Today: Section 6.1

- **Test 4: Dec. 6 - 8**
- **Final Exam: Dec. 17 - 19**
- Dates are subject to slight modification...
- **Homework** and an **EMCF** are Due on Monday.
- An **EMCF** is due on Wednesday (even though we do not have class).
- **Homework** and an **EMCF** are due on the Monday following the break.

Review

Theorem: (The mean value theorem for integrals.) Suppose f is a continuous function on the interval $[a,b]$. Then there is a value c so that $a < c < b$, and

$$f(c) = \underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}_{\text{the average value of } f \text{ on the interval } [a,b]}$$

Review Example: Give the average value of the function $f(x) = x^2$ on the interval $[-1,2]$, and determine the number of values where f achieves this average value on this interval.

$$= \frac{1}{2-{-1}}\int_{-1}^{2} x^2\,dx = \frac{1}{3}\cdot\frac{1}{3}x^3\Big|_{-1}^{2}$$

$$= \frac{1}{9}\left[8 - {-1}\right] = 1$$

Solve $x^2 = 1$

$x = \pm 1$.

Both are in the interval $[-1,2]$.

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Review Example: Suppose you know $\boxed{\int_0^2 (f(x) - 2x)\,dx = 3}$. Give the average value of f on the interval $[0,2]$.

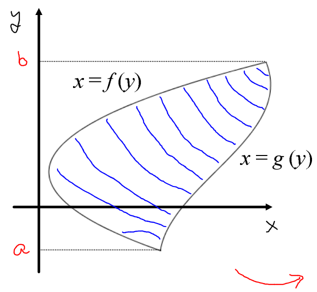
$$= \frac{1}{2-0}\int_0^2 f(x)\,dx = \frac{1}{2}\int_0^2 f(x)\,dx = \boxed{\frac{7}{2}}$$

$$\int_0^2 (f(x) - 2x)\,dx = 3 \Rightarrow \int_0^2 f(x)\,dx - \int_0^2 2x\,dx = 3$$

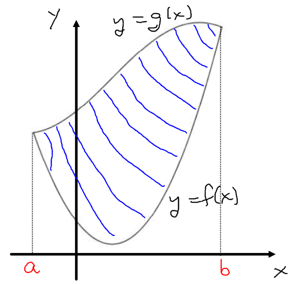
$$\therefore \int_0^2 f(x)\,dx = x^2\Big|_0^2 + 3 = 4 - 0 + 3 = 7$$

New 6.1

Question: How do we find the area of the region shown below?



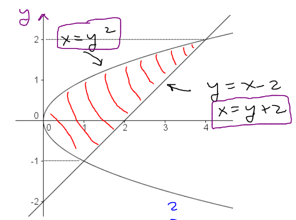
Same area



$$\text{Area} = \int_a^b \left(g(x) - f(x)\right)dx = \int_a^b \left(g(y) - f(y)\right)dy$$

Left Right

Example: Give the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$.



$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1$ or $y = 2$

$$\text{Area} = \int_{-1}^{2}\left((y+2) - (y^2)\right)dy$$

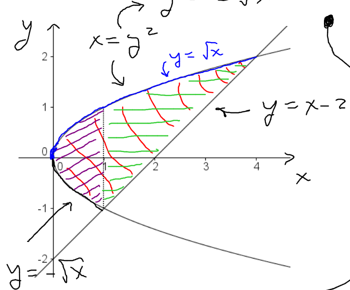
$$= \left(\tfrac{1}{2}y^2 + 2y - \tfrac{1}{3}y^3\right)\Big|_{-1}^{2}$$

$$= \left(2 + 4 - \tfrac{8}{3}\right) - \left(\tfrac{1}{2} - 2 + \tfrac{1}{3}\right)$$

$$= 8 - 3 - \tfrac{1}{2} = \tfrac{9}{2}.$$

Example: Rewrite the solution to the previous area problem using integrals in terms of x.

$y = \pm\sqrt{x}$



Area = Area (purple) + Area (green)

Note: $x = y^2$ and $y = -1 \Rightarrow x = 1$

$x = y^2$ and $y = 2 \Rightarrow x = 4$

$$\text{Area} = \int_0^1 \left(\sqrt{x} - -\sqrt{x}\right)dx + \int_1^4 \left(\sqrt{x} - (x-2)\right)dx$$