Net Area and Integrals

- We are in chapter 5.
- I posted a video with some upper sum and lower sum examples.
- The written portion of Test 3 is graded. Add the Test 3 and Test 3 FR columns in the gradebook to obtain your grade.

Grade Information by Section

15819		15825		15836		15841	
503 students		486 students		324 students		121 students	
median - 69		median - 78.5		median - 71		median - 66	
mean - 67.5		mean - 75		mean - 69.6		mean - 63.6	
0-49	0.17	0-49	0.09	0-49	0.16	0-49	0.25
50-59	0.15	50-59	0.10	50-59	0.13	50-59	0.13
60-69	0.18	60-69	0.13	60-69	0.18	60-69	0.19
70-79	0.18	70-79	0.20	70-79	0.18	70-79	0.20
80-89	0.18	80-89	0.24	80-89	0.18	80-89	0.16
90-100	0.12	90-100	0.24	90-100	0.17	90-100	0.07

Grade Distribution - This Class

Section 15825
486 students
median - 78.5
mean - 75

0-49	0.09
50-59	0.10
60-69	0.13
70-79	0.20
80-89	0.24
90-100	0.24
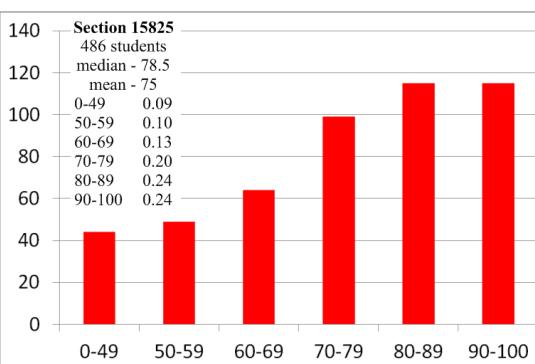
Extra Credit for Test 3

You can earn 5 additional points on Test 3 by going to

http://www.casa.uh.edu/TeacherEvaluation/

Fill out the teacher evaluation for both your lecture and lab section.

Upon completion, I will receive a note in my inbox with your student ID. Your teacher evaluation WILL NOT be forwarded with your ID. The information you give is anonymous.

Recall:
The **Riemann** Integral of f from a to b: $\int_a^b f(x)dx$

1. What do we get if f is nonnegative on the interval $[a,b]$?

Area

Area of the region bounded between the x-axis and the graph of $y = f(x)$ on the interval [a,b].

Recall:
The **Riemann** Integral of f from a to b: $\int_a^b f(x)dx$

2. What do we get in the general case?

$$= \text{Area}(\text{///}) - \text{Area}(\text{///})$$
$$= \text{Net Area}$$
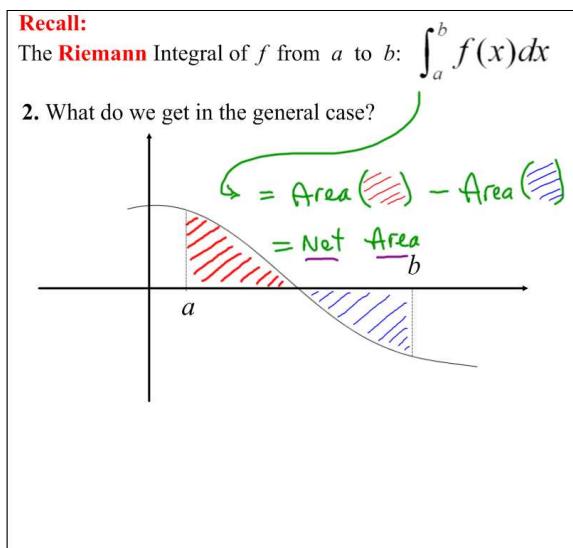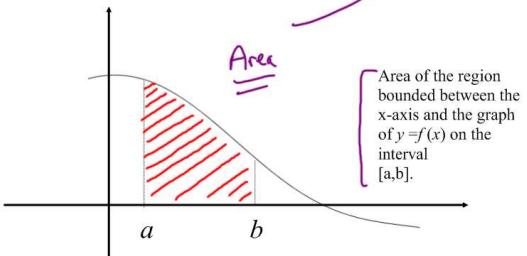
Properties of the Riemann Integral:

1. $\int_a^b \left(f(x) + g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

2. $\int_a^b \alpha f(x)dx = \alpha \int_a^b f(x)dx$, $\quad \alpha$ is a real number.

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ ✳

4. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

5. $\int_a^a f(x)dx = 0$

6. If $f(x) \geq g(x)$ on $[a,b]$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

Example: Suppose $\int_0^1 f(t)dt = 2$ and $\int_0^3 f(t)dt = -4$. Give

$$\int_1^3 3f(t)dt.$$

2. $\int_a^b \alpha f(x)dx = \alpha \int_a^b f(x)dx,$ α is a real number.

