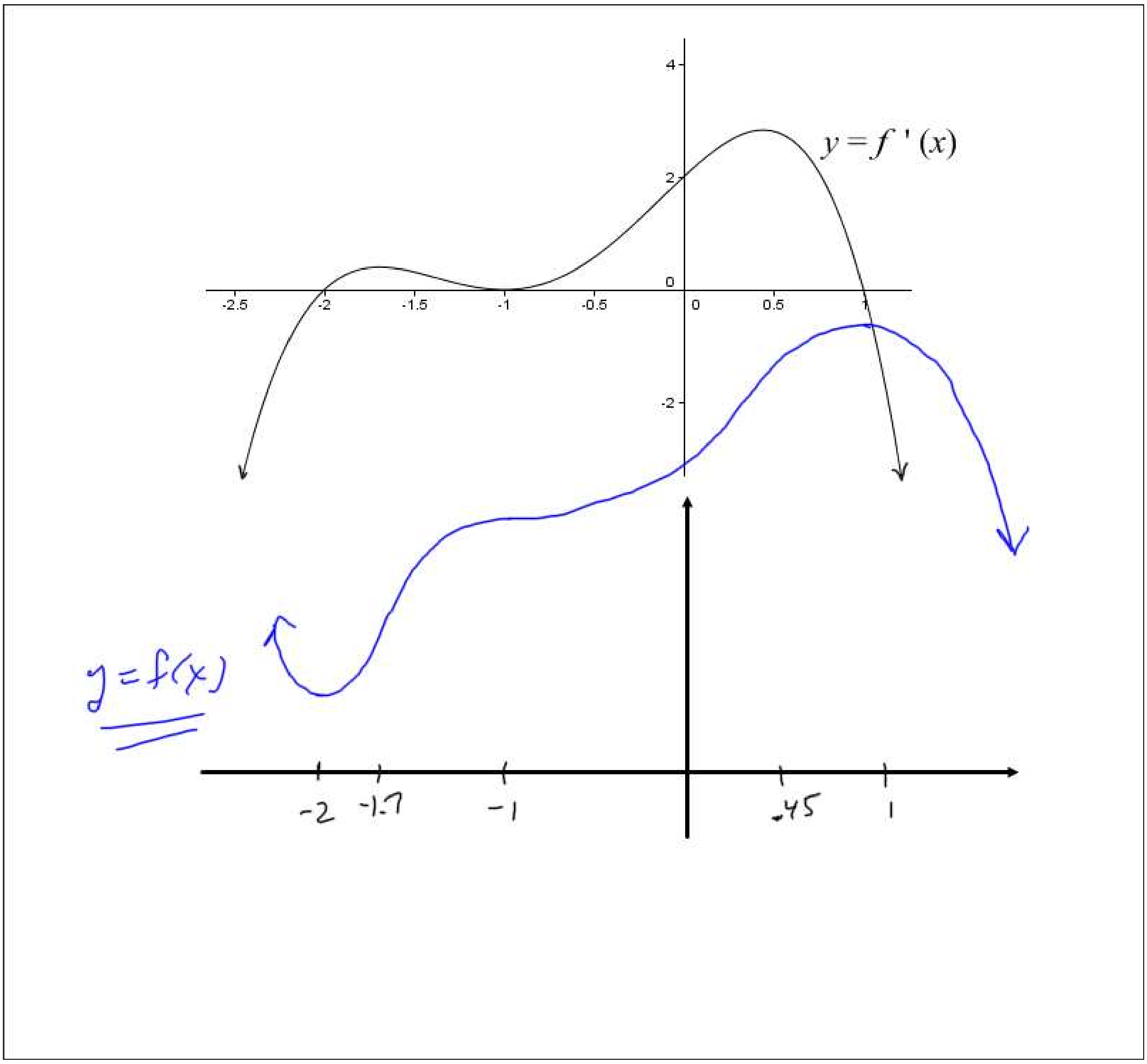
C.U. f' is increasing

C.D. f' is decreasing

$\therefore$ f is C.U. on $(-\infty, -1.7]$ and $[-1, 0.45]$.

f is C.D. $[-1.7, -1]$ and $[0.45, \infty)$.

f has inflection at $x = -1.7, -1, 0.45$



4
2
0
-2
-2.5
-1.5
-1
-0.5
0.5
1
y = f ' (x)
y = f(x)
-2
-1.7
-1
.45
1

Concept	Questions/Comments
11. Asymptotes and behavior at the edge of the domain.	Horizontal Asymptotes: $y = c$ is a H.A. iff either $\lim_{x \to -\infty} f(x) = c$ or $\lim_{x \to \infty} f(x) = C$ Vertical Asymptotes: infinite discont.

