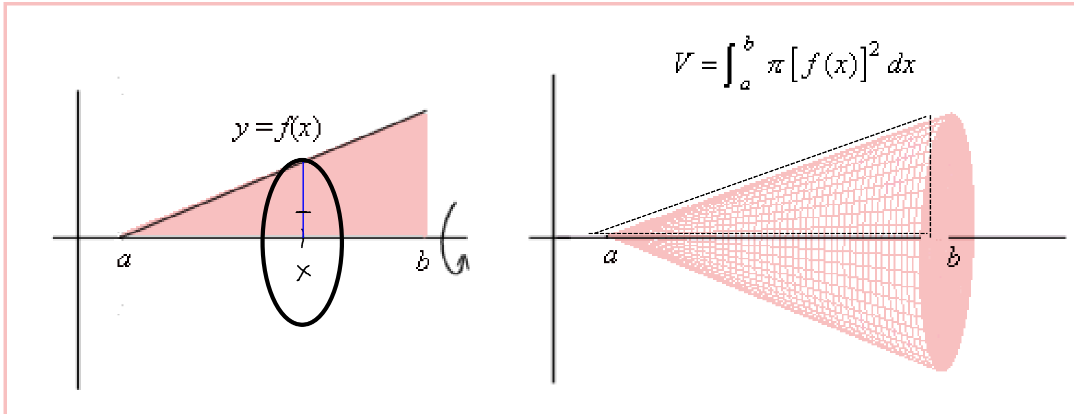
Today: Section 6.2

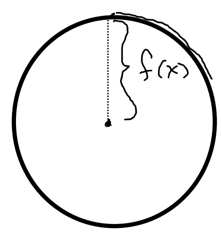
- **Test 4: Dec. 6 - 8**
- **Final Exam: Dec. 17 - 19**
- Dates are subject to slight modification...
- An **EMCF** is due on Wednesday (even though we do not have class).
- **Homework** and an **EMCF** are due on the Monday following the break.

Revolving A Region Around a Horizontal Axis



* We can fill the entire region above with vertical line segments. I have drawn one at x. If we assume the bottom portion of the segment is attached to the x axis and spin the segment around the x axis, then it generates a circular disk.

Rotated line segment: Circular disk.



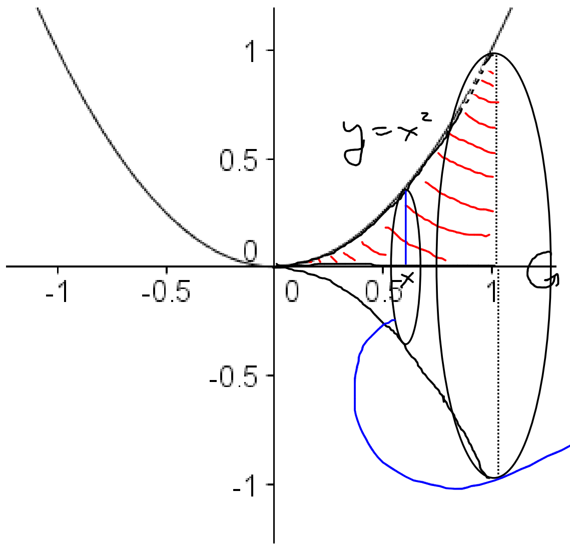
dx thickness

Face area: $\pi f(x)^2$

Infinitesimal volume of a circular disk $= \pi f(x)^2 \, dx$

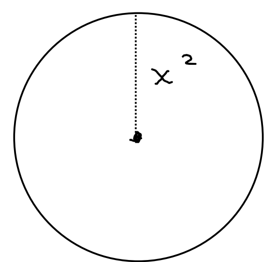
$\therefore$ Full Volume $= \int_a^b \pi f(x)^2 \, dx$

Example: The region bounded between the x-axis and the graph of $y = x^2$, over the interval [0,1], is rotated around the x axis. Give the volume that is generated.



Imagine that we fill the entire region with vertical line segments from the x-axis to the graph of $y = x^2$. Then we rotate each of these around the x-axis. Collectively, these will fill our volume.

circular disk.



Thickness

dx

Face Area

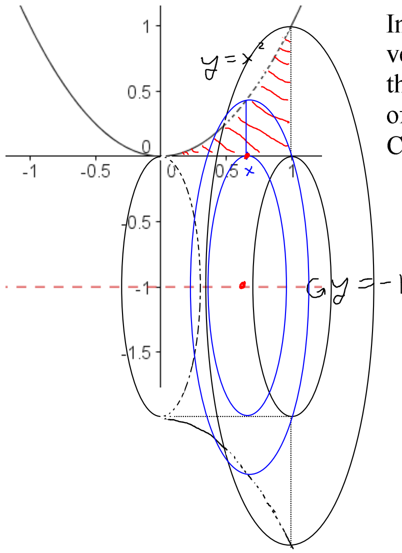
$\pi (x^2)^2$

Infi. Volume $= \pi (x^2)^2 \, dx$

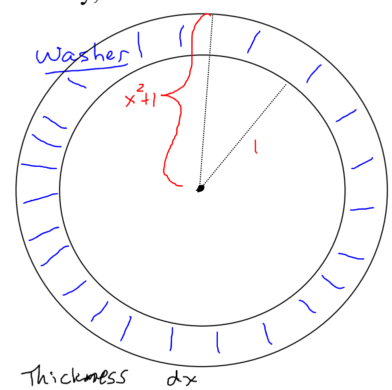
Full Volume $= \displaystyle\int_0^1 \pi x^4 \, dx = \frac{\pi}{5} x^5 \Big|_0^1$

$= \dfrac{\pi}{5}$.

