

## **Info...**

1. Test 3 is graded. **See the lecture notes from Wednesday for grade information and extra credit information.**
2. Your lowest test score will be replaced by your percentage grade on the final exam (provided it is higher).

**Sy Liebergot, Former NASA Flight Controller**

“Apollo13: The Longest Hour”

**Tuesday, November 13  
7 – 8 pm in SEC 100**

# “Apollo 13: The Longest Hour”

Sy Liebergot, Former NASA Flight Controller



**Tuesday, November 13, 2012**  
**7 – 8 p.m.**

**Science and Engineering Classroom Building (SEC), Room 100**  
**(Building 529 on the UH Campus Map)**  
**University of Houston**

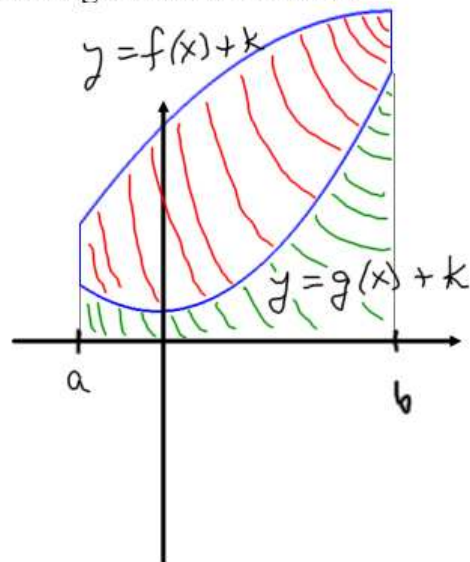
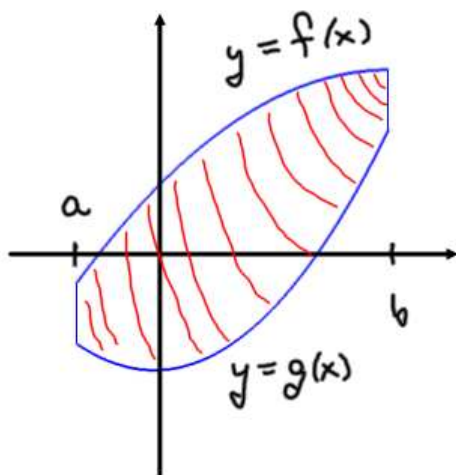
Hear what it was like to be a front-line Flight Controller in NASA’s Mission Control when a monster failure occurred during the Apollo 13 mission to the moon. Sy Liebergot will share his reactions, NASA footage, and details of the explosion and the heroic efforts to bring the crew back safely to Earth. He’ll also discuss the Apollo 13 movie’s accuracy and how he met Tom Hanks and Ron Howard.

- Sponsored by the Houston-Louis Stokes Alliance for Minority Participation and the College of Natural Sciences and Mathematics



**UNIVERSITY of HOUSTON**  
COLLEGE of NATURAL SCIENCES & MATHEMATICS

**Exploration:** How do we find the area of the region shown below?



$$f(x) \geq g(x) \text{ on } [a, b]$$

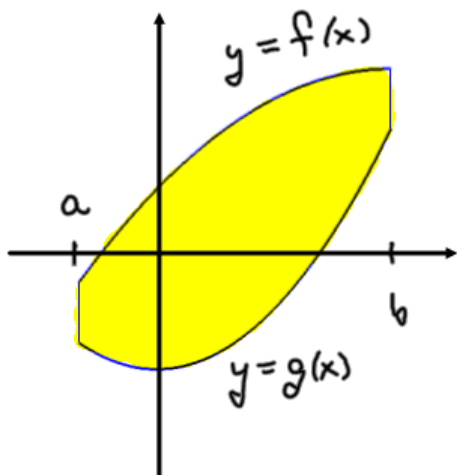
$$\text{Area}(\text{red}) = \text{Area}(\text{red}) + \text{Area}(\text{green}) - \text{Area}(\text{green})$$

A Cartesian coordinate system showing the two curves  $y=f(x)$  and  $y=g(x)$  on the interval  $[a, b]$ . The region between them is shaded with red diagonal lines. The x-axis is marked with  $a$  and  $b$ .

$$\begin{aligned}
 &= \int_a^b (f(x)+k) dx - \int_a^b (g(x)+k) dx \\
 &= \int_a^b [(f(x)+k) - (g(x)+k)] dx \\
 &= \int_a^b (f(x) - g(x)) dx
 \end{aligned}$$

The Riemann integral of top minus bottom.

