Today: Section 6.2

- **Test 4:** **Dec. 6 - 8**
- **Final Exam:** **Dec. 17 - 19**
- Dates are subject to slight modification...
- An **EMCF** is due on Wednesday (even though we do not have class).
- **Homework** and an **EMCF** are due on the Monday following the break.

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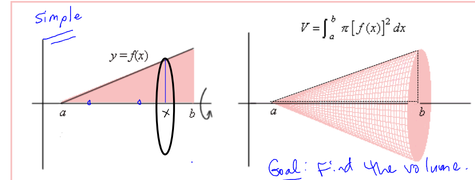
1. Give the area bounded between the x-axis and the graph of $f(x) = x^2 - 1$ on the interval [-1,2].

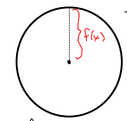
Revolving A Region Around a Horizontal Axis

Simple



We can fill the entire region above with vertical line segments. I have drawn one at x. If we assume the bottom portion of the segment is attached to the x axis and spin the segment around the x axis, then it generates a circular disk.

Circular Disk:



Thickness: dx

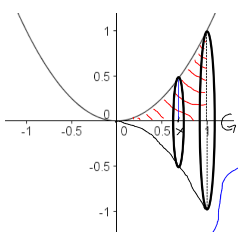
Face Area: $\pi (f(x))^2$

Infinitesimal Volume $= \pi (f(x))^2 \, dx$

Full Volume $= \int_a^b \pi (f(x))^2 \, dx$

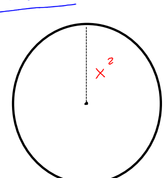
(sum the Disks)

Example: The region bounded between the x-axis and the graph of $y = x^2$, over the interval [0,1], is rotated around the x axis. Give the volume that is generated.



Fill up the region between the x-axis and the graph of $y = x^2$ from 0 to 1 with vertical line segments. Rotate each of these to fill the volume.

Circular disk:



Thickness: dx

Face Area: $\pi (x^2)^2$

Inf. Volume $= \pi (x^2)^2 \, dx$

Full volume $= \int_0^1 \pi x^4 \, dx = \frac{\pi}{5} x^5 = \frac{\pi}{5}$.

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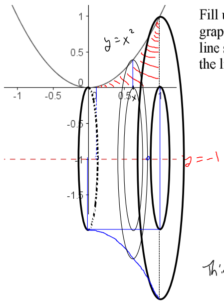
Good Notes

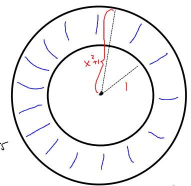
2. What is the answer to the previous example if the graph involved in the rotation is $y = x^3$?

Example: The region bounded between the x axis and the graph of $y = x^2$, over the interval [0,1], is rotated around the line $y = -1$. Give the volume that is generated.



Fill up the region between the x-axis and the graph of $y = x^2$ from 0 to 1 with vertical line segments. Rotate each of these around the line $y = -1$ to fill the volume.

Circular washer



Thickness dx

Face Area $= \pi(x^2+1)^2 - \pi \cdot 1^2$

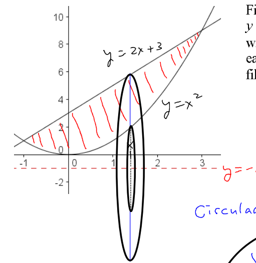
Inf. Volume $= \left(\pi(x^2+1)^2 - \pi\right)dx$

Full Volume $= \int_0^1 \left(\pi(x^2+1)^2 - \pi\right)dx$

multiply this out

$= \ldots$

Example: The region bounded between the graphs of $y = x^2$ and $y = 2x + 3$ is rotated around the line $y = -1$. Give the volume that is generated.



Fill up the region between the graph of $y = x^2$ and $y = 2x + 3$ from -1 to 3 with vertical line segments. Rotate each of these around the line $y = -1$ to fill the volume.

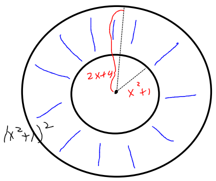
$x^2 = 2x+3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, \ x = 3$

Circular washer



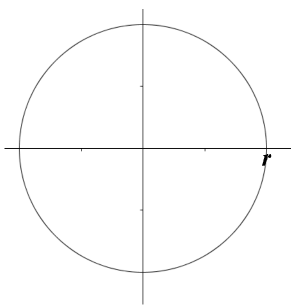
Thickness dx

Face Area $= \pi(2x+4)^2 - \pi(x^2+1)^2$

Inf. Volume $= \left(\pi(2x+4)^2 - \pi(x^2+1)^2\right)dx$

Full Volume $= \int_{-1}^{3} \left(\pi(2x+4)^2 - \pi(x^2+1)^2\right)dx$

Example: How do we derive the formula for the volume of a sphere?



$x^2 + y^2 = r^2$

See the video!!