Example: The region bounded between the x axis and the graph of $y = x^2$, over the interval [0,1], is rotated around the line $y = -1$. Give the volume that is generated.



Imagine that we fill the entire region with vertical line segments from the x-axis to the graph of $y = x^2$. Then we rotate each of these around the line $y = -1$. Collectively, these will fill our volume.



$$\text{Face Area} = \pi(x^2+1)^2 - \pi \cdot 1^2$$

$$= \pi(x^2+1)^2 - \pi$$

$$\text{Inf. volume} = \left(\pi(x^2+1)^2 - \pi\right)dx$$

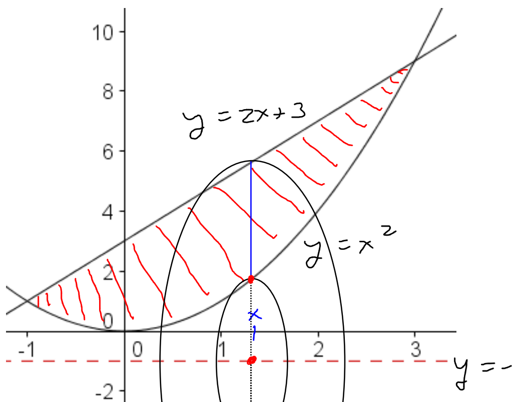
$$\text{Full volume} = \int_0^1 \left(\pi(x^2+1)^2 - \pi\right)dx$$

$$= \pi\int_0^1 (x^4 + 2x^2 + 1 - 1)\,dx$$

$$= \pi\left(\frac{1}{5}x^5 + \frac{2}{3}x^3\right)\Big|_0^1$$

$$= \pi\left(\left(\frac{1}{5} + \frac{2}{3}\right) - 0\right) = \frac{13\pi}{15}.$$

Example: The region bounded between the graphs of $y = x^2$ and $y = 2x + 3$ is rotated around the line $y = -1$. Give the volume that is generated.



$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

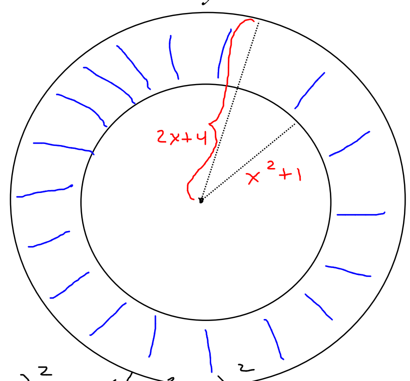
$(x-3)(x+1) = 0$

$x = -1, \quad x = 3.$

Imagine that we fill the entire region with vertical line segments from the graph of $y = x^2$ to the graph of $y = 2x + 3$. Then we rotate each of these around the line $y = -1$. Collectively, these will fill our volume.

washer

(a circular disk with a hole punched out of the middle)



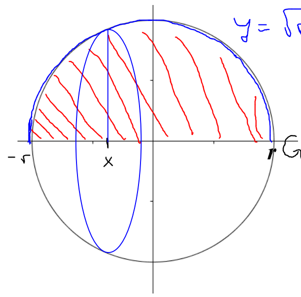
Thickness dx

Face Area $= \pi(2x+4)^2 - \pi(x^2+1)^2$

Inf. Volume $= \left(\pi(2x+4)^2 - \pi(x^2+1)^2\right)dx$

Full Volume $= \displaystyle\int_{-1}^{3} \left(\pi(2x+4)^2 - \pi(x^2+1)^2\right)dx$

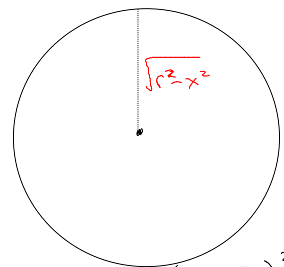
Example: How do we derive the formula for the volume of a sphere?



Note: We can generate the sphere by rotating the region between the x-axis and the upper portion of the circle $x^2 + y^2 = r^2$, around the x-axis.

Imagine that we fill the entire region with vertical line segments from the x-axis to the upper portion of the circle. Then we rotate each of these around the x-axis. Collectively, these will fill our sphere.

circular disk



Thickness

dx

Face Area

$\pi\left(\sqrt{r^2-x^2}\right)^2$

$$\text{Inf. Volume} = \pi\left(\sqrt{r^2-x^2}\right)^2 dx$$

$$\text{Full Volume} = \int_{-r}^{r} \pi\left(\sqrt{r^2-x^2}\right)^2 dx$$

$$= \int_{-r}^{r} \pi(r^2 - x^2)\,dx = \pi\left(r^2x - \frac{1}{3}x^3\right)\Bigg|_{-r}^{r}$$

$$= \pi\left[\left(r^3 - \frac{1}{3}r^3\right) - \left(-r^3 + \frac{1}{3}r^3\right)\right]$$

$$= \pi\left[2r^3 - \frac{2}{3}r^3\right]$$

$$= \frac{4}{3}\pi r^3$$