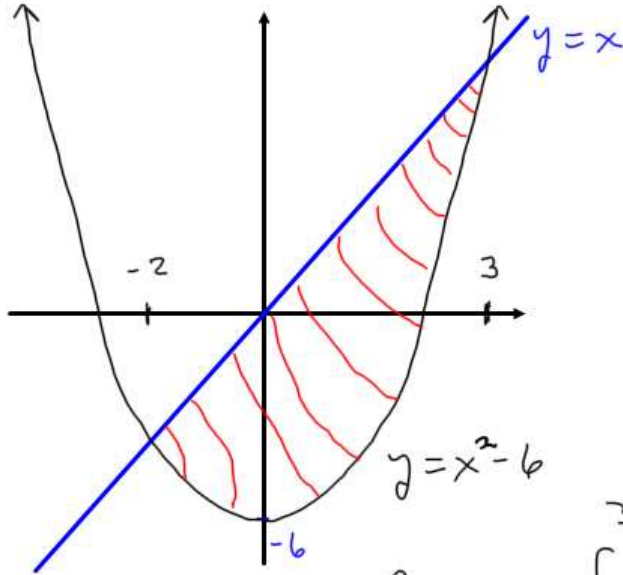
**Area Formula:** If  $f(x)$  and  $g(x)$  are continuous functions on the interval  $[a,b]$ , and  $f(x) \geq g(x)$  for all  $a \leq x \leq b$ , then the area bounded between the graphs of  $f(x)$  and  $g(x)$  on the interval  $[a,b]$  is given by



$$\int_a^b (f(x) - g(x)) dx$$

$$\int_a^b (\text{Top} - \text{Bottom}) dx$$

**Example:** Find the area bounded by the graphs of  $y=x$  and  $y=x^2-6$ .



$$x^2 - 6 = x$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = -2, x = 3$$

$$\text{Area} = \int_{-2}^3 (\text{Top} - \text{Bottom}) dx$$

$$= \int_{-2}^3 (x - (x^2 - 6)) dx$$

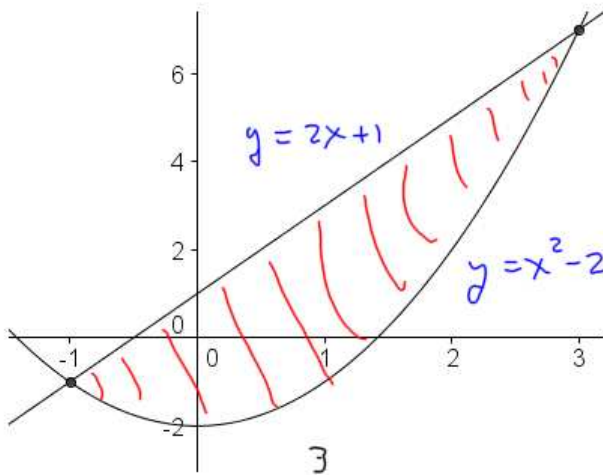
$$= \int_{-2}^3 (x - x^2 + 6) dx$$

$$= \left( \frac{1}{2}x^2 - \frac{1}{3}x^3 + 6x \right) \Big|_{-2}^3$$

$$= \left( \frac{9}{2} - 9 + 18 \right) - \left( 2 + \frac{8}{3} - 12 \right)$$

$$= 19 + \frac{9}{2} - \frac{8}{3} = 19 + \frac{11}{6} = 20.8\overline{33}$$

**Example:** Find the area bounded by the graphs of  $y = 2x + 1$  and  $y = x^2 - 2$ .



$$x^2 - 2 = 2x + 1$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = -1, x = 3$$

