**Today: Section 6.1**

- **Test 4:** **Dec. 6 - 8**
- **Final Exam:** **Dec. 17 - 19**
- Dates are subject to ~~slight~~ modification... note the change
- **Homework** and an **EMCF** are Due on Monday.
- An **EMCF** is due on Wednesday (even though we do not have class).
- **Homework** and an **EMCF** are due on the Monday following the break.

short

**Happy Birthday Christine!!**

**Review**

**Theorem:** (The mean value theorem for integrals.) Suppose $f$ is a continuous function on the interval $[a,b]$. Then there is a value $c$ so that $a < c < b$, and

$$f(c) = \underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}_{\text{the average value of } f \text{ on } [a,b]}$$

**Free Friday!!**
**Popper P29**

1. $3+4=$

**Review Example:** Give the average value of the function $f(x)=x^2$ on the interval $[-1,2]$, and determine the number of values where $f$ achieves this average value on this interval.

$$= \frac{1}{2--1}\int_{-1}^{2} x^2\,dx = \frac{1}{3}\cdot\frac{1}{3}x^3\Big|_{-1}^{2}$$

$$= \frac{1}{9}(8 - -1) = \boxed{1}$$

Solve $x^2 = 1$ for $-1 \le x \le 2$.

$x = \pm 1$ ← both are in this interval.

2

**Review Example:** Suppose you know $\int_0^2 (f(x)-2x)\,dx = 3$. Give the average value of $f$ on the interval $[0,2]$.

**Free Friday!!**
**Popper P29**

2. $2+2=$

$$= \frac{1}{2-0}\underbrace{\int_0^2 f(x)\,dx}_{?} = \frac{1}{2}\cdot 7 = \boxed{\frac{7}{2}}$$

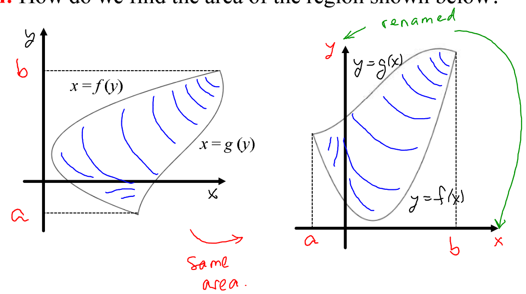
$$\int_0^2 (f(x)-2x)\,dx = 3$$

$$\int_0^2 f(x)\,dx - \int_0^2 2x\,dx = 3$$

$$\int_0^2 f(x)\,dx = x^2\Big|_0^2 + 3 = 4-0+3 = \boxed{7}$$

New

**Question:** How do we find the area of the region shown below?




$$\text{Area} = \int_a^b \left(g(x) - f(x)\right)dx = \int_a^b \left(g(y) - f(y)\right)dy$$

dummy variable      right      left

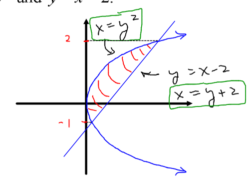
**Example:** Give the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$.

**Free Friday!! Popper P29**

3. $3 + 3 =$

**Free Friday!! Popper P29**

4. $2 + 5 =$




Solve

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1$ and $y = 2$

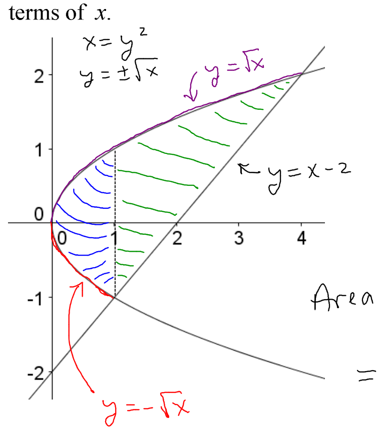
$$\therefore \text{Area} = \int_{-1}^{2} (\text{right} - \text{left})\,dy = \int_{-1}^{2} \left((y+2) - y^2\right)dy$$

$$= \left(\tfrac{1}{2}y^2 + 2y - \tfrac{1}{3}y^3\right)\Big|_{-1}^{2}$$

$$= \left(2 + 4 - \tfrac{8}{3}\right) - \left(\tfrac{1}{2} - 2 + \tfrac{1}{3}\right)$$

$$= 8 - 3 - \tfrac{1}{2} = \boxed{\tfrac{9}{2}}$$

**Example:** Rewrite the solution to the previous area problem using integrals in terms of $x$.




$x = y^2$ and $y = -1$

$\Rightarrow x = 1$

$x = y^2$ and $y = 2$

$\Rightarrow x = 4$

$$\text{Area} = \text{Area}(\ ) + \text{Area}(\ )$$

$$= \int_0^1 \left(\sqrt{x} - -\sqrt{x}\right)dx + \int_1^4 \left(\sqrt{x} - (x-2)\right)dx$$