Concept	Questions/Comments
12. Graphing	

1. Domain
2. Asymptotes and behavior for x near the "edges" of the domain.
3. First Derivative
   - critical numbers
   - slope chart
   - intervals of increase
   - intervals of decrease
   - classify c.n.
4. Second Derivative
   - intervals of concavity
   - inflection
5. Graph it!! (plot plots associated with the information above, along with the y - intercept, and the x - intercept(s) if they are easily found.

Example: Graph $f(x) = \dfrac{x^2}{3-2x}$ ← rational function

1. <u>Domain:</u> Note $3-2x=0$ iff $x=\frac{3}{2}$.

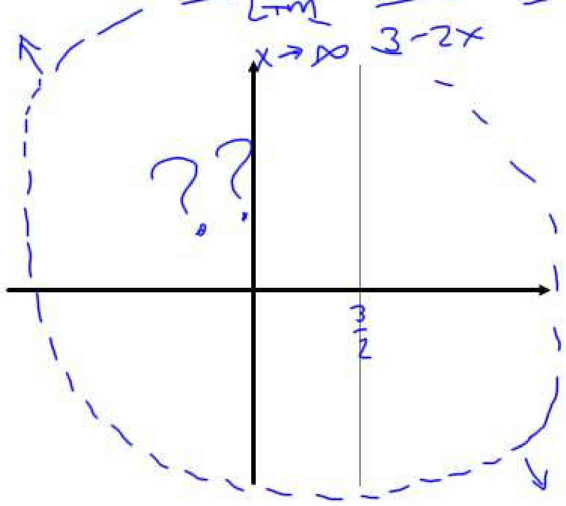
   The domain is $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$.

2. <u>V.A.</u>: f has a V.A. at $x=\frac{3}{2}$.

$$\lim_{x\to-\infty} \frac{x^2}{3-2x} = \lim_{x\to-\infty} \frac{x^{\cancel{2}}}{\cancel{x}\left(\frac{3}{x}-2\right)}$$

$$= \lim_{x\to-\infty} \frac{x}{\underbrace{\frac{3}{x}}_{\to 0}-2} = +\infty$$

$$\lim_{x\to\infty} \frac{x^2}{3-2x} = \lim_{x\to\infty} \frac{x}{\underbrace{\frac{3}{x}}_{\to 0}-2} = -\infty$$



$$f(x) = \frac{x^2}{3-2x}$$

$$x \neq \frac{3}{2}$$

3. $$f'(x) = \frac{(3-2x)2x - x^2 \cdot (-2)}{(3-2x)^2}$$

$$f'(x) = \frac{6x - 4x^2 + 2x^2}{(3-2x)^2}$$

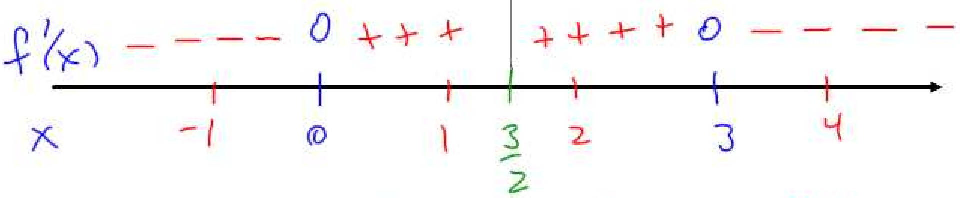
$$f'(x) = \frac{2x(3-x)}{(3-2x)^2}$$

exists for all x except $x = \frac{3}{2}$.

