Info

- Homework and EMCFs are posted.
- The quiz in lab/workshop on Friday will cover area and u-substitution.
- Visit the Alpha Lambda Delta Bake Sale in the PGH Breezeway starting at 10am.

Review

$u = 3x^2+2$
$du = 6x\,dx$

Example: $\displaystyle\int_{-1}^{2} \frac{x}{\sqrt{3x^2+2}}\,dx = \frac{1}{6}\int_{-1}^{2} \underbrace{(3x^2+2)}_{u}^{-\frac{1}{2}} \cdot \underbrace{6x\,dx}_{du?}$

Chain rule: $\displaystyle \frac{d}{dx}F(u) = \boxed{F'(u)\frac{du}{dx}}$

Expression dependent upon u

$u = 3x^2+2$
$x = 2 \Rightarrow u = 14$
$x = -1 \Rightarrow u = 5$

$\displaystyle = \frac{1}{6}\int_{5}^{14} u^{-\frac{1}{2}}\,du$

$\displaystyle = \frac{1}{6}\cdot 2u^{1/2}\Big|_{5}^{14}$

$\displaystyle = \frac{1}{3}\left(\sqrt{14}-\sqrt{5}\right).$

P28

1. $\displaystyle\int_{0}^{\pi/3} \sin(3x)\,dx =$

Example: $\displaystyle\int_{0}^{\pi/4} \sin(2x)\underbrace{\left(\cos(2x)+2\right)}_{u?}^{3}\,dx =$

$\displaystyle = -\frac{1}{2}\int_{0}^{\pi/4} \underbrace{(\cos(2x)+2)}_{u}^{3} \cdot \underbrace{(-2)\sin(2x)\,dx}_{du?}$

$u = \cos(2x)+2$
$du = -2\sin(2x)\,dx$
$x = \pi/4 \Rightarrow u = 2$
$x = 0 \Rightarrow u = 3$

$\displaystyle = -\frac{1}{2}\int_{3}^{2} u^3\,du = -\frac{1}{2}\cdot\frac{1}{4}u^4\Big|_{3}^{2}$

$\displaystyle = -\frac{1}{8}(16-81)$

$= 65/8.$

P28

2. $\int_0^1 \frac{1}{\sqrt{3x+1}}\,dx =$

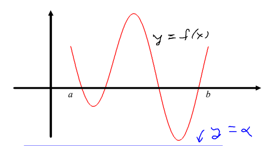
Example: Suppose f is a continuous function and $\int_{-1}^{3} f(x)\,dx = 5$ and $\int_{-1}^{2} f(x)\,dx = 6$. Give the value for $\int_{2}^{3} f(x)\,dx$.

Example in the video

Average Value of a Function

Last topic in Chapter 5.

Question: The graph of f is shown below. Is there a value of y so that $y(b - a)$ is smaller than the net area bounded by the graph of f over the interval $[a,b]$? $= \int_a^b f(x)\,dx$



Value of $y \equiv \alpha$

Need α so that

$\alpha(b-a) < \int_a^b f(x)\,dx$

$\alpha \equiv$ constant

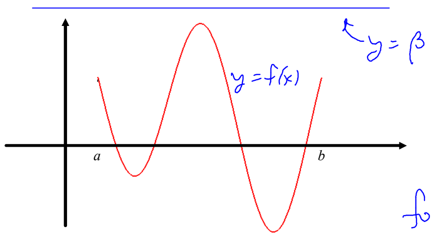
$\int_a^b \alpha\,dx = \alpha x \Big|_a^b = \alpha(b-a)$

$\int_a^b \alpha\,dx < \int_a^b f(x)\,dx$

If $g(x) < f(x)$ then $\int_a^b g(x)\,dx < \int_a^b f(x)\,dx$

Yes. Just pick $\alpha < f(x)$ for all $a \le x \le b$.

Question: The graph of *f* is shown below. Is there a value of *y* so that *y*(*b* - *a*) is larger than the net area bounded by the graph of *f* over the interval [*a*,*b*]?



Yes

Just pick

$\beta > f(x)$

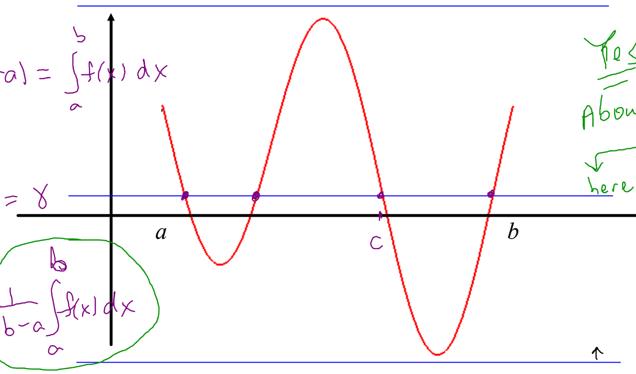
for $a \leq x \leq b$.

Question: The graph of *f* is shown below. Is there a value of *y* so that *y*(*b* - *a*) is equal to the net area bounded by the graph of *f* over the interval [*a*,*b*]?

$\gamma(b-a) = \int_a^b f(x)\,dx$

$f(c) = \gamma$

$\gamma = \frac{1}{b-a}\int_a^b f(x)\,dx$



Yes

About here

Theorem: (The mean value theorem for integrals.) Suppose f is a continuous function on the interval $[a,b]$. Then there is a value c so that $a < c < b$, and

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

The average value of f(x) on [a,b].

Note: The value $f(c)$ is called the average value (or mean value) of the function f over the interval $[a,b]$.

pf: Define $F(x) = \int_a^x f(t)\,dt$

The MVThm for derivatives says there is a value c so that $a < c < b$ and

$$F'(c) = \frac{F(b) - F(a)}{b-a}$$

Note: Fund theorem of calculus

$\Rightarrow F'(x) = f(x)$.

So, $f(c) = \frac{1}{b-a}\left[\int_a^b f(t)\,dt - \overset{0}{\overset{\parallel}{\int_a^a f(t)\,dt}}\right]$

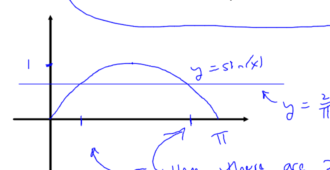
i.e. $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$. #

Example: Give the average value of $f(x) = \sin(x)$ on the interval $[0,\pi]$.

$\boxed{\text{Average Value of } f(x) = \sin(x) \text{ on } [0,\pi]} = \frac{1}{\pi - 0}\int_0^\pi \sin(x)\,dx$

$= \frac{1}{\pi}\left(-\cos(x)\right)\Big|_0^\pi$

$= \frac{1}{\pi}\left(1 - -1\right) = \frac{2}{\pi}$



Here, there are 2 places where f(x) takes on the average value.

Example: Give the average value of $f(x) = 3x^2 - x$ on the interval $[-1,3]$, and determine the value c where f has this value on the interval $(-1,3)$.

$$= \frac{1}{3--1}\int_{-1}^{3}(3x^2-x)\,dx = \cdots = 6$$

Find c so that $\boxed{-1 < c < 3}$

and $3c^2 - c = 6$

$$3c^2 - c - 6 = 0$$

$$c = \frac{1 \pm \sqrt{1+72}}{6} = \frac{1 \pm \sqrt{73}}{6}$$

$$c = \cancel{\frac{1-\sqrt{73}}{6}} \quad \text{or} \quad \boxed{c = \frac{1+\sqrt{73}}{6}}$$

Note: $-1 < \frac{1+\sqrt{73}}{6} < 3$.