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

$$\int_0^3 f(t)dt = \int_0^1 f(t)dt + \int_1^3 f(t)dt$$

$$-4 = 2 + \int_1^3 f(t)dt$$

$$\Rightarrow \int_1^3 f(t)dt = -6$$

$$\therefore \int_1^3 3f(t)dt = 3\int_1^3 f(t)dt = 3(-6) = -18$$

Recall: The first part of the **Fundamental Theorem of Calculus**

Theorem: If f is a continuous function on the interval $[a,b]$, and F is an anti derivative of f, then

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

New: The second part of the **Fundamental Theorem of Calculus**

Theorem : If f is a continuous function, then → on $[a,b]$

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

why? Spse there is an anti-deriv. for $f(t)$. Call it $F(t)$. Then

$$\int_a^x f(t)dt = F(t)\Big|_a^x = F(x) - F(a)$$

$$\therefore \quad \frac{d}{dx}\int_a^x f(t)dt = \frac{d}{dx}\left[F(x) - F(a)\right] \quad \text{number}$$

$$= F'(x) = f(x)$$

Examples: $\frac{d}{dx}\int_0^x \sin(t)dt = \sin(x)$

$\boxed{\frac{d}{dx}\int_a^x f(t)dt = f(x)}$

Note: $\int_0^x \sin(t)dt = -\cos(t)\Big|_0^x = -\cos(x)+1$

$\frac{d}{dx}(-\cos(x)+1) = \sin(x).$

$$\frac{d}{dx}\int_0^x \frac{1}{\sin(t)+2}dt = \frac{1}{\sin(x)+2}$$

$$\frac{d}{dx}\int_0^{x^2} \sin(t)dt = \frac{d}{du}\int_0^u \sin(t)dt \cdot \frac{du}{dx}$$

chain rule

$$= \sin(u)\cdot\frac{du}{dx}$$

$$= \sin(x^2)\cdot 2x = 2x\sin(x^2)$$

ex.

$$\frac{d}{dx}\int_{?}^{3}\frac{1}{t^4+6}\,dt \quad \bigcirc\!\!\!X$$

$$\frac{d}{dx}\int_a^{u}f(t)\,dt = f(x)$$

4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$$= \frac{d}{dx}\left[-\int_3^{x}\frac{1}{t^4+6}\,dt\right]$$

$$= -\frac{d}{dx}\int_3^{x}\frac{1}{t^4+6}\,dt$$

$$= -\frac{1}{x^4+6}.$$

The final part of the **Fundamental Theorem of Calculus**

Theorem : If f is a continuous function and u is a differentiable function, then

$$\frac{d}{dx}\int_a^{u(x)}f(t)\,dt = f\big(u(x)\big)u'(x)$$

$$\frac{d}{du}\int_a^{u}f(t)\,dt \cdot \frac{du}{dx} = f(u)\frac{du}{dx}$$

chain rule

Nov 7-6:38 AM

Examples: $\frac{d}{dx}\int_0^{2x}\sin^2(t)\,dt = \sin^2(2x)\cdot 2$

$$= 2\sin^2(2x)$$

$$\frac{d}{dx}\int_{3x}^{0}\frac{1}{\sin(t)+2}\,dt = -\frac{d}{dx}\int_0^{3x}\frac{1}{\sin(t)+2}\,dt$$

$$\frac{d}{dx}\int_a^{u(x)}f(t)\,dt = f(u(x))u'(x)$$

$$= -\frac{1}{\sin(3x)+2}\cdot 3$$

$$\frac{d}{dx}\int_{2x}^{x^2}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt = \qquad \bigcirc\!\!\! x>0$$

$$= \frac{d}{dx}\left[\int_{2x}^{1}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt + \int_1^{x^2}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt\right]$$

$$= -\frac{d}{dx}\int_1^{2x}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt + \frac{d}{dx}\int_1^{x^2}\frac{1}{\sqrt{t}+\cos(t)+3}\,dt$$

$$= -\frac{1}{\sqrt{2x}+\cos(2x)+3}\cdot 2 + \frac{1}{\sqrt{x^2}+\cos(x^2)+3}\cdot 2x$$

Example: Find a function f so that $2x+\cos(3x) = \int_x^{1}f(t)\,dt$

$$2x+\cos(3x) = \int_x^{1}f(t)\,dt$$

$$\frac{d}{dx}\big(2x+\cos(3x)\big) = \frac{d}{dx}\int_x^{1}f(t)\,dt$$

$$2 - 3\sin(3x) = -\frac{d}{dx}\int_1^{x}f(t)\,dt$$

$$2 - 3\sin(3x) = -f(x)$$

$$\therefore\ f(x) = -2 + 3\sin(3x).$$