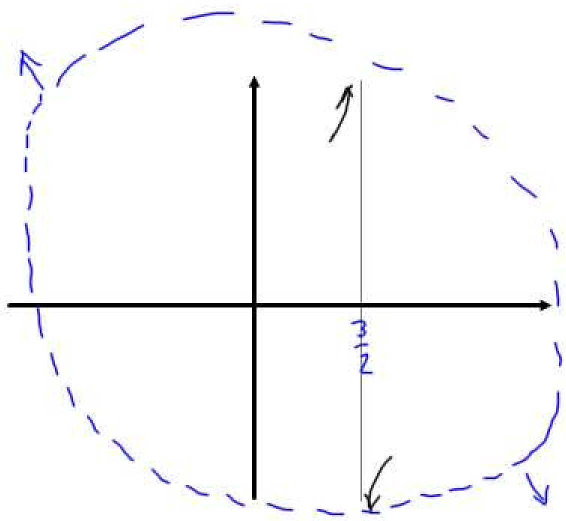
$$f'(x) = 0 \iff 2x(3-x) = 0$$

$$\boxed{x = 0\ ,\ x = 3} \quad \text{c.n.}$$

slope chart



$f'(-1) = -$ $f'(1) = +$ $f'(2) = +$ $f'(4) = -$



$$f'(x) = \frac{2x(3-x)}{(3-2x)^2} = \frac{6x-2x^2}{(3-2x)^2}$$

4. $$f''(x) = \frac{(3-2x)^{\cancel{2}}(6-4x) - (6x-2x^2)\cdot 2\,\cancel{(3-2x)}(-2)}{(3-2x)^{\cancel{4}\,3}}$$

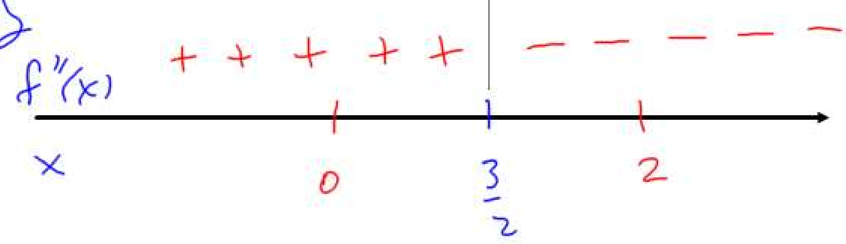
$$= \frac{(3-2x)(6-4x) + 24x - 8x^2}{(3-2x)^3}$$

$$= \frac{18 - \cancel{12x} - \cancel{12x} + \cancel{8x^2} + \cancel{24x} - \cancel{8x^2}}{(3-2x)^3}$$

$$f''(x) = \frac{18}{(3-2x)^3}$$

Exists for all x except $x = \frac{3}{2}$.
$f''(x)$ is NEVER 0.

Concavity chart



$f''(0) = +$ $f''(2) = -$

Note: f does not have inflection at 3/2 since 3/2 is not in the domain of f.

f is C.U. on $(-\infty, \frac{3}{2})$
f is C.D. on $(\frac{3}{2}, \infty)$.