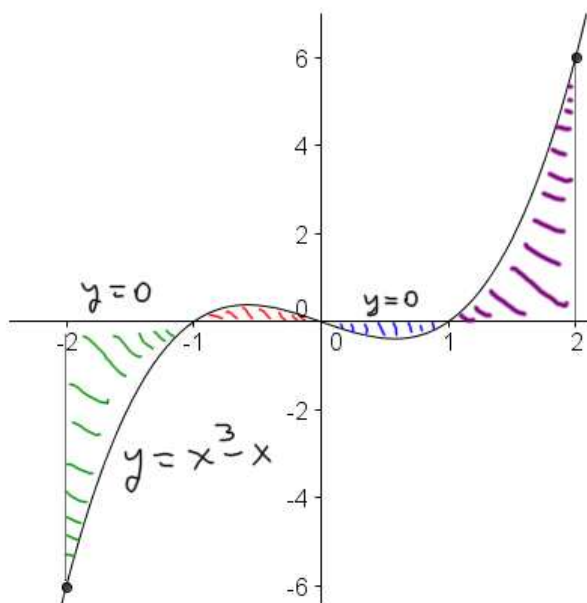
$$\text{Area} = \int_{-1}^3 (\text{Top} - \text{Bottom}) dx = \int_{-1}^3 (2x + 1 - (x^2 - 2)) dx$$

$$= \int_{-1}^3 (2x - x^2 + 3) dx$$

= ... you finish this.



**Example:** Give a formula in terms of integrals for the area between the  $x$ -axis and the graph of  $y = x^3 - x$  for  $x$  between  $-2$  and  $2$ .

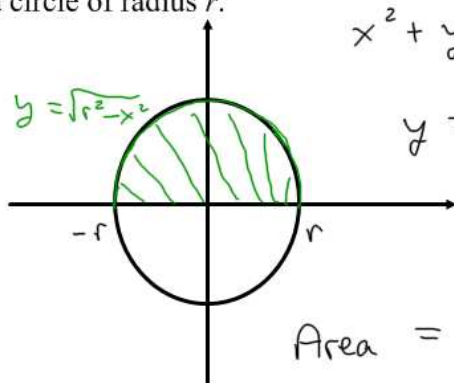


$$\text{Area} = \text{Area}(\text{green}) + \text{Area}(\text{red}) + \text{Area}(\text{blue}) + \text{Area}(\text{purple})$$

$$= \int_{-2}^{-1} (0 - (x^3 - x)) dx + \int_{-1}^0 (x^3 - x) dx + \int_0^1 (0 - (x^3 - x)) dx + \int_1^2 (x^3 - x) dx$$

$$= \int_{-2}^{-1} (-x^3 + x) dx + \int_{-1}^0 (x^3 - x) dx + \int_0^1 (-x^3 + x) dx + \int_1^2 (x^3 - x) dx$$

**Example:** Use an integral to derive the formula for the area of a circle of radius  $r$ .



$$x^2 + y^2 = r^2$$

$$y = \pm \sqrt{r^2 - x^2}$$

$$\text{Area} = 2 \text{ Area}(\text{shaded})$$

$r > 0$

$$= 2 \int_{-r}^r \sqrt{r^2 - x^2} \, dx$$

$$= 2r \int_{-r}^r \sqrt{1 - \left(\frac{x}{r}\right)^2} \, dx$$

$$\frac{x}{r} = \sin(\theta)$$

↙ new variable

$$= 2r \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2(\theta)} \, r \cos(\theta) \, d\theta$$

$$x = r \sin(\theta) \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$dx = r \cos(\theta) \, d\theta$$

$$= 2r \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r \cos^2(\theta) \, d\theta$$

$$\cos^2(\theta) = \frac{1}{2} + \frac{1}{2} \cos(2\theta)$$

$$= 2r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left( \frac{1}{2} + \frac{1}{2} \cos(2\theta) \right) \, d\theta$$

$$= 2r^2 \left( \frac{1}{2} \theta + \frac{1}{4} \sin(2\theta) \right) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= 2r^2 \left[ \left( \frac{\pi}{4} + 0 \right) - \left( -\frac{\pi}{4} + 0 \right) \right] = 2r^2 \cdot \frac{\pi}{2}$$

$$= \underline{\underline{\pi r^2}}$$