$$f(x)=\frac{x^2}{3-2x}$$

5. Graph.

c.n. at

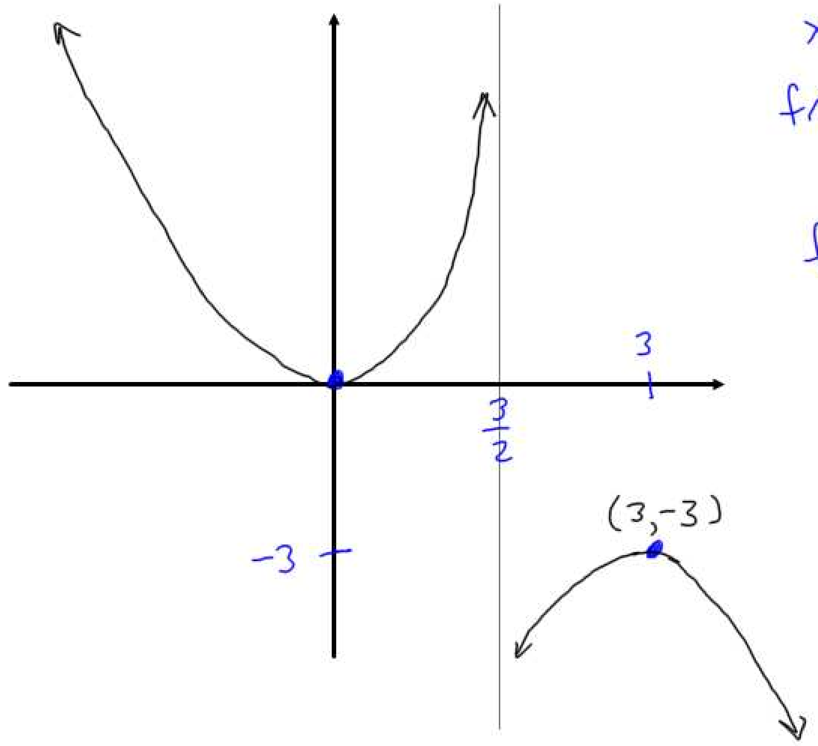
$x=0$

$x=3$

$f(0)=0$

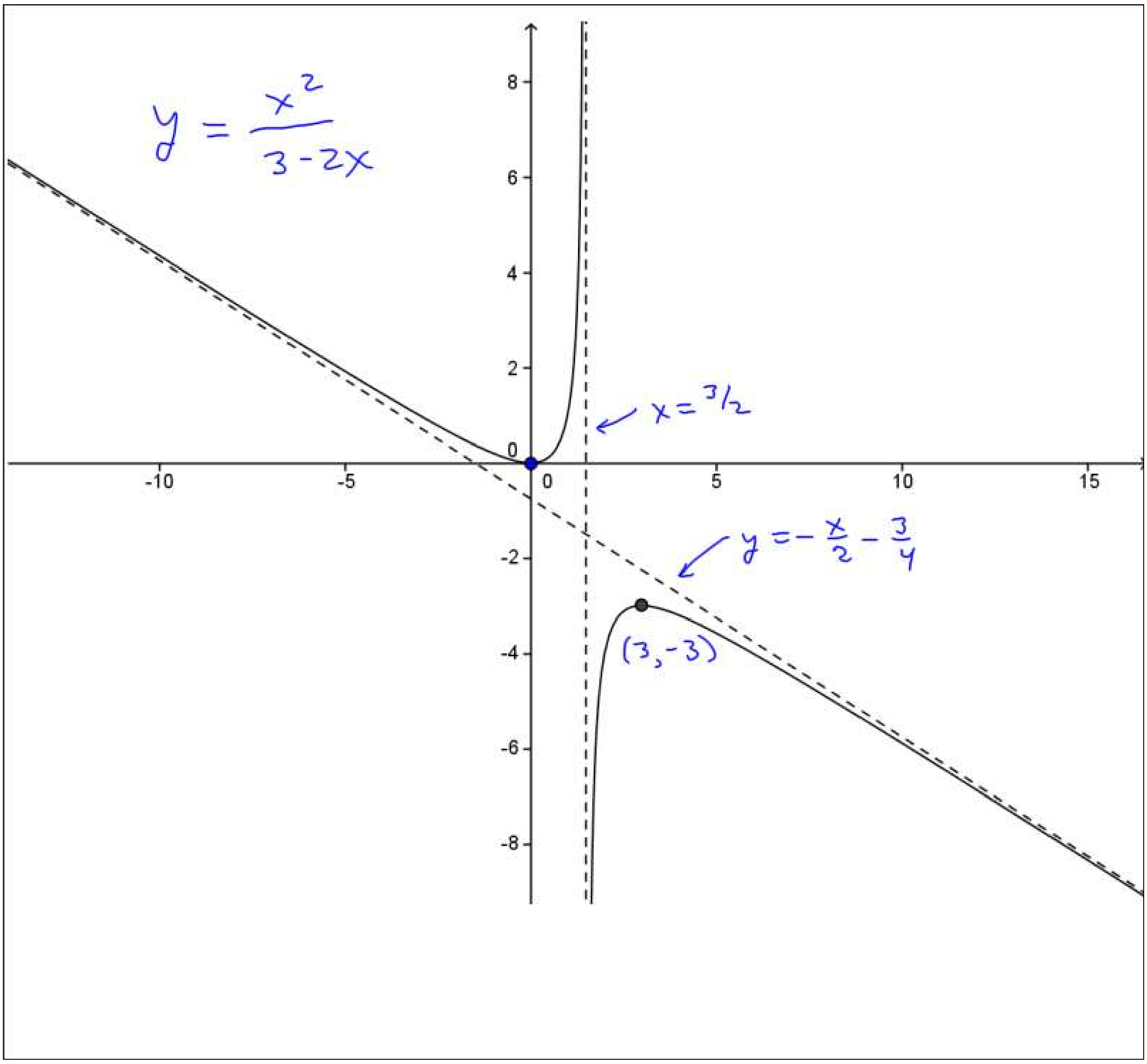
$(0,0)$

$f(3)=-3$



$$f(x)=\frac{x^2}{3-2x} = \frac{x^2}{-2x+3} = -\frac{x}{2}-\frac{3}{4}+\frac{9/4}{-2x+3}$$

$$\begin{array}{r|l} & -\frac{x}{2} - \frac{3}{4} \\ -2x+3 & x^2 + 0x + 0 \\ & -\left(x^2 - \frac{3}{2}x\right) \\ \hline & \frac{3}{2}x + 0 \\ & -\left(\frac{3}{2}x - \frac{9}{4}\right) \\ \hline & 9/4 \end{array}$$



$y = \frac{x^2}{3-2x}$
$x = 3/2$
$y = -\frac{x}{2} - \frac{3}{4}$
$(3, -3)$